Geometrical Optics

Refraction at Spherical Surface

Optical Path Length (OPL) from S to P is

\[ \text{OPL} = n_1 l_0 + n_2 l_i \]

where

\[ l_0 = \sqrt{R^2 + (s_o + R)^2 - 2R(S_o + R)\cos\Phi} \]

\[ l_i = \sqrt{R^2 + (s_i - R)^2 + 2R(S_i - R)\cos\Phi} \]

By Fermat’s Principle

\[ \frac{d(OPL)}{d\Phi} = 0 = \frac{n_1 R(s_o + R)\sin\Phi}{2l_o} - \frac{n_2 R(s_i - R)\sin\Phi}{2l_i} = 0 \]

or

\[ \frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{1}{R} \left( \frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o} \right) \]

which means that not all rays from S pass P.

**Paraxial Approximation**

For small \( \Phi \), \( l_i = s_i \), \( l_o = s_o \), the equation becomes

\[ \frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \]

which is independent of the OPL, so all rays pass through P.

Define object focal length \( f_o = \frac{n_1}{n_2 - n_1} R s_o \) when \( s_i = \infty \).

Define image focal length \( f_i = \frac{n_2}{n_2 - n_1} R s_i \) when \( s_o = \infty \).
Sign conventions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{m0}$, $f_0$</td>
<td>+ left of $V$</td>
</tr>
<tr>
<td>$x_0$</td>
<td>+ left of $F_0$</td>
</tr>
<tr>
<td>$s_{v}, f_1$</td>
<td>+ right of $V$</td>
</tr>
<tr>
<td>$x_1$</td>
<td>+ right of $F_1$</td>
</tr>
<tr>
<td>$R$</td>
<td>+ if $C$ is right of $V$</td>
</tr>
<tr>
<td>$\gamma = \gamma_1$</td>
<td>+ above optical axis</td>
</tr>
</tbody>
</table>

*This table anticipates the imminent introduction of a few quantities not yet spoken of.

Thin Lenses
From the first surface,
\[
\frac{n_m + n_i}{s_\ell} = \frac{n_i - n_m}{s_\ell R_1}
\]
For simplicity, assume \( s_\ell \) is negative, i.e., a virtual image.
Since \( |s_\ell| = |s_\ell| + d \), by the sign convention, \( s_\ell = -s_\ell + d \).
Thus, from the second surface,
\[
\frac{n_p + n_m}{-s_\ell} = \frac{n_i - n_m}{s_\ell R_2}
\]
Again, by sign convention, \( R_2 < 0 \). Adding the two equations, we have
\[
\frac{n_m + n_m}{s_\ell} = (n_i - n_m)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + \frac{n \cdot d}{(s_\ell - d)s_\ell}
\]
By thin lens approximation \( (d \to 0) \), we have
\[
\frac{1}{s_o} + \frac{1}{s_i} = \frac{n_i}{n_m} - 1 \left(\frac{1}{R_1} - \frac{1}{R_2}\right)
\]
This is Thin-Lens Equation, or Lensmaker’s Formula.
Since \( \lim_{s_o \to \infty} s_i = \lim_{s_i \to \infty} s_o \), we can write the equation as
\[
\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}
\]
where
\[
\frac{1}{f} = (n_i - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)
\]
f is the focal point.
Focal Points and Planes

Transverse Magnification \[ M_T = \frac{y_i}{y_o} = \frac{s_t}{s_o} = \frac{x_t}{x_o} = \frac{f}{f} \]
TABLE 5.2 Meanings Associated with the Signs of Various Thin Lens and Spherical Interface Parameters

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Sign</th>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_o)</td>
<td>Real object</td>
<td>Real object</td>
<td>Virtual object</td>
</tr>
<tr>
<td>(s_i)</td>
<td>Real image</td>
<td>Real image</td>
<td>Virtual image</td>
</tr>
<tr>
<td>(f)</td>
<td>Converging lens</td>
<td>Converging lens</td>
<td>Diverging lens</td>
</tr>
<tr>
<td>(y_o)</td>
<td>Erect object</td>
<td>Erect object</td>
<td>Inverted object</td>
</tr>
<tr>
<td>(y_i)</td>
<td>Erect image</td>
<td>Erect image</td>
<td>Inverted image</td>
</tr>
<tr>
<td>(M_T)</td>
<td>Erect image</td>
<td>Erect image</td>
<td>Inverted image</td>
</tr>
</tbody>
</table>

TABLE 5.3 Images of Real Objects Formed by Thin Lenses

<table>
<thead>
<tr>
<th>Convex</th>
<th></th>
<th>Image</th>
<th>Location</th>
<th>Type</th>
<th>Location</th>
<th>Orientation</th>
<th>Relative Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\infty &gt; s_o &gt; 2f)</td>
<td>Real</td>
<td>(f &lt; s_i &lt; 2f)</td>
<td>Inverted</td>
<td>Minified</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s_o = 2f)</td>
<td>Real</td>
<td>(s_i = 2f)</td>
<td>Inverted</td>
<td>Same size</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f &lt; s_o &lt; 2f)</td>
<td>Real</td>
<td>(\infty &gt; s_i &gt; 2f)</td>
<td>Inverted</td>
<td>Magnified</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s_o = f)</td>
<td>(\pm \infty)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(s_o &lt; f)</td>
<td>Virtual</td>
<td>(</td>
<td>s_i</td>
<td>&gt; s_o)</td>
<td>Erect</td>
<td>Magnified</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Concave</th>
<th></th>
<th>Image</th>
<th>Location</th>
<th>Type</th>
<th>Location</th>
<th>Orientation</th>
<th>Relative Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anywhere</td>
<td>Virtual</td>
<td>(</td>
<td>s_i</td>
<td>&lt;</td>
<td>f</td>
<td>)</td>
<td>Erect</td>
</tr>
<tr>
<td>Anywhere</td>
<td>(s_o &gt;</td>
<td>s_i</td>
<td>)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Longitudinal Magnification

\[ M_L = \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2 \]
Thin-Lens Combinations

For $L_1$,
\[
\frac{1}{s_{\text{ul}}} = \frac{1}{f_1} - \frac{1}{s_{\text{ol}}}
\]
Let $s_{\text{ol}} > f_1$ and $f_1 > 0$.

For $L_2$
\[
s_{\text{ol}} = d - s_{\text{ul}}
\]
\[
\frac{1}{s_{\text{ol}}} = \frac{1}{f_2} - \frac{1}{s_{\text{ol}}}
\]
Thus,
\[
s_{\text{ol}} = \frac{f_2 d - f_2 s_{\text{ol}} f_1 / (s_{\text{ol}} - f_1)}{d - f_2 s_{\text{ol}} f_1 / (s_{\text{ol}} - f_1)}
\]
and
\[
M_T = M_{T1} M_{T2} = \frac{f_1 s_{\text{ol}}}{d (s_{\text{ol}} - f_1) - s_{\text{ol}} f_1}
\]

Front focal length:
\[
\frac{1}{s_{\text{ol}}}_{\text{fl}} = \frac{1}{f_1} - \frac{1}{d - f_2} - \frac{d (f_1 + f_2)}{f_1 (d - f_2)} = \frac{f_1 (d - f_2)}{d - f_1 + f_2}
\]

Back focal length:
\[
\frac{1}{s_{\text{ol}}}_{\text{bf}} = \frac{1}{f_2} - \frac{1}{d - f_1} - \frac{d (f_1 + f_2)}{f_2 (d - f_1)} = \frac{f_2 (d - f_1)}{d - f_1 + f_2}
\]
If \( d = 0 \), \( f = \frac{b f l}{f f} = \frac{1}{f_2 + \frac{1}{f_1}} \)

**Stops (5.3)**

**Aperture stop**: any element that determines the amount of light reaching the image.

**Field stop**: the element limiting the size or angular breadth of the object that can be imaged by the system.

**Field Stop**: The size of the image plane.

**Pupils**: for determining the light of cone entering the image plane.

**Entrance Pupil**: the image of the aperture stop as seen from an axial point on the object side through those elements before the stop. Determine the cone of the light entering the image plane with respect to the object.

**Exit Pupil**: the image of the aperture stop as seen from an axial point on the image side through those elements before the stop. Determine the cone of the light entering the image plane with respect to the image plane.

\( E_{xp} \): the point at center of the exit pupil.

\( E_{np} \): the point at center of the entrance pupil.

**Chief Ray**: any ray from an off-axis object point that passes through the center of the aperture stop. Either this ray or its extending line passes \( E_{xp} \) or \( E_{np} \).