Geometrical Optics

Refraction at Spherical Surface

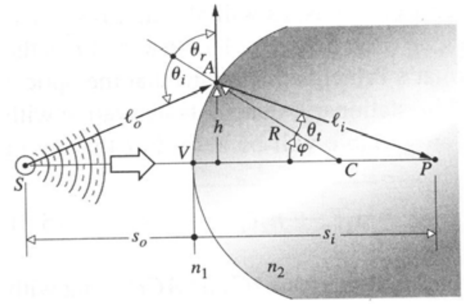


Figure 5.6
Refraction at a spherical interface. Conjugate foci.

Optical Path Length (OPL) from S to P is

$$OPL = n_1 l_0 + n_2 l_i$$

where

$$l_0 = \sqrt{R^2 + (s_o + R)^2 - 2R(S_o + R)\cos\phi}$$
$$l_i = \sqrt{R^2 + (s_i - R)^2 + 2R(S_i - R)\cos\phi},$$

By Fermat's Principle

$$\frac{d(OPL)}{d\phi} = 0 \Rightarrow \frac{n_1 R(s_o + R)\sin\phi}{2l_o} - \frac{n_2 R(s_i - R)\sin\phi}{2l_i} = 0$$

or

$$\frac{n_1}{l_o} + \frac{n_2}{l_i} = \frac{1}{R}\left(\frac{n_2 s_i}{l_i} - \frac{n_1 s_o}{l_o}\right)$$

which means that not all rays from S pass P.

Paraxial Approximation

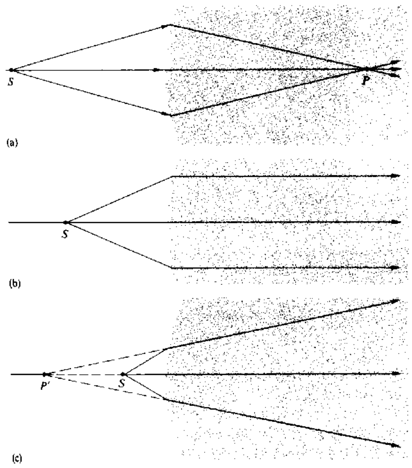
For small ϕ, $l_i \approx s_i$, $l_o \approx s_o$, the equation becomes

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

which is independent of the OPL, so all rays pass through P.

Define object focal length $f_o = \frac{n_1}{n_2 - n_1} R \equiv s_o$ when $s_i = \infty$.

Define image focal length $f_i = \frac{n_2}{n_2 - n_1} R \equiv s_i$ when $s_o = \infty$



Sign conventions

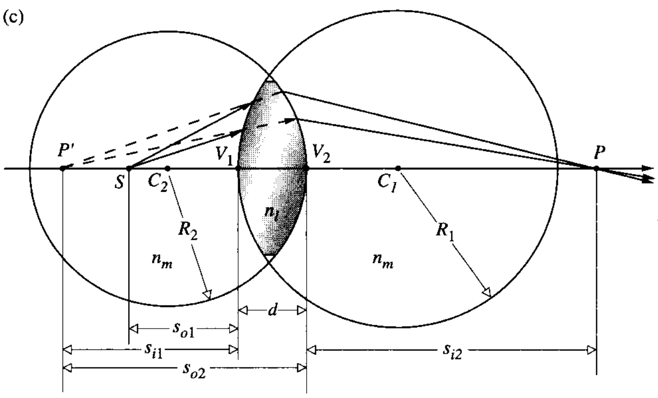
TABLE 5.1 Sign Convention for Spherical Refracting Surfaces and Thin Lenses* (Light Entering from the Left)

s_o, f_o	+ left of V
x_o	+ left of F_o
s_i, f_i	+ right of V
x_i	+ right of F_i
R	+ if C is right of V
y_o, y_i	+ above optical axis

*This table anticipates the imminent introduction of a few quantities not yet spoken of.

Thin Lenses

(c)



From the first surface,

$$\frac{n_m}{s_{o1}}+\frac{n_l}{s_{i1}}=\frac{n_l-n_m}{R_1}.$$

For simplicity, assume s_{i1} is negative, i.e., a virtual image.

Since $|s_{o2}|=|s_{i1}|+d$, by the sign convection, $s_{o2}=-s_{i1}+d$.

Thus, from the second surface,

$$\frac{n_p}{-s_{i1}+d}+\frac{n_m}{s_{i2}}=\frac{n_m-n_l}{R_2}$$

Again, by sign convention, $R_2<0$. Adding the two equations, we have

$$\frac{n_m}{s_{o1}}+\frac{n_m}{si_2}=(n_l-n_m)(\frac{1}{R_1}-\frac{1}{R_2})+\frac{n_l d}{(s_{il}-d)s_{i1}}$$

By thin lens approximation ($d\rightarrow 0$), we have

$$\frac{1}{s_o}+\frac{1}{s_i}=(\frac{n_l}{n_m}-1)(\frac{1}{R_1}-\frac{1}{R_2})$$

This is Thin-Lens Equation, or Lensmaker's Formula.

Since $\lim_{s_o\rightarrow\infty} s_i=f=\lim_{s_i\rightarrow\infty} s_o$, we can write the equation as

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f},$$

where

$$\frac{1}{f}=(\frac{n_l}{n_m}-1)(\frac{1}{R_1}-\frac{1}{R_2}).$$

f is the focal point.

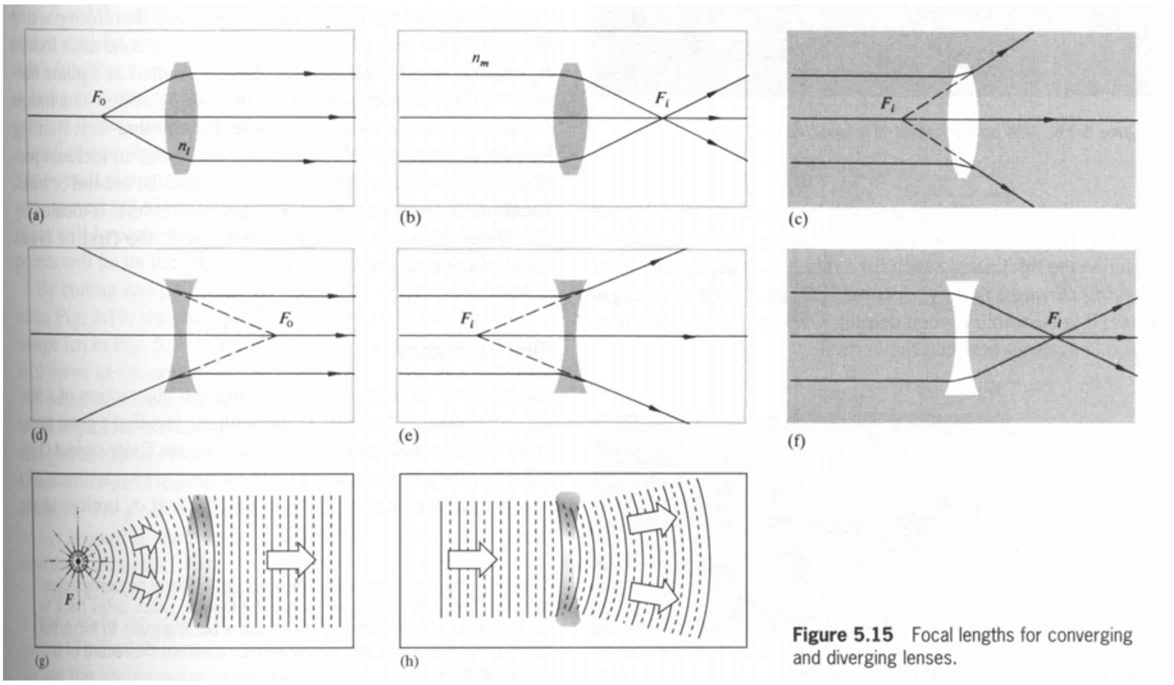


Figure 5.15 Focal lengths for converging and diverging lenses.

Focal Points and Planes

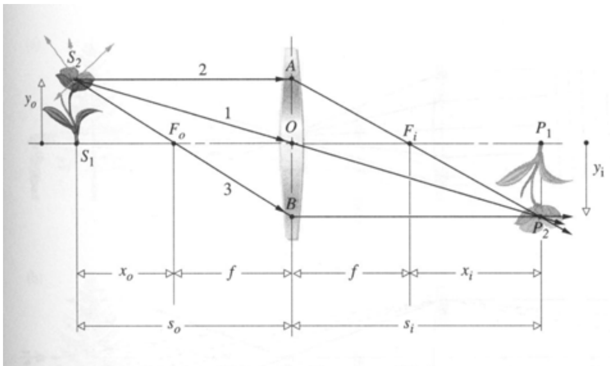


Figure 5.22 Object and image location for a thin lens.

Transverse Magnification $M_T \equiv \frac{y_i}{y_o} = -\frac{s_i}{s_o} = -\frac{x_i}{f} = -\frac{f}{x_o}$

TABLE 5.2 Meanings Associated with the Signs of Various Thin Lens and Spherical Interface Parameters

Quantity	Sign +	Sign −
s_o	Real object	Virtual object
s_i	Real image	Virtual image
f	Converging lens	Diverging lens
y_o	Erect object	Inverted object
y_i	Erect image	Inverted image
M_T	Erect image	Inverted image

TABLE 5.3 Images of Real Objects Formed by Thin Lenses

Convex

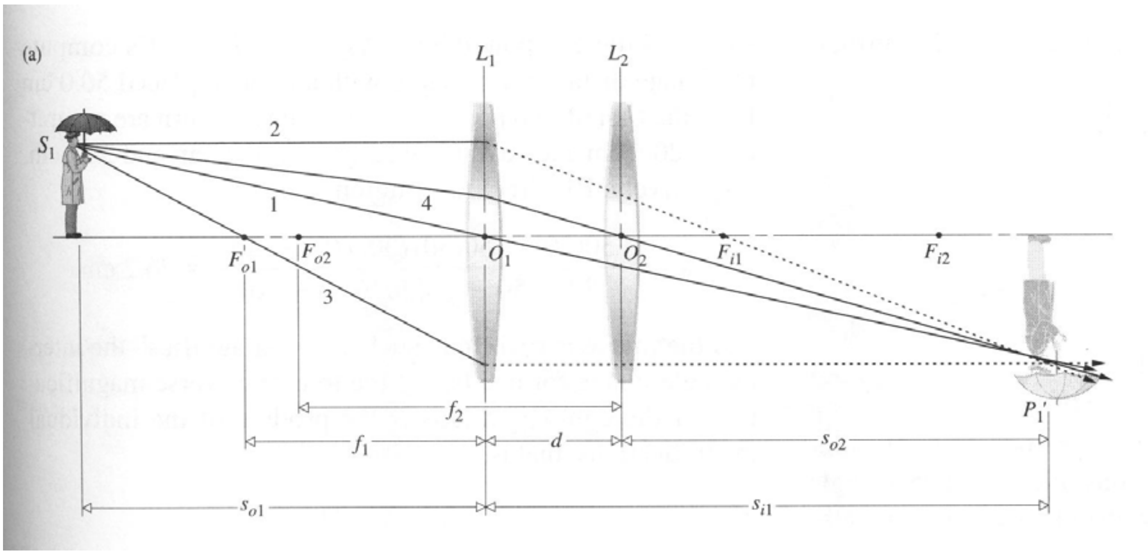
Object Location	Image Type	Image Location	Image Orientation	Image Relative Size
$\infty > s_o > 2f$	Real	$f < s_i < 2f$	Inverted	Minified
$s_o = 2f$	Real	$s_i = 2f$	Inverted	Same size
$f < s_o < 2f$	Real	$\infty > s_i > 2f$	Inverted	Magnified
$s_o = f$		$\pm\infty$		
$s_o < f$	Virtual	$\lvert s_i\rvert > s_o$	Erect	Magnified

Concave

Object Location	Image Type	Image Location	Image Orientation	Image Relative Size
Anywhere	Virtual	$\lvert s_i\rvert < \lvert f\rvert$, $s_o > \lvert s_i\rvert$	Erect	Minified

Longitudinal Magnification

$$M_L \equiv \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$$



Thin-Lens Combinations

For L_1,

$$\frac{1}{s_{i1}}=\frac{1}{f_1}-\frac{1}{s_{o1}}$$

Let $s_{o1}>f_1$ and $f_1>0$.

For L_2

$$s_{o2}=d-s_{i1}$$

$$\frac{1}{s_{i2}}=\frac{1}{f_2}-\frac{1}{s_{o2}}$$

Thus,

$$s_{i2}=\frac{f_2 d-f_2 s_{o1}f_1/(s_{o1}-f_1)}{d-f_2-s_{o1}f_1/(s_{o1}-f_1)}$$

and

$$M_T=M_{T1}M_{T2}=\frac{f_1 s_{i2}}{d(s_{o1}-f_1)-s_{o1}f_1}$$

Front focal length:

$$\left.\frac{1}{s_{o1}}\right|_{s_{i2}=\infty}=\frac{1}{f_1}-\frac{1}{d-f_2}=\frac{d-(f_1+f_2)}{f_1(d-f_2)} \Rightarrow bfl=\frac{f_1(d-f_2)}{d-(f_1+f_2)}$$

Back focal length:

$$\left.\frac{1}{s_{i2}}\right|_{s_{o1}=\infty}=\frac{1}{f_2}-\frac{1}{d-f_1}=\frac{d-(f_1+f_2)}{f_2(d-f_1)} \Rightarrow bfl=\frac{f_2(d-f_1)}{d-(f_1+f_2)}$$

If $d \rightarrow 0$, $f = bfl = ffl = \frac{f_2 f_1}{f_2 + f_1} \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

Stops (5.3)

Aperture stop: any element that determines the amount of light reaching the image.
Field stop: the element limiting the size or angular breadth of the object that can be imaged y the system.

Field Stop: The size of the image plane.

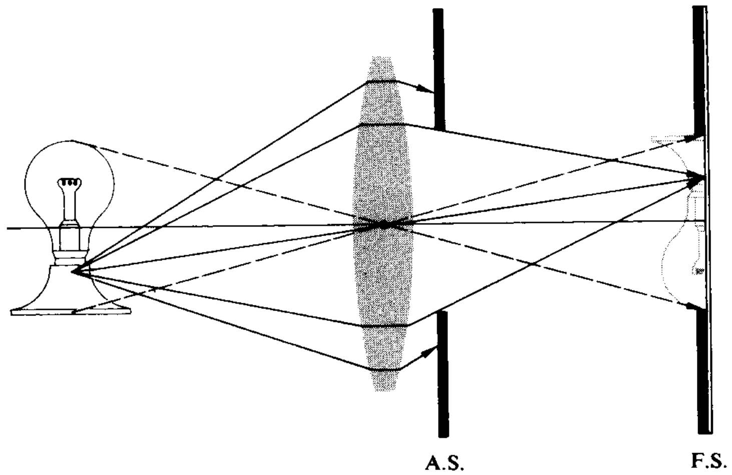


Figure 5.33 Aperture stop and field stop.

Pupils: for determining the light of cone entering the image plane.

Entrance Pupil: the image of the aperture stop as seen from an axial point on the object side through those elements before the stop. Determine the cone of the light entering the image plane with respect to the object.

Exit Pupil: the image of the aperture stop as seen from an axial point on the image side through those elements before the stop. Determine the cone of the light entering the image plane with respect to the image plane.

E_{xp}: the point at center of the exit pupil.
E_{np}: the point at center of the entrance pupil.

Chief Ray: any ray from an off-axis object point that passes through the center of the aperture stop. Either this ray or its extending line passes E_{xp} or E_{np}.