The Experimental Basis of Quantum Theory

Thornton and Rex, Ch. 3

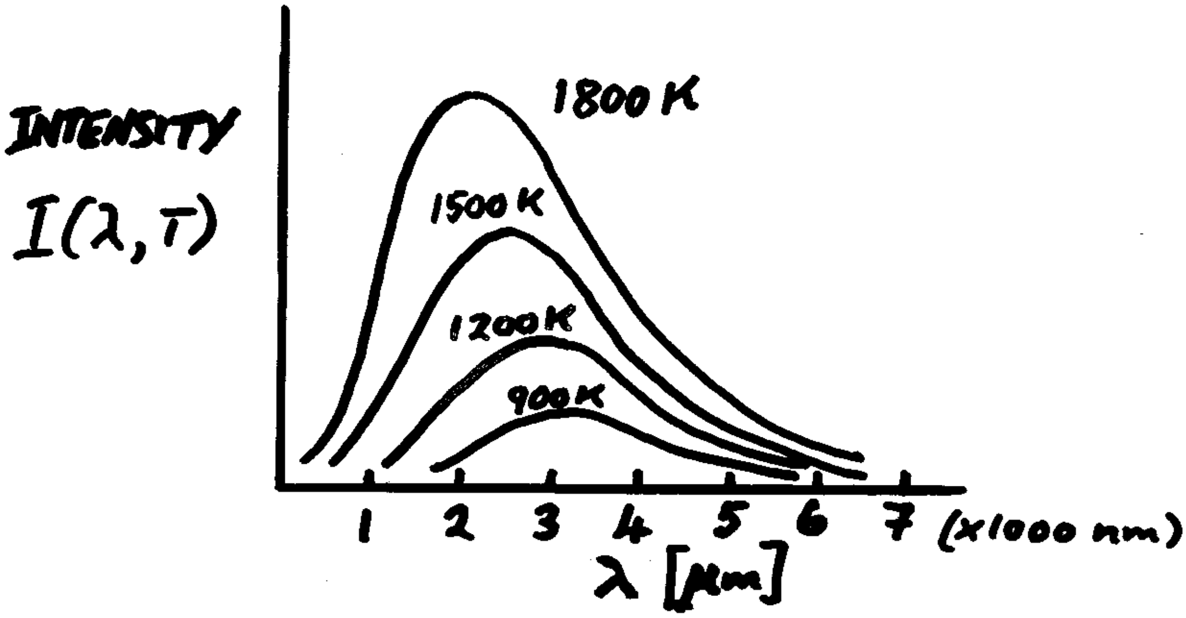
Blackbody Radiation

As an object gets hot, it radiates energy.

How does the radiation depend on temperature?

What is the distribution among frequencies (or wavelengths)?

BLACKBODY RADIATION



- MAXIMUM OF THE DISTRIBUTION SHIFTS TO SMALLER WAVELENGTHS AS THE TEMPERATURE INCREASES :-

$$\lambda_{MAX} \cdot T = 2.898 \times 10^{-3} \text{ m.K}$$

<u>WIEN'S DISPLACEMENT LAW</u>

- TOTAL POWER $= R(T)$ $$= \int_0^\infty I(\lambda, T)\, d\lambda = \varepsilon \sigma T^4$$

<u>STEFAN-BOLTZMANN LAW</u>

WITH ε = EMISSIVITY AND σ = S-B CONSTANT

$$= 5.67 \times 10^{-8} \ \mathrm{W/m^2 K^4}$$

Max Planck, prof. at University of Berlin, attempted an explanation in 1900, adding one new and strange assumption:

Each oscillation mode can not absorb just any amount of energy. Each mode can only absorb energy in packets of fixed size.

It described the data perfectly!

The size of each energy packet (now termed a quantum) is proportional to the frequency of the mode:

$$E = h\nu$$

The proportionality factor (Planck's constant) is

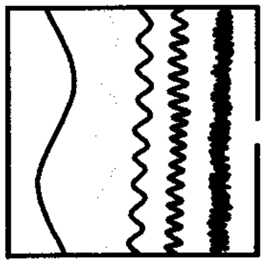
$$h = 6.63 \times 10^{-34}\ \mathrm{J\ s}.$$

The above formula was first presented at a meeting of the German Physical Society on Dec. 14, 1900.
The birthday of Quantum Physics!

Max Planck derived:

$$I(\lambda,T) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

Planck's Radiation Law



Important new assumption:

The energy in each frequency mode ν can only come in integer multiples of some fundamental unit (quanta): $E = h\,\nu$

with

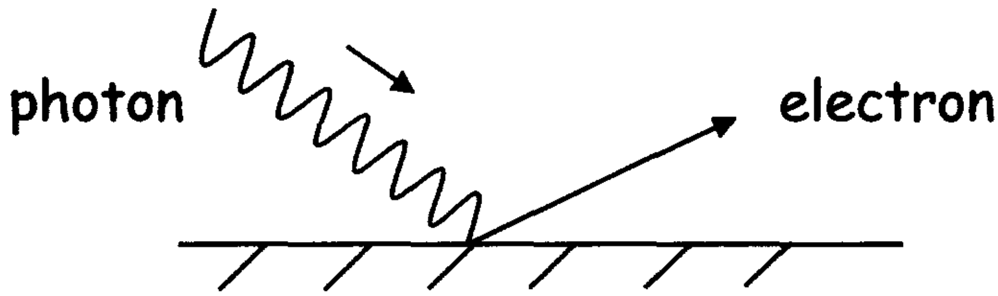
$$h = 6.6261 \times 10^{-34}\ \text{J} \cdot \text{s}$$
$$= 4.1357 \times 10^{-15}\ \text{eV} \cdot \text{s}$$

The S-B law and Wien's law both follow from Planck's formula.

The Photoelectric Effect

1887 - Hertz:

Visible or UV light on metal surface may release electrons



Classical theory says energy of electrons should increase with intensity of light.

However, this was not the case.

Experiments (Lenard) showed:

1) KE of photoelectrons depends only on ν (frequency) of light, independent of I (its intensity).

2) # of photoelectrons is proportional to I

3) For a given metal, there is a minimum ν, below which no photoelectrons are emitted.

4) The photoelectrons are emitted instantaneously, independent of I.

Classical theory could not explain these observations.

1905 - Einstein explained:

Electromagnetic radiation transferred in discrete bundles of energy ("photons"). The energy of each photon is

$$E = h\nu$$

The KE of an emitted photoelectron is

$$KE = h\nu - \phi$$

- $h\nu$ is Energy of the incident photon
- ϕ is the binding energy of electron to the metal surface (the <u>work function</u>).

The KE of the electrons can be measured by applying a positive voltage to the surface and noting when the photoelectric current stops. This voltage is called the <u>stopping potential.</u>

3.34 WHAT IS THE THRESHOLD FREQUENCY FOR PHOTOELECTRONS FROM LITHIUM ($\phi = 2.9\,eV$) ?

$$E_{PHOTON} = KE_{ELECTRONS} + \phi$$

AT THRESHOLD $KE_{ELECTRONS} = 0$

$$\therefore \quad E_{PHOTON} = \phi = 2.9\ eV$$

$$\therefore \ h\nu = 2.9\ eV$$

$$\therefore \ \nu = \frac{2.9}{h} = \frac{2.9}{4.14 \times 10^{-15}}$$

$$\therefore \ \nu = 7.00 \times 10^{14}\ Hz$$

$(\lambda = c/\nu = 429\ nm)$

WHAT IS THE STOPPING POTENTIAL IF $\lambda = 400\ nm$?

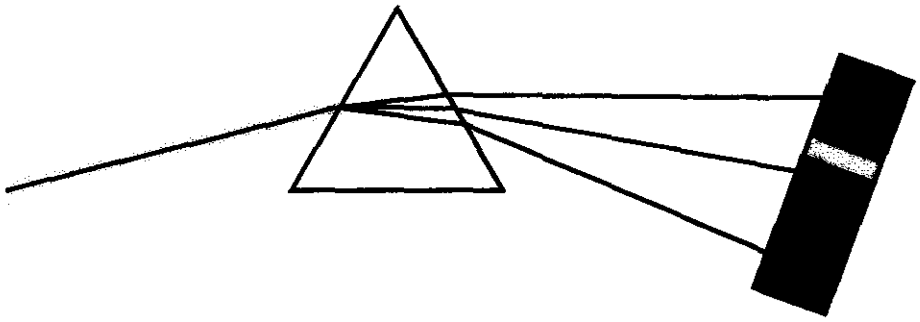
$$E_{PHOTON} = \frac{1243}{400} = 3.11\ eV$$

$$KE_{ELECTRONS} = E_{PHOTON} - \phi$$

$$= 3.11 - 2.9 = 0.21\ eV$$

SO ELECTRONS WILL BE STOPPED WITH A POTENTIAL OF $0.21\ V$.

Spectral Lines



- 1814-1824: Von Fraunhofer discovered absorption lines in sun.

- 1850's: Kirchhoff discovered characteristic emission lines of elements

- 1859: Kirchoff and Bunsen discovered new elements, Cesium and Rubidium, by first observing their spectral lines.

- 1885: Balmer found a formula for the wavelengths of the spectral lines in Hydrogen:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{4} - \frac{1}{n^2}\right)$$

where $n=3,4,5,\ldots$

The constant R_H is Rydberg's constant:

$$R_H = 1.096776 \times 10^7\ \mathrm{m}^{-1}$$

- 1890: Rydberg found a more general formula for the spectral lines of Hydrogen:.

$$\frac{1}{\lambda} = R_H \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

where both n and m are integers with $n>m$.

Visible light is in the approximate range
$400 \text{ nm} < \lambda < 700 \text{ nm}$

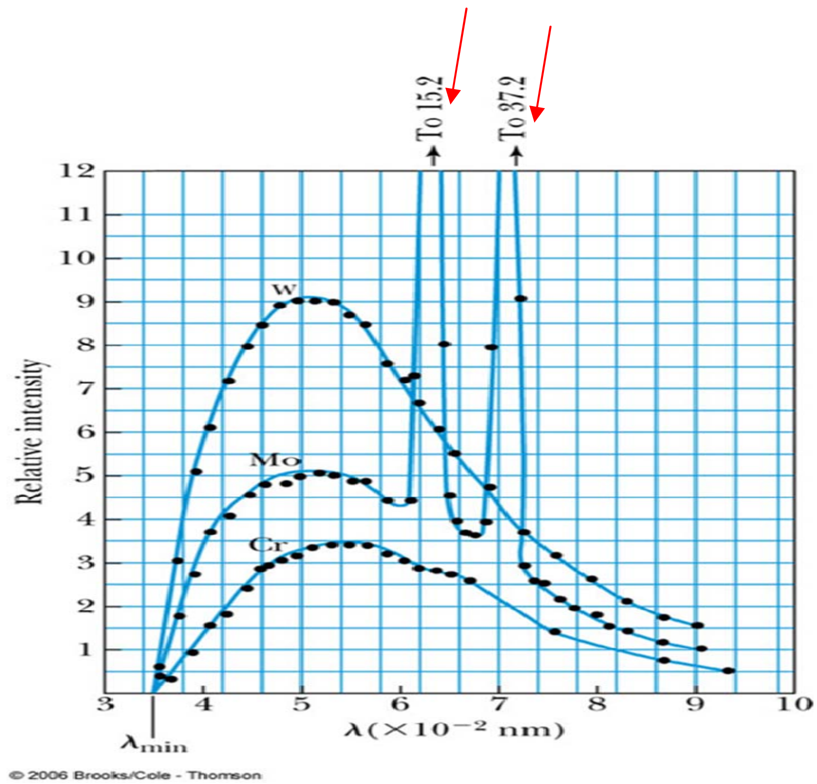
m=1: Lyman *series* => Ultraviolet (UV)
($\lambda < 122$ nm)

m=2: Balmer series
n=3: $\lambda = 657$ nm
n=4: $\lambda = 486$ nm
n=5: $\lambda = 434$ nm
n=6: $\lambda = 410$ nm
n>7: Ultraviolet ($\lambda < 397.1$ nm)

m=3: Paschen series => Infrared (IR)
($\lambda > 820$ nm)

X Ray Spectra

Electrons bombarding a high-Z target
Incident electron KE: $K_o = eV_o$



Wavelength distribution of X-Rays

"Lines": knock *atomic* e's out of metal anode
(see spectral lines in a few slides)

Smooth continuum:

Deceleration of incident e's in anode radiate X rays
Distribution of energies: bremsstrahlung

Xrays: $E = h\nu = hc/\lambda$

For a given incident KE, largest loss = smallest λ

λ min = hc/Ko or Ko = 1240/ λ min (eV nm)

Example: cutoff wavelength of .035 nm corresponds to acceleration voltage V of
Ko = eV = 1240 eV nm / .035 nm, so V = 3550 Volts .

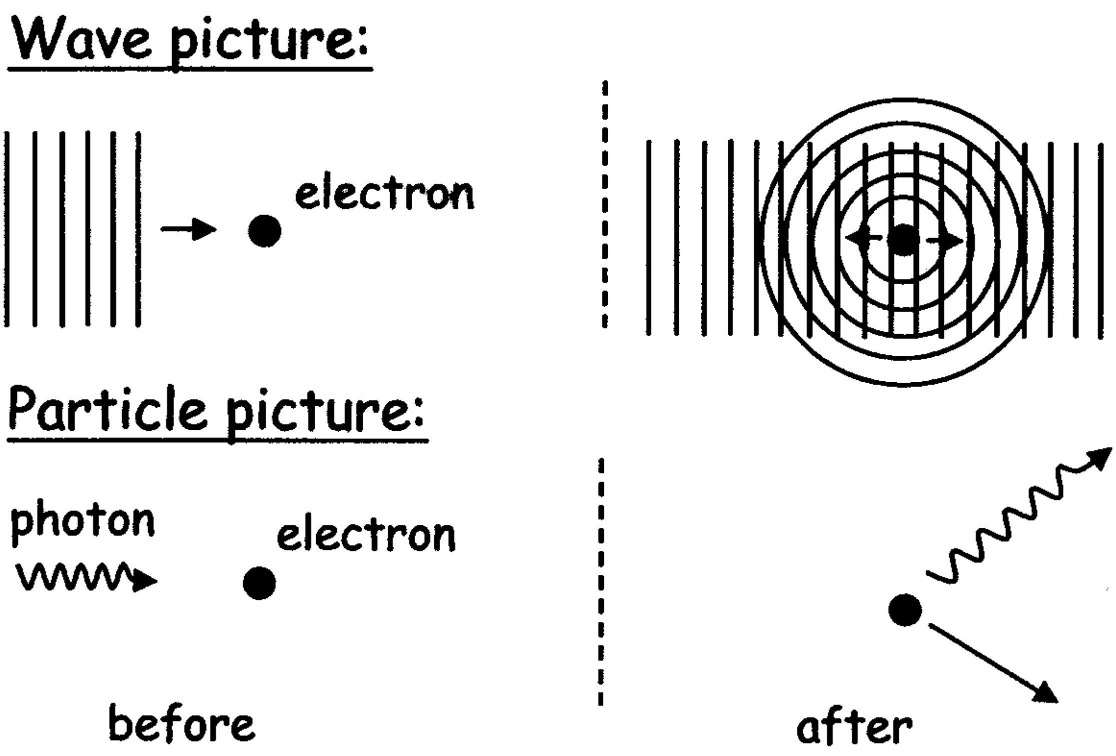
Note: hc is very flexible:
coordinated change of units in energy and wavelength

hc	= 1240 eV	nm	nano
	= 1240 keV	pm	pico
	= 1240 MeV	fm	femto

Compton Scattering

• Photoelectric effect, X-ray spectra suggest that photons, as well as being electromagnetic waves, also act like particles.

• This effect is even more pronounced in scattering of X-rays off electrons.



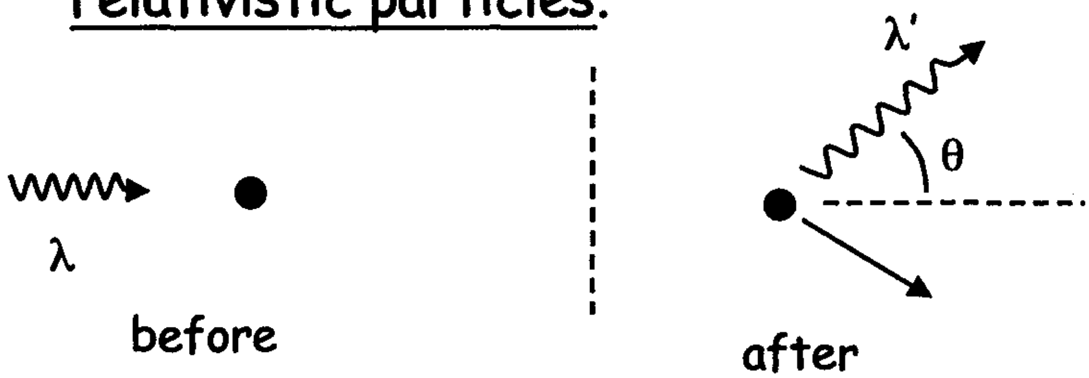
- 1923 -
  Experiments by Arthur Compton.
  He observed:

  Some component of the scattered wave (especially at backward-scattering angles) has a longer wavelength than the incoming wave.

$$\lambda' > \lambda$$

- This could not be understood purely in terms of light as a wave.

- Compton showed that it was understandable as a <u>scattering of relativistic particles</u>.



The formula for the shift in wavelength is

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

with

λ' the scattered wavelength

λ the incoming wavelength

θ the scattering angle

m the electron mass

The combination

$h/(mc) = 2.43 \times 10^{-12}$ m $= 2.43 \times 10^{-3}$ nm

is called the Compton wavelength of the electron.

Compton Effect (c=1)

p, p′ refer to photon, m and q refer to electron

E cons: $p + m = p' + \sqrt{(q^2 + m^2)}$ (1)

Px cons: $p = p' \cos\theta + q \cos\varphi$ (2)

Py cons: $0 = p' \sin\theta + q \sin\varphi$ (3)

Eliminate φ: add squares of (2), (3)

$(p-p' \cos\theta)^2 + (p' \sin\theta)^2 = q^2 = p^2 + p'^2 - 2pp' \cos\theta$

Or defining $\Delta p = p - p'$ we find

$q^2 = (\Delta p)^2 + 2pp' (1-\cos\theta)$ (4)

Eliminate q with square of (1):

$(\Delta p)^2 + 2\, m\, \Delta p + \cancel{m^2} = q^2 + \cancel{m^2}$; equate to rhs of (4)

$2\, m\, \Delta p = 2pp' (1-\cos\theta)$ ($(\Delta p)^2$'s cancel) (5)

$1/p' - 1/p = (1-\cos\theta)/m$ (6)

Restore c's: $1/p'c - 1/pc = (1-\cos\theta)/mc^2$

Use $pc = hc/\lambda$ (only NOW use λ at all!)

$$\lambda' - \lambda = (h/mc)\,(1-\cos\theta)$$

4 vector rules

We are used to 3-d vectors like **r**, **p**

Geometry: invariance under rotations

magnitude (length) of vector

dot product of 2 vectors

magnitude = √ [dot product of vector with self]

Generalize to 4 dimensions:

Notice that the relativistic interval

$-\Delta s^2 = c^2t^2 - x^2 - y^2 - z^2$

(invariant under Lorentz Transform)

Apply LT and verify…

Note also that a similar fact applies for E and p:

$$m^2c^4 = E^2 - p_x{}^2c^2 - p_y{}^2c^2 - p_z{}^2c^2$$

This leads to the extremely useful definition of a relativistic 4-vector invariant under LT's.

$a^\mu = (a_t, a_x, a_x, a_z)$

$X^\mu = (ct, \mathbf{r})$ $\quad\quad\quad\quad$ $P^\mu = (E/c, \mathbf{p})$

$(c\,P^\mu = (E, \mathbf{pc})$ if prefer E units)

In $c = 1$ notation, just $(t,\mathbf{r})$ and $(E,\mathbf{p})$

Invariant dot product generalizes 3d dot product

$A.B = a_t b_t - \underline{\mathbf{a}}.\underline{\mathbf{b}}$

To denote 4-vector I here use P (caps);

(personally, I use $\wp$ (script) when calculating by hand)

$P = (e,\underline{p})$ $\quad$ 2nd slot a vector (underscore or arrow or bold);

$\underline{p}$ is vector or p magnitude depending on context

$P.P' = EE' - (\underline{p}.\underline{p}')$ $\quad$ mixed-sign dot product (invariant!)

$P^2 = e^2 - p^2 = m^2$ $\quad$ special case of dot product

compare: $- s^2 = (\Delta ct)^2 - (\Delta x)^2$

Compton Effect with 4-vector conservation:

$P=(p,\underline{p})$ and $M=(m,\underline{0}) \rightarrow P', Q$ (scattered γ, electron)

Much easier with 4-vectors!

$P + M = P' + Q$ so,

using $\Delta P = P - P'$ and $M = (m,\underline{0})$

$$Q = \Delta P + M \qquad (1\text{-}3)$$

P and E conservation in 1 line!

Squaring:

$$Q^2 = \cancel{m^2} \quad = \Delta P^2 + 2M.\Delta P + \cancel{m^2} \;,$$

so we have

$$0 = \Delta P^2 + 2M.\Delta P$$

now $\Delta P^2 = P^2 + P'^2 - 2P.P'$

for a photon,

$$P = (p,\underline{p}) \text{ and } P^2 = 0$$

$$0 = 0 + 0 - 2P.P' + 2M.\Delta P$$

Now use $M.\Delta P = m\Delta E$

$= m\Delta p$ (photon)

$$0 = -2pp'(1 - \cos\theta) + 2m\Delta p \quad \rightarrow \quad (5)$$

COM frame M^2 for fixed target: $(e,\underline{p})$ and $(m,\underline{0})$

$$M_e^2 = (P+M)^2 = P^2 + m^2 + 2P.M = 2m^2 + 2em = 2m(e+m)$$

$$M_e = \sqrt{2m(k+2m)} \sim \sqrt{2mk} \qquad (k >> m)$$

COM frame M^2 for collider: $(e,\underline{p})$ and $(e,-\underline{p})$

$$M_e^2(cm) = (P+P')^2 = 2\,m^2 + 2P.P'$$

$$= 2m^2 + 2(\,e^2 - \underline{p}.(-\underline{p})\,) \qquad = 4e^2$$

$$M_e = 2e = 2(k+m) \sim 2k \qquad (k >> m)$$