Phy251 Fall 2010 Practical Laboratory 2
Moment of Inertia

Torque

A torque, τ, is generated when a force, F, acts on an object some distance, R, from a point (typically a pivot point) around which the object can rotate. The magnitude of the torque depends on the angle at which the force is applied, however, in this laboratory the force will always be applied at a 90-degree angle to a line drawn from the pivot to the location where the force is applied. In this case the torque is given by,

$$\tau = FR\,. \tag{1}$$

An unbalanced torque applied to an object will cause it to change its rotational speed. The angular acceleration, α (units are radians/s^2), is the rate at which the rotational speed changes. The unbalanced torque and the resulting angular acceleration are related by

$$\tau = I\alpha\,, \tag{2}$$

where the constant, I, is known as the Moment of Inertia of the object. Clearly, with a constant torque, the angular acceleration will be largest when the Moment of Inertia of the object is the smallest, and vise-versa.

Moment of Inertia

The Moment of Inertia of an object correlates its mass and how the mass is distributed over the object. For a point mass, m_1, located on a mass-less arm a distance, r, from a pivot, the moment of inertia is given by, $I_1 = m_1 r^2$. For two arms symmetrically located

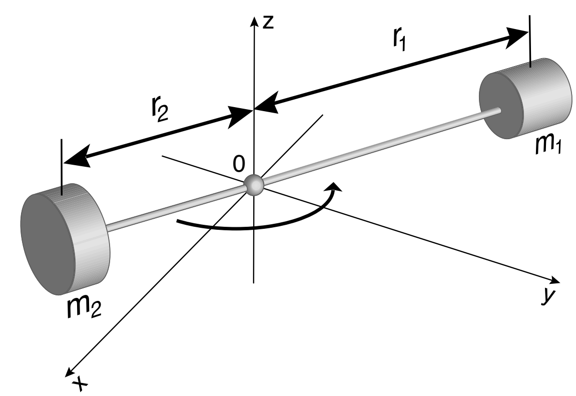

Figure 1. Shown are two masses on opposing arms extending from a pivot point.

about a pivot point, with point masses m_1 and m_2, located at distances r_1 and r_2 from the pivot, as shown in Figure 1, the Moment of Inertia is given by, $I = m_1 r_1^2 + m_2 r_2^2$. When the arms are not mass-less and/or the masses are not point masses, the moment of inertia will include a constant, I_0, which accounts for the distribution of mass in each of these components, so that the Moment of Inertia becomes, $I = m_1 r_1^2 + m_2 r_2^2 + I_0$. The equation is simplified if the masses are at the same distance, r, from the pivot resulting in the equation for the Moment of Inertia of the system,

$$I = \left(m_1 + m_2\right)r^2 + I_0, \tag{3}$$

that will be investigated in this laboratory.

Testing the theoretical behavior of the Moment of Inertia

Applying a known torque to this system and observing the resulting angular acceleration, Equation 2 allows the Moment of Inertia to be determined. According to Equation 3, a plot of this *measured* Moment of Inertia vs. r^2, the distance squared, should result in a *straight line with slope equal to the sum of the masses*, and with intercept, I_0, the moment of inertia associated with the rotating arms, base, and extended nature of the two masses.

Experimental Procedure

The apparatus, shown in Figure 2, is used to measure the angular acceleration of a pair of masses, arms, and supporting base when subjected to a fixed torque, τ, caused by the string tension force, T, acting on the surface of the central pulley with radius, R. The tension is generated by the weight of the hanging mass. The mass drops with a linear acceleration, a, causing the tension to be smaller than the weight, $T = Mg - Ma$.

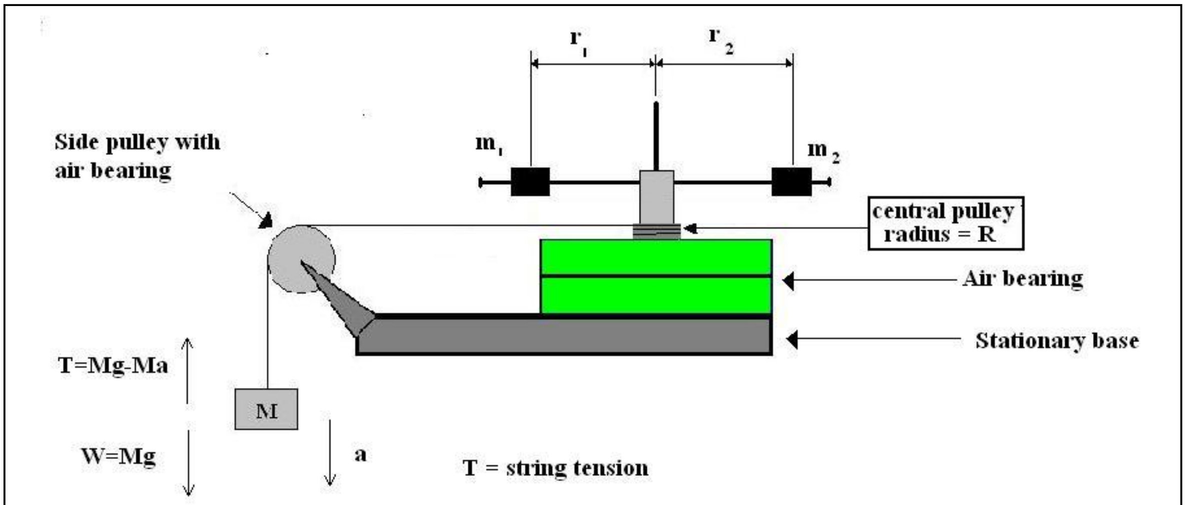

Figure 2. Shown is an apparatus to apply a known torque to a system of masses.

The vertical acceleration, a, is obtained using the time, t, it takes for the mass (M) to fall from the starting elevation, $y = \frac{1}{2}at^2$ ($a = 2y/t^2$). The angular acceleration, α, is obtained from the linear acceleration using $\alpha = a/R$.

Follow the procedure on the Excel spreadsheet to measure the moment of inertia for the six mass locations. For the $r = 0$ point, be sure to stack the two masses in the same orientation they were in while on the arms. Make a plot of the measured Moment of Inertia and each distance squared, determine the slope and intercept (w/uncertainties) of a straight line, best fit to the data and compare its parameters with the expected values.

Uncertainties on variables

In principle the uncertainties on the variables, m_1, m_2, M, R, and y (the starting elevation for the falling mass) can all contribute to the uncertainties on the measurements of the torque and angular acceleration, and thereby the measured Moment of Inertia. Based on previous experimentation, we expect that the fractional uncertainty in the time, t, to fall to the floor dominates the other random fractional uncertainties. The equation for the uncertainty in the Moment of Inertia, δI, shown in the Excel sheet makes this assumption.

Questions (provide uncertainty and units for every quantity derived from a measurement)

1. Compare the slope of your best-fit straight line with expected value. Are they consistent?
2. Does the intercept of the line agree with the measured value of I_0 at $r = 0$?
3. The best-fit straight line passes through the error bars of how many of the data points? If the theory and error bars are correct, should the error bars of all 6 data points be crossed in nearly every experiment? Explain!

Checklist

Your lab report should include the following four items:

1) Your spreadsheet
2) The formula view of the spreadsheet
3) The graph of the measured I (with error bars) at each r^2, and a linear fit (with uncertainties on slope and intercept) to the data.
4) Answers to questions

Compatibility of measurements with expected values

To test the compatibility of two measurements, $d_1 \pm \delta d_1$ and $d_2 \pm \delta d_2$, find $\Delta = |d_1 - d_2|$, and if $\Delta < \delta d_1 + \delta d_2$, the two measurements are compatible.

Moment of Inertia

Measure grey fields	Calculate yellow fields

	Value	Units
Rotating mass m_1:		gm
Rotating mass m_2:		gm
Falling mass M:		gm
Radius of axle R:	1.270	cm
Gravitational acceleration g:	980	cm/sec^2
Starting elevation, y, of mass M		cm
Uncertainty in time, δt	0.2	sec

Determination of the moment of inertia

Mount the masses on the rod and measure the time it takes the mass M to fall to the floor for the 6 indicated positions of the masses along the rod. Calculate the linear acceleration a, the angular acceleration α, the tension in the string T, the torque τ, the moment of inertia I, and its uncertainty, δI, for each configuration using equations given below.

Trial	r	r^2	t	$a = 2y / t^2$	$\alpha = a / R$	$T = M(g-a)$	$\tau = TR$	$I = \tau/\alpha$	$\delta I = I\,(2\delta t/t)$
	cm	cm^2	sec	cm/sec^2	rad/sec^2	dyne	dyne-cm	gm*cm^2	gm*cm^2
1	10.00	100.00							
2	9.00	81.00							
3	8.00	64.00							
4	6.00	36.00							
5	4.00	16.00							
6	0.00	0.00							

Plot the measured values of I (with error bars) vs. r^2, and determine the slope and intercept (w/uncertainties) of a best fit straight line to the data.

Example Spreadsheet