Physics 431 – **The Final Exam**
(3:00-5:00 pm, 12/16/2009)

TIME ALLOTTED: 120 MINUTES

Name: ____

PID: __________________ **Pass code:** __________________

Signature: ____

Closed Book. Two 8 1/2" X 11" sheets of notes (double sided is allowed), graded lab reports, and a scientific calculator permitted. No Laptop/Netbook or similar devices.

Total 260 points
Your score x 0.10 will count toward your final grade.

You may NOT leave the room until finished with the exam. Show all work on these pages and circle your answers (you may use the back of the page if necessary). No partial credit will be given for true/false and fill-in-the-blank questions.

You may review your graded final exam in my office 4238 BPS after 9 am on December 18. Your final grade will be posted by 4pm on December 18.

Good luck!

Constants you might need:

Planck's constant, h = 6.62 x 10^{-34} J s; $\hbar = 1.05457266 \times 10^{-34}$ Js

Permittivity of free space, ε_o = 8.85 x 10^{-12} F/m

Permeability of free space, μ_o = 1.26 x 10^{-6} H/m

Speed of light in vacuum, c = 2.998 x 10^{8} m/s

Electron charge, e = 1.6 x 10^{-19} C

Electron volt, 1 eV = 1.6 x 10^{-19} J

Photon energy E = hυ; Photon momentum $\hbar k$

1 radian = 57.3 degrees (Do not mess up "radian" vs. "degree" in calculations.)

1 light year = 9.4605284 × 10^{15} m

True/False Questions (8 points, 2 points for each question)

____(1) For normal dispersion (such glass lenses used in Phy 431 Optics Lab), the index of refraction is larger for blue light than for red light.

____(2) The phase velocity of light in dispersive media can exceed the speed of the light in vacuum.

____(3) At near normal incidence about 4% of the light is reflected back off each air-glass interface.

____(4) The retina contains two types of photoreceptors, rods and cones. The rods intermingle uniformly over most of the retina. They are more numerous and sensitive than the cones.

Fill in the Blank Questions (12 points, 2 points for each question)

(1) A ray of green laser beam (λ=532 nm in air) enters a drop of water (refractive index n=1.52). The wavelength of the laser light in water is ____________ nm.

(2) A camera exposure setting with shutter speed of 1/64 sec and aperture setting f/4 gives an equivalent film exposure to _________sec shutter speed and f/2 aperture setting.

(3) Assume a myopic eye (nearsightedness) cannot see clearly objects located at distances greater than 50 cm. A ***positive / negative*** corrective lens with a focal length of __________ cm is required. [circle positive or negative]

(4) A hyperopic (farsightedness) person can see clearly objects no closer than 150 cm. For such a person to read this page clearly at the normal distance of 25 cm, a ***positive / negative*** corrective lens with a focal length of __________ cm is required. [circle positive or negative]

(5) $\frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ = ___________________________ [Specify the value and units]

(6) As x→0, sin(x)/x→________ ; $J_1(x)/x$→_________ (J_1 is the Bessel function of the first kind, order one) [1 point each]

Short Questions (60points)

1) (a) Find the magnitude and phases of the complex quantities for $\psi(x,t)=e^{i(kx-\omega t)}e^{i\delta}$ and $\psi(y,t)=2e^{iky}e^{+i\omega t}+2e^{iky}e^{-i\omega t}$. [3 points]

   (b) Determine the direction of propagation of the plane wave $\psi(x,y,z,t)=A\cos\left(\frac{1}{\sqrt{14}}(kx+2ky+3kz)-\omega t\right)$. (Express it in terms of a unit vector in Cartesian coordinates) [2 points]

2) When light passes through two polarizers whose axes of transmission are parallel (defined as 0 degree), a photodetector reads 30 units. If a third polarizer is inserted in between at 45 degrees, what will the detector read? [5 points]

3) Briefly describe the causes of the following aberrations in an optical system: (a) spherical aberration, (b) chromatic aberration, (c) astigmatism. [6 points]

   Define "chief ray" in an optical system. [2 points]

4) A glass block having an index of 1.55 is covered with a layer of water of refractive index 1.33. For light traveling in the glass, what is the critical angle at the interface? [6 points]

5) A student constructed a telescope in the laboratory. The lens he used as the objective was labeled as D=25 mm, f/16. The lens he used as the eye piece was labeled as 20 mm. Determine the magnification of the telescope. [6 points]

6) A soap film surrounded by air has an index of refraction of 1.34. If a region of the film appears bright red (use λ_0=633 nm in vacuum/air) in normally reflected light, what is its minimum thickness there? [6 points]

7) How many wavelengths wide must a single slit be if the central bright lobe of the Fraunhofer diffraction spans an angular distance of 60 degrees from the optic axis? [6 points]

8) A HeNe laser emits at a power level of 10 mW. Assuming the laser beam is Gaussian with a beam waist w_0=1 mm at the laser output, determine (a) the total photon flux (i.e. # of photons/sec) [3 points] and (b) the average intensity (irradiance) after the beam propagate 100 meters. (use divergence angle $\theta = \frac{\lambda}{\pi w_0}$ and approximate the beam waist at z=100 m with $w \approx z \times \theta$) [5 points]

9) A transmission grating is used to resolve the sodium D lines doublet (589.592 nm and 588.995 nm) in the first-order diffraction spectrum.

(i) How many rulings are needed? [4 points]

(ii) The grating and a screen are positioned at the front and back focal plane, respectively, of a positive focusing lens. If the focal length of the lens is 20 cm, and the total width of the grating is 2 cm, what is the linear separation at the focal plane between the two D lines? [6 points] [Hint: For the first-order diffraction spectrum, the principle maximum occurs at $\lambda = h\sin\theta$ and the angular separation $\Delta\theta = \frac{\Delta\lambda}{h\cos\theta}$. h is the distance between two adjacent slits.]

1. Thin lens imaging (10 points)

Consider a bi-convex lens for which the refractive index is n=1.5 and the radii of curvature of the front and back surfaces are $|R_1|=|R_2|=5$ cm, respectively.

(A) Calculate the focal length of the lens. [3 points]

(B) An object is located at an unknown distance to the left lens. If a virtual upright image is formed at 25 cm to the left of the lens, calculate the position of the object and the lateral magnification of the image. [7 points]

2. Dispersing Prism/Glass Plate (15 points).

A narrow beam of white light passes through a prism at an angle of 70º from normal. The apex angle of the prism is 50º. The index of refraction of the prism is 1.51 for red light and 1.56 for violet light.

(A) At what angle does the red and violet light emerge? What is the angular spread of the emerging light? **[**Hint: $\delta = \theta_1 - \alpha + \sin^{-1}\left[\sin\alpha\sqrt{n^2 - \sin^2\theta_1} - \cos\alpha\sin\theta_1\right]$**]**

(B) If you place a screen 100 cm away from the prism, by what distance will the red light be separated from the violet light? (neglect the size of the prism and use small angle approximation)

(C) You want to disperse the white light without using a prism, so you decide to use a piece of flat glass. How thick a piece of glass would you need to place in the beam of white light to separate the red and violet light by 1mm, using the same incident angle of 70º?

[Hint: $d = \dfrac{t\sin(\theta_1 - \theta_2)}{\cos(\theta_2)} = t\sin\theta_1\left[1 - \sqrt{\dfrac{1-\sin^2(\theta_1)}{n^2 - \sin^2(\theta_1)}}\right]$]

3. Microscope and Telescope (20 points).

(A) A compound microscope has magnification of 100 (viewing with a fully relaxed eye). The microscope has standardized tube-length of 160 mm, and a 20× objective lens with numerical aperture NA=0.55. [12 points]

(1) What is the focal length of the objective lens?
(2) What is the focal length of the eyepiece lens?
(3) What is the minimal distance between two points it they were to be resolved by the microscope objective? [use the Rayleigh's criterion for λ= 550nm, visible light illumination]

(B) **Determined** the minimal size (radius of aperture) of a telescope required to resolve the components of a double star whose linear separation is 100 million km and whose distance from the earth is 10 light years. **Explain** the *minimum resolvable angular separation* (or *angular limit of resolution*). (take λ= 550nm). [8 points]

4. **Interference (30 points)**

(A) Design an antireflection coating for a normal incident light of 550 nm wavelength on a glass substrate with the refractive index of 1.52. **Specify the thickness and refractive index** of the coating material.

(B) Two sheets of flat plate glass 25 cm long are separated at one end by a 100-μm- diameter human hair, thereby forming a thin wedge-shape air film. **How many fringes per centimeter** will be observed under normal illumination from a Hg lamp source (λ_0=546.1 nm, green line).

(C) **Draw** a diagram showing a Michelson interferometer. Be sure to clearly label all of the critical components such as the mirrors, beam splitter, compensator, light source, and detector/eye, etc.

(D) A thin sheet of fluorite (CaF2) of index of 1.434 is inserted normally in one arm of a Michelson interferometer. Under illumination with sodium D light, λ_0=589.29 nm, 35 fringes are seen to be displaced. **Determine** the thickness of the sheet.

(E) Suppose that a Michelson interferometer is illuminated by a source emitting a doublet of vacuum wavelengths, λ_1 and λ_2. As one of the mirrors is moved, the fringes periodically disappear and then reappear. A displacement Δd of the mirror cause a one-cycle variation in the visibility when $\Delta d = \dfrac{\lambda_1 \lambda_2}{2\Delta\lambda}, \quad where \quad \Delta\lambda \equiv \lambda_1 - \lambda_2$. **Calculate** the displacement Δd when the sodium D doublet (589.592 nm and 588.995 nm) is used as the light source.

5. **Diffraction: Single Slit, Double Silts & Rectangular Apertures (30 points)**
   In a diffraction experiment, a collimated beam of laser from a green diode laser (λ=532nm) is to be used.

(A) The laser output beam is approximately circular with a diameter of 2 mm. You need a beam expander to produce a collimated beam of 10 mm for the diffraction experiments. You are given six lenses with the following focal lengths (negative lenses: f=-1 cm, -2 cm, and -5cm; positive lenses: f= +2cm, +10 cm, and +15 cm). **Choose** two lenses to set up such a beam expander (Galilean telescope type). **Specify** the distance between these two lenses and **Draw** a ray diagram to illustrate the beam expansion.

(B) In the 1st experiment, a hole of R=0.5 mm radius is used as an aperture.
Explain Fraunhofer and Fresnel diffraction regimes based on the Huygens-Fresnel principle. **Determine** whether Fresnel or Fraunhofer approximation applies when the screen-to-aperture distance z is (i) 10 cm (ii) 2 m.
[Hint: consider z vs. $\frac{\pi R^2}{\lambda}$ & z^3 vs. $\frac{\pi R^4}{4\lambda}$]

(C) In the 2nd experiment, the collimated beam falls normally on a slit 50 μm wide. A lens of 50 cm focal length placed just behind the slit focuses the diffraction light on a screen located at the focal distance.
Calculate the distance from the center of the diffraction pattern (central maximum) to the first minimum.

(D) In the 3rd experiment, a double slit is used as an aperture. It is found that the fourth secondary maximum is missing.
Determine the ratio of slit width ***a*** to slit separation ***b***. [consider Fraunhofer diffraction]

(E) A square aperture (width=a) is used for the final experiment. **Sketch** the diffraction pattern**. Determine** the value of intensity ratio I/I_0 for the first diagonal maximum of the resultant Fraunhofer diffraction pattern. I_0 is the intensity at the central maximum. (The diagonal maxima occur on the line $\alpha=\beta=1/2k\times a\times\sin(\theta)=1.4303\pi$).

6. **Optical Fibers. (25points)**

An optical fiber has cladding index n1 = 1.485, and core index n2 = 1.50. It is used with a laser at wavelength 1.55 μm. The core d = 8 μm. The number of modes in a stepped-index fiber is provided by the expression $N_m \approx \frac{1}{2}\left(\pi d \times \mathrm{NA} / \lambda_0\right)^2$.

(A) Determine the numerical aperture of the fiber. What is the maximum acceptance angle? What would happen to a ray incident at 45 degrees?

(B) What is the maximum d for single mode operation at this wavelength?

(C) For the given d = 8 μm, what is the minimum wavelength for single mode operation?

(D) Determine the intermodal delay (in ns/km) for such a stepped-index fiber.

(E) Given an attenuation of 0.2 dB/km, how far can a signal travel along it before the power level drops by half?

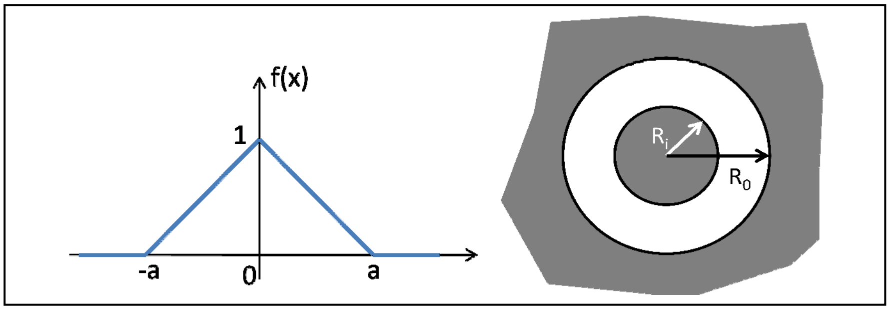
7. Gaussian Beam Optics (25 points)

A HeNe laser cavity is 25 cm long. The beam waist occurs at the laser output with radius w_o=1 mm. It is a lowest order Gaussian beam. The reflectivity of the back mirror is R_1=100% and the reflectivity of the output mirror R_2 = 96%. The laser wavelength is λ= 633 nm.

(A) What is the radius of curvature of the output mirror?
(B) What is the Rayleigh range, z_R of the beam?
(C) Determine the beam waist at a distance of 5 cm and 50 cm from the output mirror.
(D) Calculate the radius of curvature of the wavefront of the beam at a distance of 5 cm and 50 cm from the output mirror.
(E) A 10 mm focal length lens is placed at a distance of 50 cm from the output of the laser. Assume the lens diameter is large enough to capture the entire beam. At what distance from the lens will the focused beam waist occur, and what will be the value of the Gaussian beam radius at the focus? [Hint: Make a suitable approximation and check it's validity at the end of your calculation.]

8. Fourier Optics / Fraunhofer Diffraction (25 points)

(A) Determine the Fourier transform of the triangular pulse shown below. Make a sketch of your answer. Label the **first and second minimums** in units of $k_x = 2\pi/\lambda \times \sin\theta$ on the curves. [The Fourier transform of f(x), $F\left(k_x\right) = \int_{-\infty}^{+\infty} f\left(x\right) e^{ik_x x} dx$.]



(B) A transparent ring on an otherwise opaque mask as shown above is used as an aperture for a diffraction experiment. Assuming uniform normally incident plane-wave illumination. The aperture is circular and has a central obscuration disk (**R_0/R_i=2).** A screen is position at a distance z away from the aperture. The irradiance/intensity at the center of the diffraction pattern, P, on the screen is **I_0**.

(i) Find an expression in the Fraunhofer diffraction pattern of the aperture, and

(ii) Determine the irradiance at P when the central obscuration is removed.

[Hint: The electric field amplitude of the Fraunhofer diffraction pattern of a circular aperture (radius R) under uniform illumination U_0 is $U\left(x, y, z\right) = U_0 \times A \times \frac{e^{-i(kz)}}{-i\lambda z} \times \left[\frac{2J_1\left(\rho\right)}{\rho}\right]$, where $A=\pi R^2$ is the area of the aperture, R the radius of the aperture, $\rho = kR\sin\theta$, k the wavenumber, θ the direction of the diffracted ray. The irradiance/intensity $I \propto |U|^2$].