Dispersing Prism [Reading Hecht 5.5]

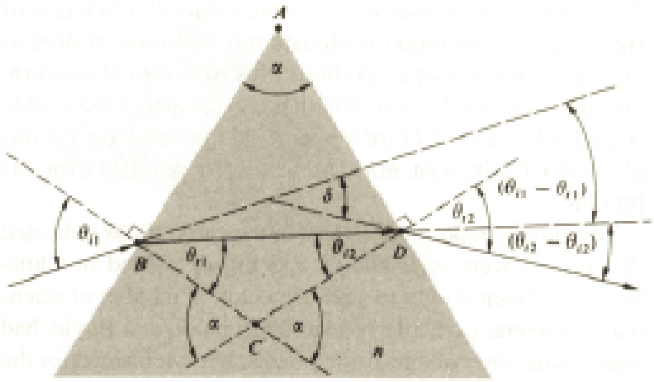
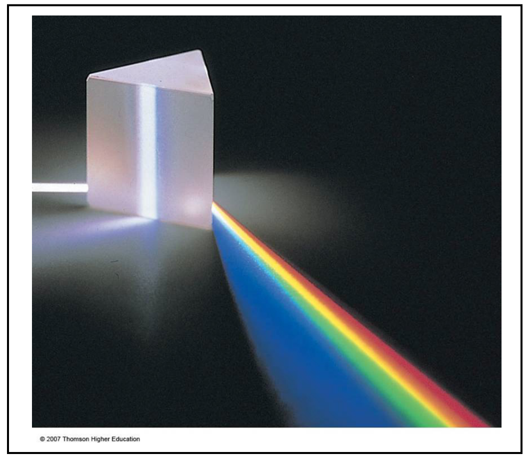


Figure 5.56 Geometry of a dispersing prism.



The total deviation:

$$\delta = (\theta_{i1} - \theta_{t1}) + (\theta_{t2} - \theta_{i2})$$

Since $\alpha = \theta_{t1} + \theta_{i2}$,

$$\delta = \theta_{i1} + \theta_{t2} - \alpha$$

From Snell's law

$$\theta_{t2} = \sin^{-1}(n \sin\theta_{i2}) = \sin^{-1}[(\sin\alpha)(n^2 - \sin^2\theta_{i1})^{1/2} - \sin\theta_{i1}\cos\alpha]$$

The deviation is then

$$\delta = \theta_{i1} + \sin^{-1}[(\sin\alpha)(n^2 - \sin^2\theta_{i1})^{1/2} - \sin\theta_{i1}\cos\alpha] - \alpha$$

δ Increases with n. For visible light, n increases as frequency increases. Therefore, blue light deviates more than red light.

At minimum deviation angle

$$\frac{d\delta}{d\theta_{i1}} = 1 + \frac{d\theta_{t2}}{d\theta_{i1}} = 0 \rightarrow \frac{d\theta_{t2}}{d\theta_{i1}} = -1$$

Also

$$\alpha = \theta_{t1} + \theta_{i2} \rightarrow d\theta_{t1} = -d\theta_{i2}$$

Taking derivatives of Snell's law at the two interface, we get,

$$\cos\theta_{i1} d\theta_{i1} = n\cos\theta_{t1} d\theta_{t1}$$
$$\cos\theta_{t2} d\theta_{t2} = n\cos\theta_{i2} d\theta_{i2}$$

therefore

$$\frac{\cos\theta_{i1}}{\cos\theta_{t2}} = \frac{\cos\theta_{t1}}{\cos\theta_{i2}} \text{ or } \frac{1 - \sin^2\theta_{i1}}{1 - \sin\theta_{t2}} = \frac{n^2 - \sin^2\theta_{i1}}{n^2 - \sin^2\theta_{t2}}$$

Since $n \neq 1$, we have $\theta_{i1} = \theta_{t2}$, and $\theta_{t1} = \theta_{i2}$

At the minimum deviation angle δ_m,

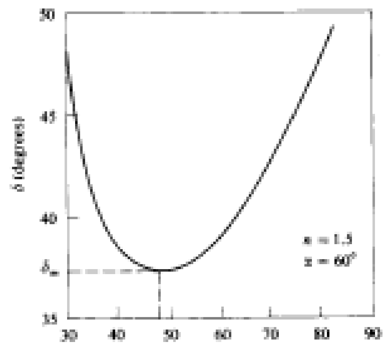
$$\theta_{i1} = (\delta_m + \alpha)/2,$$

$$\theta_{t1} = \alpha/2,$$

and

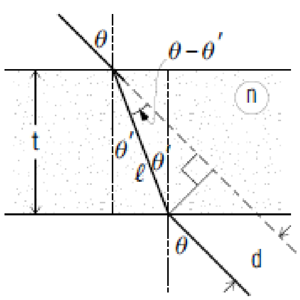
$$n = \frac{\sin[(\delta_m + \alpha)/2]}{\sin\alpha/2}$$

One of the most accurate technique for determining the refractive index.



The deviation increases with decreasing index n. For most materials n increases with decreasing λ. This is the basis for the splitting of white light into colors by the prism.

Lateral displacement through a plane parallel glass plate



$$d = \frac{t\sin\left(\theta - \theta'\right)}{\cos\left(\theta'\right)}$$

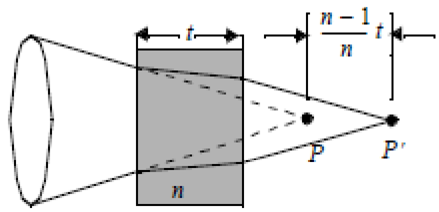
Using the trigonometric identity
$\sin(\theta - \theta') = \sin\theta\cos\theta' - \cos\theta\sin\theta'$;
we get:
$d/t = \sin\theta[1 - \cos\theta/(n\cos\theta')]$
where:
$\cos\theta' = \sqrt{1 - (\sin^2\theta)/n^2}$

This can be used to laterally displace an image. One application of this very simple device is in a specialized high speed camera. The film has to move so fast that it is driven continuously (rather than actually stopping briefly for each frame as in a conventional camera). A rotating plate is used to make the image track the moving film during exposure of a given frame to prevent blur.

But the plate introduces aberrations.

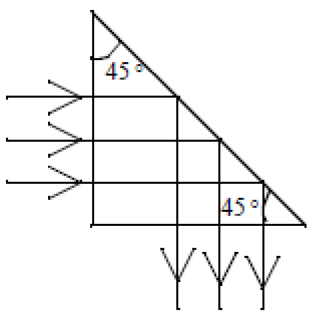
- Chromatic effect: longitudinal and lateral displacements depend on n which is λ dependent.
- For a plate used in convergant or divergent light, the amount of displacement is greater for larger angles which gives spherical aberration.

Plane parallel plate placed in between a lens and its focus:



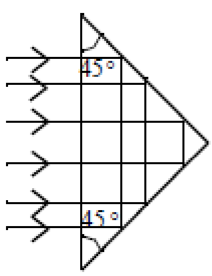
A simple calculation based on the paraxial approximation shows that the focus is displaced by amount $\boxed{\frac{n-1}{n}t}$. However, at steeper incidence angles, the focal shift becomes a function of the incidence angle, which leads to spherical aberration

Right Angle Prism



common building block in non-dispersive prism devices

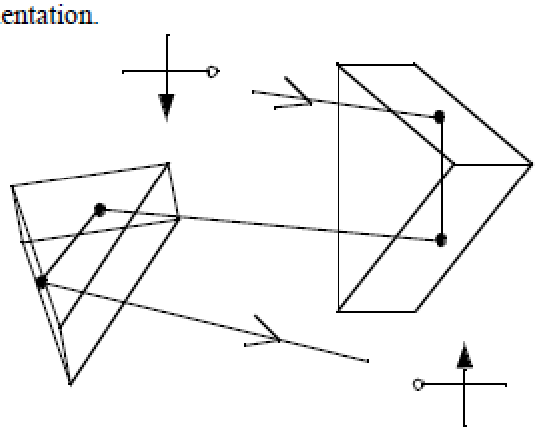
Porro prism



retro reflector (only folds back on itself in one meridian)

Erecting Prisms

Most telescopes produce an inverted image (both U-D, L-R) to the eye. Erecting prisms re-invert the image to the proper orientation.



2 porro prisms used together.

Generally contacted