Physics 431 - **Midterm Exam (3:00-4:00 pm 10/20/2009)**

TIME ALLOTTED: 60 MINUTES

Name: ____

SID: ___________________________

Signature:____

CLOSED BOOK. ONE 8 1/2" X 11" SHEET OF NOTES (double sided is allowed), AND SCIENTIFIC POCKET CALCULATOR PERMITTED (No laptop/netbook etc.)

Mark 4 to 6 problems (check boxes) you would like to be graded below:

	1. (20) ☐	2. (15) ☐	3. (20) ☐	4 .(20) ☐	5. (20) ☐	6 .(25) ☐	7 .(15) ☐	8.(20) ☐	9.(20) ☐
Score									

If you score more than 100 points, the extra points divided by 10 will count toward your final grade.

Fundamental constants and equations you might need:

Planck's constant, h = 6.62 x 10^{-34} J s; $\hbar = 1.05457266 \times 10^{-34}$ Js

Permittivity of free space, ε_o = 8.85 x 10^{-12} F/m

Permeability of free space, μ_o = 1.26 x 10^{-6} H/m

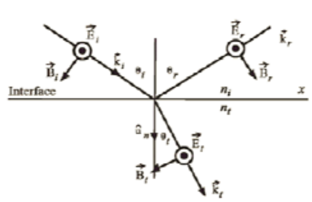
Speed of light in vacuum, c = 2.998 x 10^{8} m/s

Electron charge, e = 1.6 x 10^{-19} C

Electron volt, 1 eV = 1.6 x 10^{-19} J

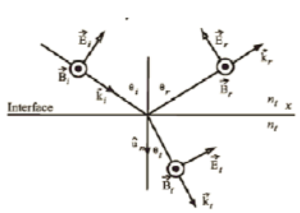
Photon energy E = hυ; Photon momentum $\hbar k$

Fresnel's equations:



$$r_\perp = \left(\frac{E_{0r}}{E_{0i}}\right)_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$t_\perp = \left(\frac{E_{0t}}{E_{0i}}\right)_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$



$$r_\parallel = \left(\frac{E_{0r}}{E_{0i}}\right)_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$t_\parallel = \left(\frac{E_{0t}}{E_{0i}}\right)_\parallel = \frac{2n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$

1. EM Waves and Polarization (20 points)

A. (10 points) Imagine an electromagnetic plane wave in a vacuum whose **E**-field (in SI units) is given by

$$E_x = 10^2 \sin \pi \left(3\times10^6 z - 9\times10^{14} t\right) \quad \text{[V/m]}$$

$$E_y = E_z = 0$$

Determine the wavenumber k, velocity (speed and propagation direction), initial phase, period, frequency, wavelength, energy, E-field amplitude, intensity $\left|\langle \mathbf{S} \rangle\right| = I \equiv \left|\langle \mathbf{E}(t)\times\mathbf{H}(t)\rangle\right| = \frac{c\varepsilon_0}{2}E^2$, and polarization. (Don't forget to write out units (eg. m^{-1}, s^{-1}, etc.)

B. (10 points) Describe the state of polarization of the following two waves (specify E-field direction or rotation at a fixed point of space z):

$$(1) \quad \mathbf{E} = \hat{x}E_0\cos(kz-\omega t) + \hat{y}E_0\cos(kz-\omega t)$$

$$(2) \quad \mathbf{E} = \hat{x}E_0\cos(kz-\omega t-\pi/2) + \hat{y}E_0\cos(kz-\omega t)$$

$$(3) \quad \mathbf{E} = \hat{x}2\times E_0\cos(kz-\omega t-\pi) + \hat{y}E_0\cos(kz-\omega t)$$

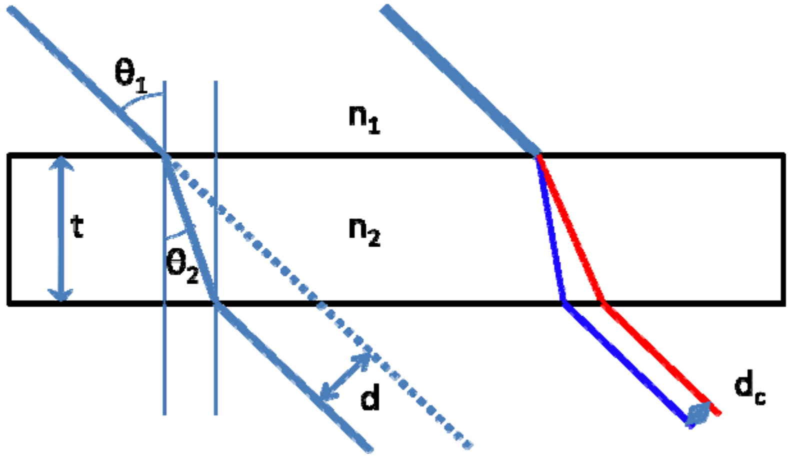
2. Total internal refraction/Snell's law/Brewster's angle [15 points]

A ray of light travels from air into glass with an angle of incidence θ_i.
It is observed that the angle between the reflected and refracted rays is 90^o.

a. Sketch a ray diagram showing the incident, reflected and refracted rays at the interface.
b. Determine the index of refraction n (relative to air) of the glass. Your answer will be a function of θ_i.
c. For a ray going from the glass to air what is the maximum angle of incidence, in the glass, for which there will be a refracted ray in the air? (express as a function of refractive index n)
d. For angles greater than the angle determined in part (c), if there is no refracted ray emerging into air what happens to the light that is incident upon the interface? For n=1.5, what is this critical angle?

3. Snell's Law [20 points]

A light ray traverses a planar slab of glass (one with parallel sides), as shown in the sketch.



(a) Use Snell's Law at each surface to determine d as a function of t, where d is the lateral displacement of the ray and t is the thickness of the slab. Express your final answer as a function of n, the index of refraction of the glass, and θ, the angle of incidence of the ray. Note: $\sin(\theta_1 - \theta_2) = \sin\theta_1 \cos\theta_2 - \cos\theta_1 \sin\theta_2$.

(b) Determine d for a narrow beam of monochromatic light beam: given n_1=1 and n_2 = 1.6, t = 8.0 cm, θ = 45°.

(c) For an exceedingly narrow beam of white light incident at the same angle, determine the approximate diameter d_c of the emerging beam for the red and violet light. The index of refraction for red light is 1.51 and for violet light it's 1.55.

(d) Use the answer to part (a) to determine d(t) for small but nonzero θ using the small angle approximation ((i.e. $0 < \theta \ll 1$)

4. Application of Fresnel's equations / Polarization (20 points)

A. Express the amplitude reflection coefficients ($r_\perp$ and $r_{||}$) in terms of incident angle θ_i and the ratio of the refractive indices of the two media $n_{ti} \equiv \frac{n_t}{n_i}$.

B. Determine the values of the amplitude reflection coefficient for light incident at 6^{0o} on an air-glass interface, $_{nti}$=1.50.

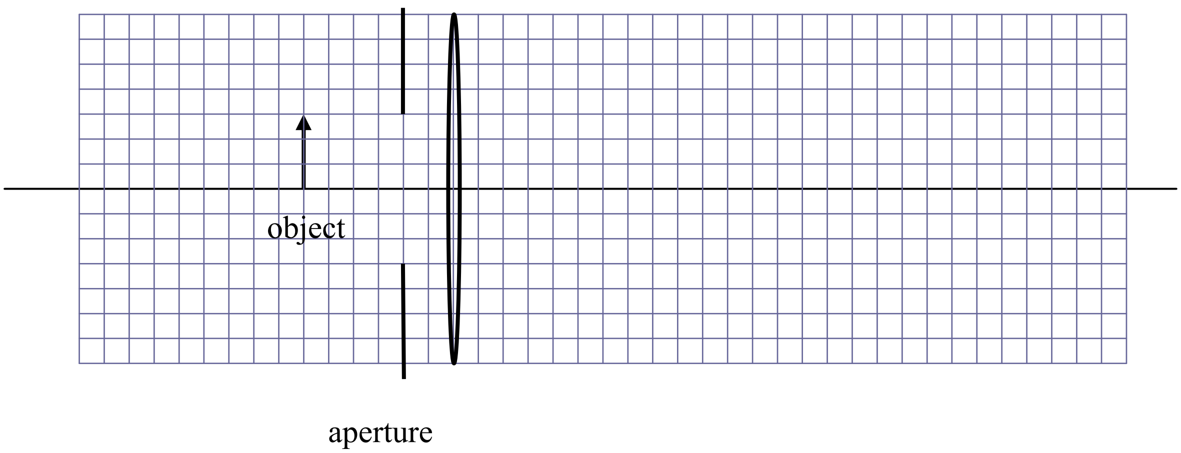
C. A beam of natural (unpolarized) light is incident on an air-glass interface (n_{ti}=1.50) at 60^o. Determine the degree of polarization of the reflected beam. (The degree of polarization $V \equiv \dfrac{R_\perp I_{0\perp} - R_{||} I_{0||}}{R_\perp I_{0\perp} + R_{||} I_{0||}}$, where $R = r^2$ is the reflectance and I_0 the intensity of the incident light.).

D. Determine the amplitude transmission coefficients $t_\perp$ and $t_{||}$ as well as the transmittance $T_\perp$ and $T_{||}$. (Caution: $T \neq t^2$)

5. Lens Maker Formula/Imaging Formula (20 points)

A thin lens with a diameter of 7 cm and a focal length of +2.0 cm has a 3.0 cm diameter stop located 1.0 cm in front of it. An object 1.50 cm high is located with its lower end on the axis, 3.0 cm in front of the lens.

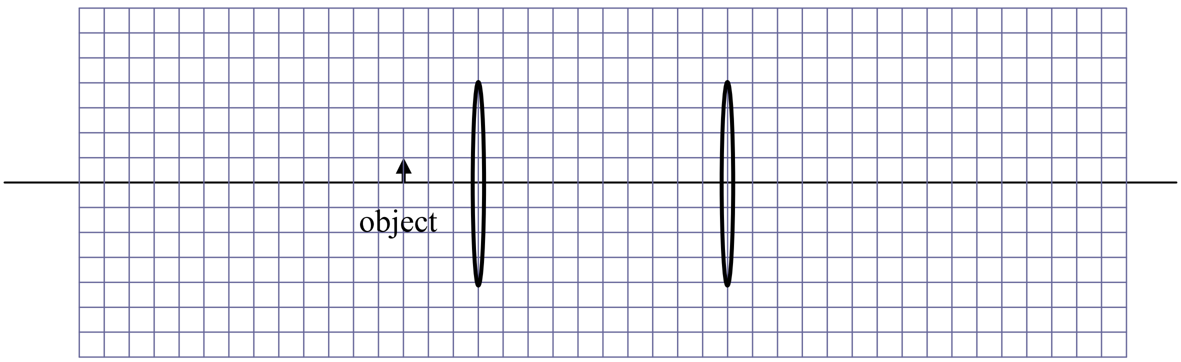
1) Determine and sketch below the position and size of the entrance and exit pupils.
2) Determine and sketch below the position of the image point and the magnification.
3) Sketch the chief ray and two marginal rays from the tip of the object.



6. Thin lens combination/Microscope (25 points)

A homemade microscope has a thin positive front lens L_1 of 2-cm focal length, 10 cm behind which is another positive lens L_2, with a 5-cm focal length. Both lenses have a diameter of 4 cm.

1) Locate the image of an object 1 cm high and 3 cm from the front lens, and compute the magnification.
2) Indicate the location and size of the entrance pupil and exit pupil.
3) Construct a ray diagram showing two marginal rays for a single axial object point (i.e. from the bottom of the object).



7. Thin lens combination (15 points)

A convex thin lens with focal length 30 cm and a concave thin lens with focal length -20cm are placed in contact. What is the focal length of the combination? Determine the image properties (location and real or virtual) of an object at a distance 40 cm from the lens pair.

8. Mirror and Imaging (20 points)

(a) An object is 1 m from a spherical concave mirror with radius of curvature 40 cm. Does a real image form? Determine the image distance from the vertex of the mirror.

(b) A person whose face is 25 cm away looks into the bowl of a spherical soupspoon with an approximate radius of curvature of 3 cm. Determine the magnification of her image reflected by the spoon.

9. Dispersing Prism (20 points)

(a) Briefly define dispersion (3 points)

(b) If a 60-60-60^o dispersing prism is made of a particular material that has index of refraction 1.49 at 400 nm and 1.48 at 700 nm, determine the separation of these two colors on a wall 1.5 m from the center of the prism. Assume that the incoming beam, which contains both colors, is oriented perpendicular to the wall, and that it enters the entrance face of the prism making an angle of 30^o with the normal to the prism face. (17 points)