Matter Waves

Thornton and Rex, Ch. 5

Matter Waves

EM waves also behave like particles (photons).

1924 - de Broglie asked:

Can particles also behave like waves?

He suggested a relation between wavelength and momentum:

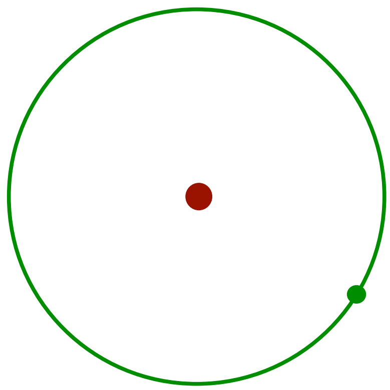
$$\boxed{\lambda = h/p}$$

No experimental evidence for this existed.

But with his matter wave idea, De Broglie could "derive" Bohr's quantization condition.

An electron orbiting an atom at radius r:

Circumference $= 2\pi r$

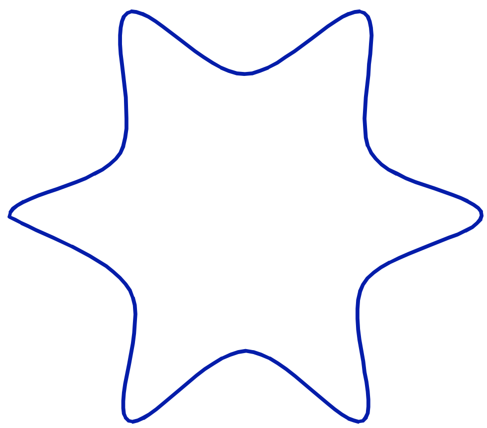


Assume stationary states correspond to standing waves of the electron.
I.e, an integral number of wavelengths must fit into the circumference:

$$2\pi r = n\lambda = nh/p = nh/mv$$

or

$$L = mvr = nh/2\pi = n\hbar$$



n=6 orbit

De Broglie's wave idea fitted naturally with Bohr's atomic model.

Encouraged with this success, de Broglie presented his ideas in his PhD thesis. With no experimental evidence for the idea, de Broglie's professors were skeptical of this radical concept. One sent a copy of the thesis to Albert Einstein. Einstein replied that the ideas certainly appeared crazy, but they were important, and the work was sound.

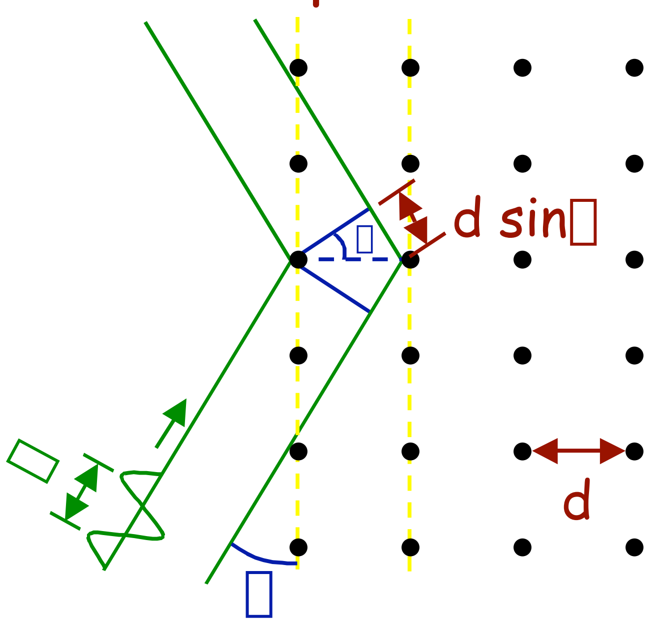
De Broglie received his PhD in 1924.
A few years later the wave nature of electrons was confirmed. In 1929 he was awarded the Nobel Prize.

Discovery of Electron Waves

1925 - Davisson and Germer were scattering electrons from metals.

On scattering electrons off crystallized Nickel, they saw peaks at certain angles.

$\Rightarrow$ Interference spectra!



Interference is constructive (i.e. peaks) when

$$2\, d \sin\theta = n\, \lambda$$

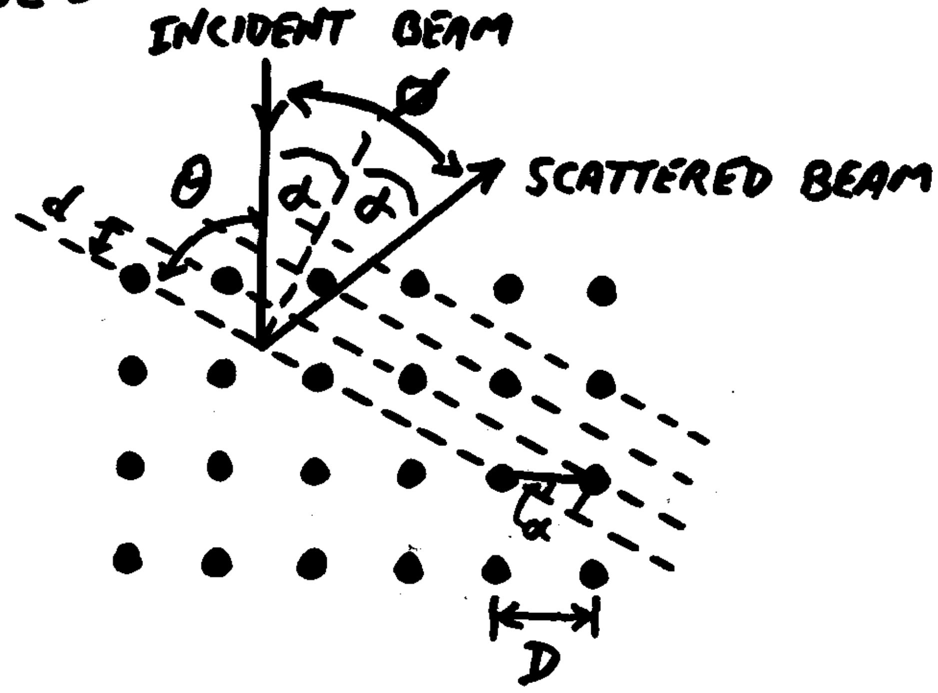
for integer n. (Bragg's law)

□ for the electron agreed with
de Broglie's formula.

This was the first experiment to reveal
the wave nature of matter.

ALL OF THE FEATURES OF X-RAY SCATTERING CAN BE SEEN IN ELECTRON SCATTERING.

NOTE, JUST TO COMPLICATE THINGS, THERE ARE 3 DIFFERENT ANGLES THAT CAN BE USED :-



CONSIDER A PARTICULAR BRAGG PLANE INCIDENT ANGLE IS θ RELATIVE TO THIS PLANE OR α RELATIVE TO THE PERPENDICULAR TO THIS PLANE :- $\alpha = 90° - \theta$

ALSO $d = D \sin \alpha$ $\quad\quad \phi = 2\alpha$

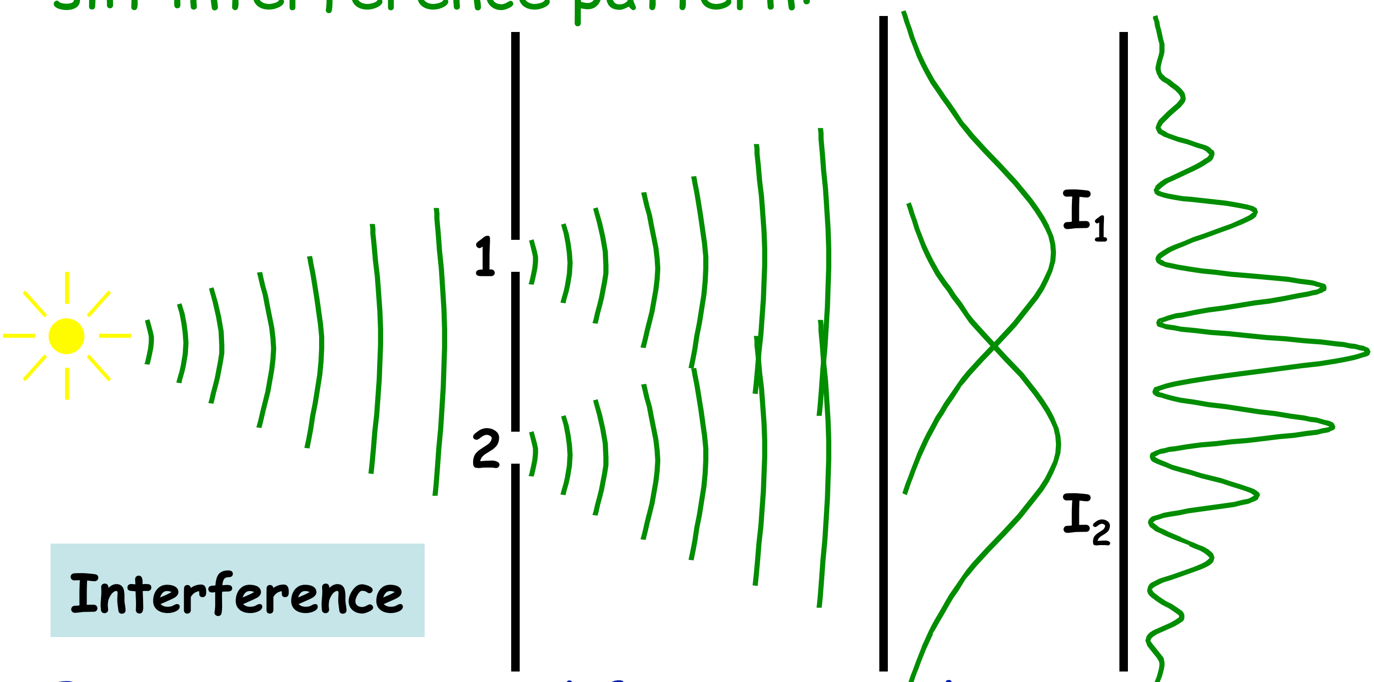
BRAGG'S LAW =>

$$n\lambda = 2d \sin\theta = 2d\cos\alpha = 2D\sin\alpha\cos\alpha$$

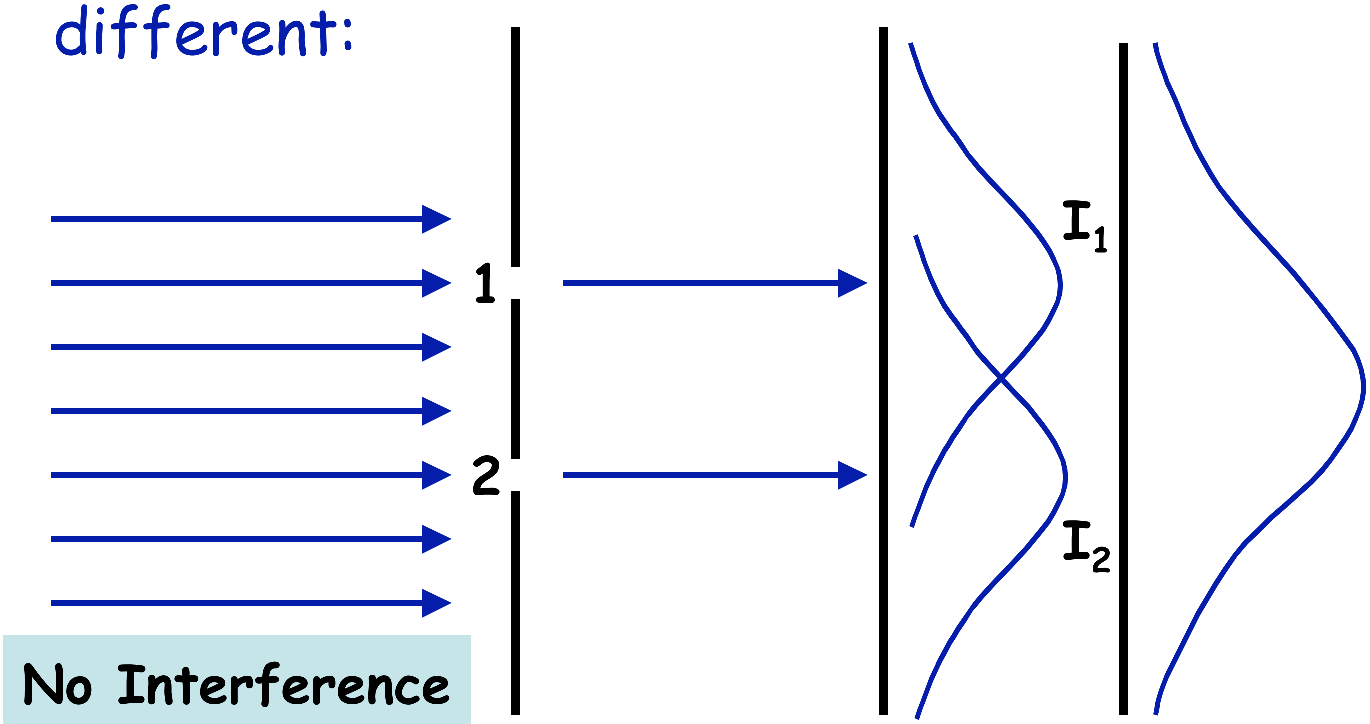
$$\therefore n\lambda = D\sin 2\alpha = D\sin\phi$$

Wave/Particle Duality

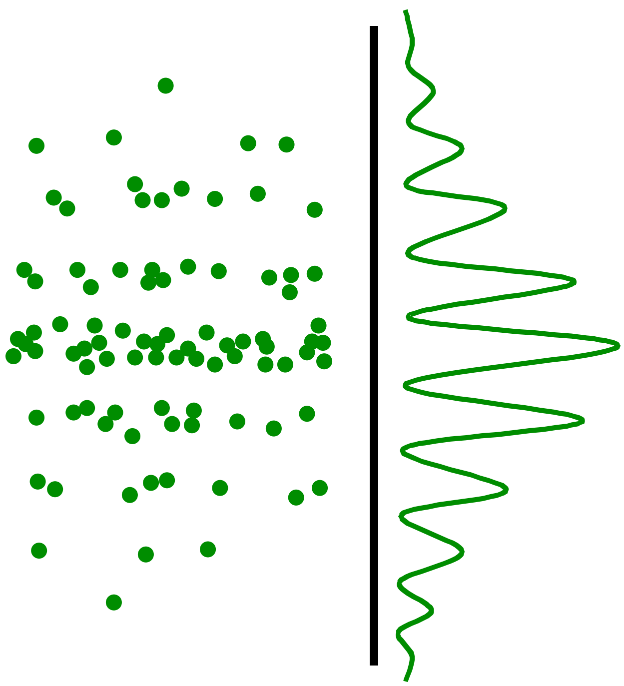
Wave nature of light from the double-slit interference pattern:



Pattern expected from particles is very different:



What happens at very low intensities?

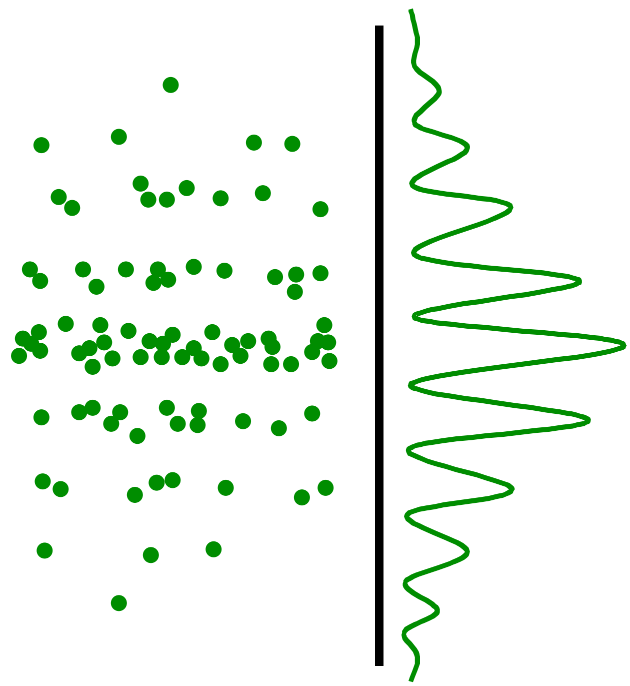


Photons hit at discrete points, gradually building up the interference pattern.

Does the photon go through slit 1 or slit 2?

Neither! (or rather, both!)

What about electrons?



They exhibit the same interference pattern (although at smaller wavelengths than for visible light.)

But the important feature of EM waves is that the wavefront is broad, and it goes through <u>both</u> slits to produce the interference pattern.

But surely the electrons don't go through both slits - they go through one or the other. You can turn the beam down so that there's a very low rate (say 1 per minute). However the distribution gradually builds up and eventually shows the interference pattern. (See Fig. 5.18)

<u>However, if we devise a method of observing which slit the electron goes through, 1 or 2, then there is no interference.</u>

Bohr's Principle of Complementarity

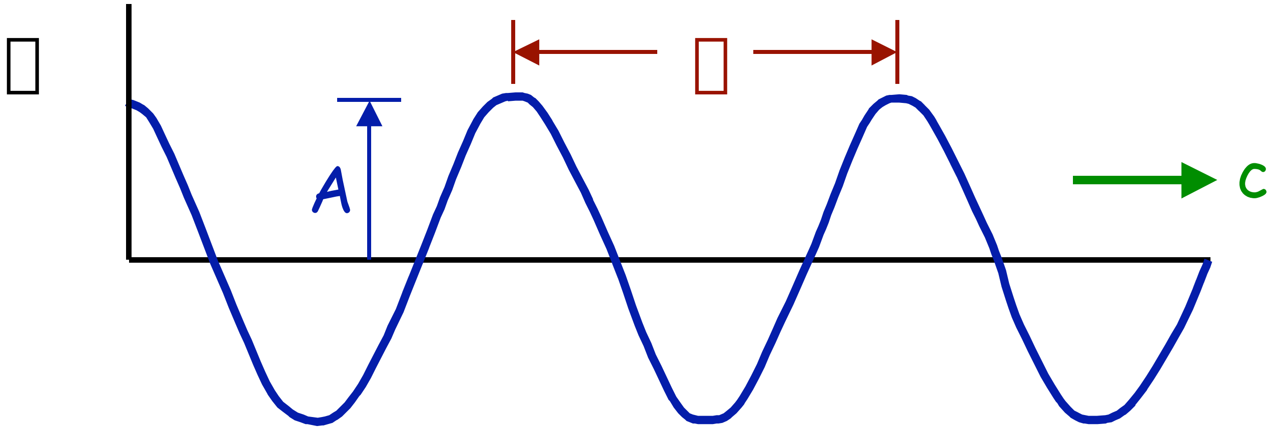
It is not possible to simultaneously describe physical observables in terms of both particles and waves.

Bohr called the fact that all objects (light, electrons, etc.) have both wave-like and particle-like properties

complementarity.

Generalities about light waves

A plane wave:



$$\psi(x,t) = A \cos[2\pi(x-ct)/\lambda]$$

Amplitude: A

Wavelength: λ

Speed: c

Frequency: $\nu = c/\lambda$

It is convenient to rewrite:

$$\psi(x,t) = A \cos(kx-\omega t)$$

Wave number: $k = 2\pi/\lambda$

Angular frequency: $\omega = 2\pi\nu$

Wave relation: $c = \lambda\nu = \omega/k$

All light waves have same speed c in vacuum, independent of wave number k.

$\Rightarrow$ Not true for matter waves.

Planck: $E = h\nu = h\omega/2\pi$

$= \hbar\omega$

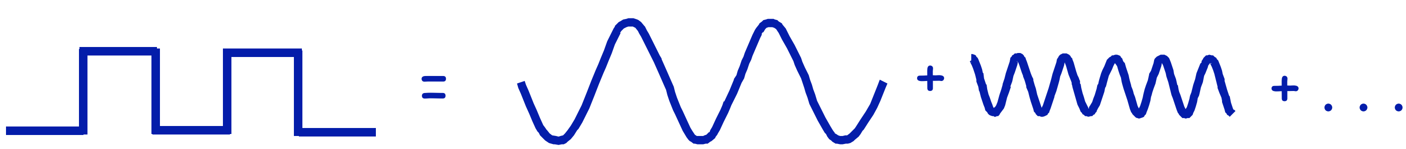
Einstein/de Broglie: $p = E/c = h/\lambda$

$= \hbar k$

A periodic wave can be constructed from a sum of plane waves:

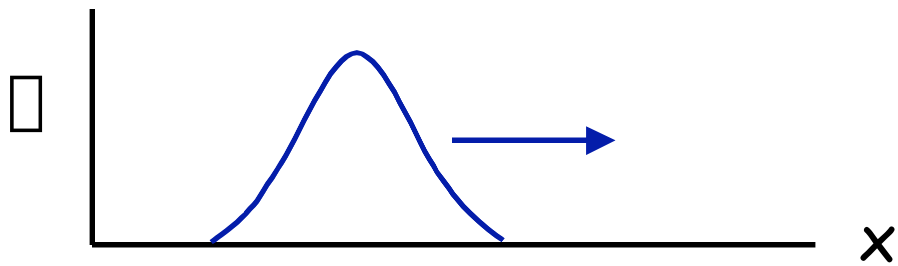
$\Rightarrow$ Fourier Series

$$\psi(x,t) = \sum A_i \cos(k_i x - \omega_i t)$$



A <u>wave packet</u> can be constructed as a continuous sum (integral) of plane waves.

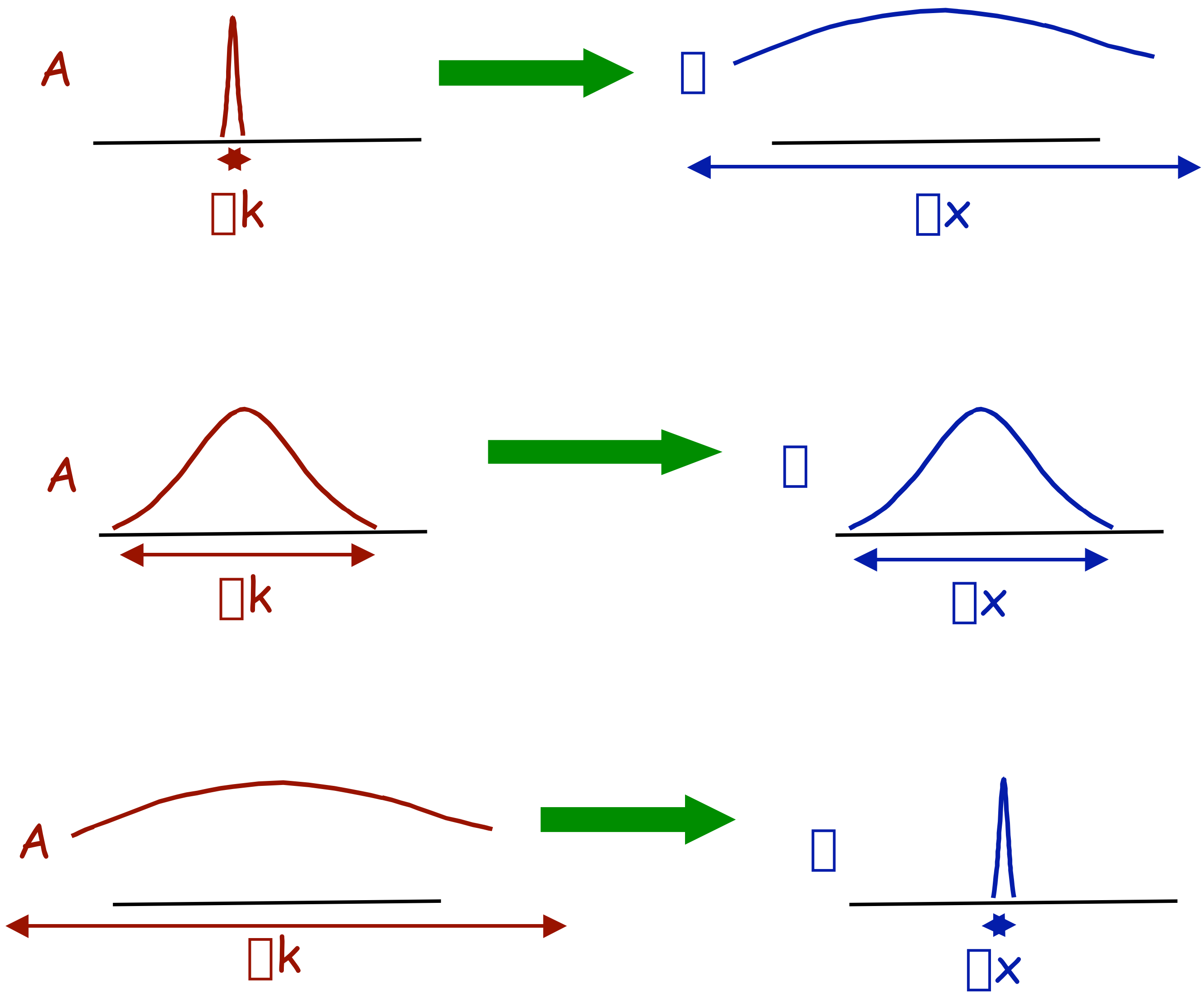
$\Rightarrow$ Fourier Transform



$$\psi(x,t) = \int A(k) \cos(kx-\omega t)\, dk$$

General fact about Fourier Transforms:

The extent Δx of the wave ψ is <u>inversely</u> related to the extent Δk of its Fourier Transform A.


A
k
x
A
k
x
A
k
x

We can write this as

$$\Delta x \, \Delta k \geq 1/2$$

Multiplying by $\hbar$ and using $p = \hbar k$ gives:

$$\Delta x \, \Delta p \geq \hbar/2$$

Heisenberg uncertainty principle

It is impossible to know precisely the position and the momentum of an object at the same time.

IN BORN'S VIEW THE ELECTRON HAD A GOOD CHANCE OF BEING FOUND IN A REGION ONLY IF THE SQUARE OF ITS WAVE AMPLITUDE, ψ, WAS LARGE THERE. THIS INTERPRETATION GIVES SOME INHERENT UNCERTAINTY IN THE POSITION OF OBJECTS.

NIELS BOHR BECAME CONVINCED OF THE USEFULNESS OF SCHRODINGER'S WAVE EQUATION AND OF THE PROBABILISTIC INTERPRETATION OF MAX BORN. HIS INSTITUTE IN COPENHAGEN BECAME THE CENTER FOR THE INTERPRETATION OF THE NEW WAVE MECHANICS.

BOHR STATED THAT THE AMPLITUDE AS CALCULATED BY SCHRODINGER'S EQUATION CONTAINS <u>ALL</u> OF THE INFORMATION (ITS POSITION, ENERGY, MOMENTUM ETC.) WE CAN OBTAIN ABOUT THE BEHAVIOR OF AN ELECTRON OR ATOM. AND THAT THIS INFORMATION CONTAINS SOME INHERENT UNCERTAINTY.

TO THIS ASSERTION, EINSTEIN, PLANCK AND EVEN SCHRODINGER AND DE BROGLIE ALL OBJECTED. THEY COULD NOT BELIEVE THAT IT WAS NOT POSSIBLE TO PREDICT PRECISELY WHERE TO FIND AN ELECTRON IN AN ATOM.

SCHRODINGER WAS PARTICULARLY UNHAPPY. HE HAD DEVELOPED HIS EQUATION IN THE HOPE OF EXPLANING THE BOHR ATOM WITHOUT THE DISCRETE JUMPS OF ENERGY THAT BOHR HAD USED. NOW THE DISCRETE JUMPS WERE STILL THERE AND AN ADDITIONAL UNCERTAINTY BESIDES!

HEISENBERG'S UNCERTAINTY PRINCIPLE

WORKING CLOSELY WITH THE COPENHAGEN PHYSICISTS, WERNER HEISENBERG, A POSTDOCTORAL FELLOW AT GOTTINGEN, HAD DEVELOPED HIS OWN APPROACH TO THE QUANTUM THEORY. HE NEVER MENTIONED PARTICLES OR WAVES, BUT SPOKE IN TERMS OF AN ABSTRACT MATHEMATICAL QUANTITY, QUANTUM STATES. THESE STATES WERE BASED ON GENERAL PROPERTIES OF MATRICES.

THE APPROACHES OF SCHRODINGER AND HEISENBERG APPEARED TO HAVE NOTHING TO DO WITH EACH OTHER WHEN THEY WERE FIRST INTRODUCED IN 1926 BUT SOON IT WAS SHOWN THAT HEISENBERG'S STATES WERE SOLUTIONS TO SCHRODINGER'S EQUATION.

HEISENBERG'S METHOD WAS PARTICULARLY APPROPRIATE FOR CONSIDERING THE INHERENT UNCERTAINTY REQUIRED BY THE PROBABILITY DESCRIPTION. HE DEVELOPED A SET OF RULES KNOWN AS UNCERTAINTY RELATIONS. THERE ARE CERTAIN PAIRS OF PHYSICAL QUANTITIES THAT CANNOT BE DETERMINED SIMULTANEOUSLY TO ANY DESIRED ACCURACY.

ONE SUCH PAIR OF VARIABLES IS ENERGY AND TIME.

ANOTHER IS POSITION AND MOMENTUM.

THUS, IT IS IMPOSSIBLE TO SPECIFY SIMULTANEOUSLY BOTH THE POSITION AND MOMENTUM OF A PARTICLE.

IF THERE IS AN UNCERTAINTY IN POSITION EQUAL TO Δx AND AN UNCERTAINTY IN MOMENTUM Δp THEN;-

$\Delta x \Delta p$ MUST BE GREATER THAN ~ $h/4\pi$

SO AS Δx GETS SMALLER, Δp GETS BIGGER

[AND VICE VERSA!]

THE DIRAC EQUATION

SCHRODINGER'S EQUATION DID NOT MEET THE REQUIREMENTS OF THE THEORY OF RELATIVITY. IT WAS NON-RELATIVISTIC.

A RELATIVISTICALLY CORRECT QUANTUM THEORY WAS DEVELOPED BY PAUL DIRAC AT CAMBRIDGE IN 1928.

THERE IS A SQUARE ROOT IN DIRAC'S EQUATION WHICH IMPLIES TWO POSSIBLE ANSWERS FOR SOME VARIABLES SUCH AS +E OR -E.

FOR A WHILE, DIRAC IGNORED THE APPARENTLY UNPHYSICAL -E SOLUTION, BUT HE DID NOTE IT IN HIS CLASSIC PAPERS. IT IS THE PREDICTION THAT EVERY PARTICLE SHOULD HAVE AN ANTIPARTICLE.

FOUR YEARS LATER, 1N 1932, THE ANTI-ELECTRON (CALLED THE POSITRON) WAS DISCOVERED.

IN 1956 THE ANTIPROTON WAS DISCOVERED.