Thick Lens

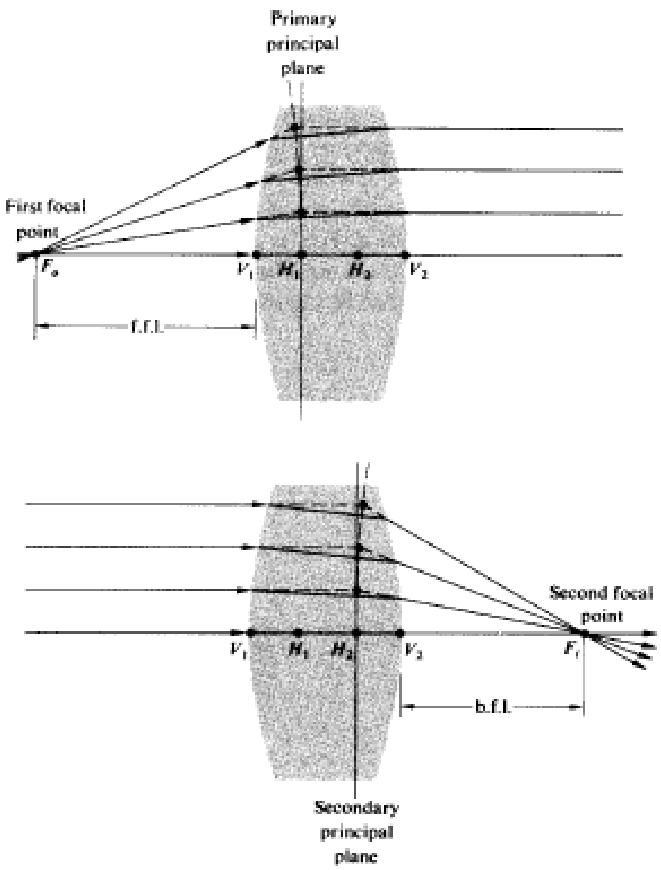


Figure 6.1 A thick lens.

Principle plane: the plane on which the extension lines of the ray incident from the first focus and the ray emerged from the lens intercept.

Secondary Plane: the same as the principle plane except that the ray is from the second focus.

First principal point H_1: the intersection of the **Principle plane** and the optical axis.

Second principal point H_2: the intersection of the **secondary plane** and the optical axis.

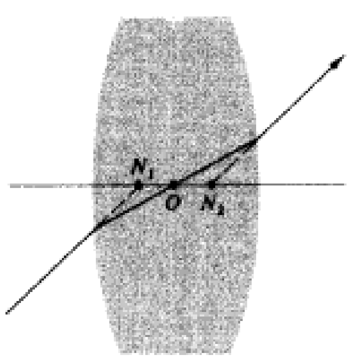


Figure 6.2 Nodal points.

Nodal points N_1 and N_2: the interception of the incident and emerged rays which pass the optical center with optical axis.

Cardinal Points: the two focal, two principal and two nodal points.

Thick Lens Formula

Single Lens

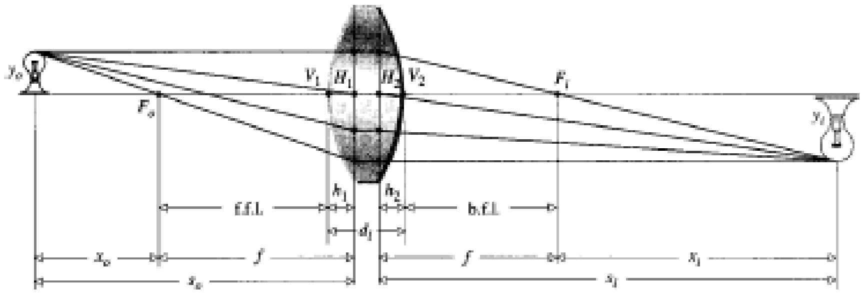


Figure 6.4 Thick-lens geometry.

If consider the thick lens as the combination of two spherical refracting surface separated by a distance d_l, the result is

$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f}$$

$$\frac{1}{f}=(n_l-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n_l-1)d_l}{n_l R_1 R_2}\right]$$

Note that s_o, s_i and f are measured from the first and second principal planes. Also the distance of the principal points and the vertices $\overline{V_1H_1}=h_1$ and $\overline{V_2H_2}=h_2$ are

$$h_1=-\frac{f(n_l-1)d_l}{R_2 n_l}$$

$$h_2=-\frac{f(n_l-1)d_l}{R_1 n_l}$$

which are positive when the principal points lie to the right of their respective vertices.

Dispersion

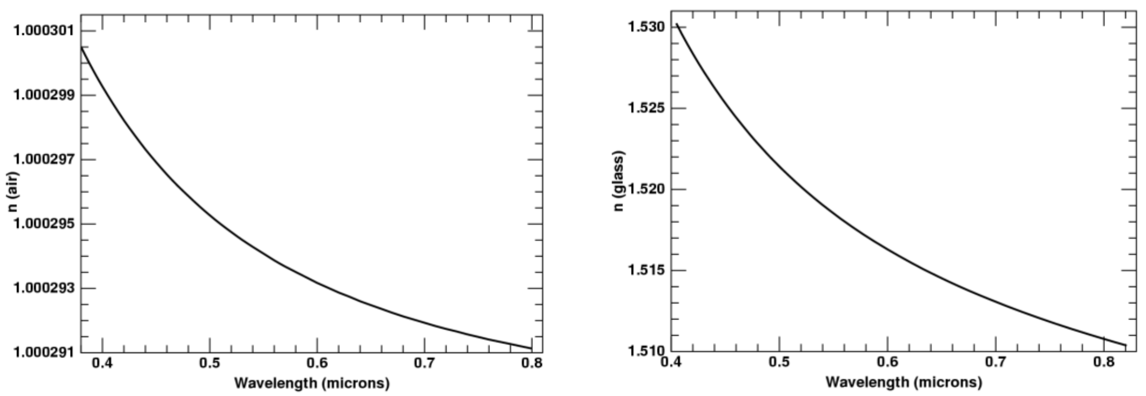
This variation of the *refractive index* with the wavelength or frequency of the light is called **dispersion**. Dispersion is a property of *all* transparent materials.

The color of green flashes is due to the dispersion of air, which makes atmospheric refraction slightly different for different parts of the spectrum. The dispersion of air, like that of water, glass, clear plastics, and most other materials, is small: the refractivity (n - 1) varies by about 1% across the visible spectrum.

Because dispersion is so small, it is negligible for many purposes. Only in special situations is the dispersion of air visible to the naked eye.

Below is the dispersion curves of air at standard conditions and a commonly used optical glass (Schott BK 7) similar to ordinary window glass.

Source: E. R. Peck and K. Reeder, Dispersion of air, JOSA **62**, 958-962 (1972)



Refractive index of air

Refractive index of glass

For 2 thin lenses in contact

$$\frac{1}{f_D} = \frac{1}{f_{D'}} + \frac{1}{f_{D''}}$$

prime: crown glass

double prime: flint glass

define lens power $P = \frac{1}{f}$ with f in meters,

P units are diopters

$$P_D = P_{D'} + P_{D''}$$

$$= (n_{D'} - 1)\left(\frac{1}{r_1{}'} - \frac{1}{r_2{}'}\right) + (n_D{}'' - 1)\left(\frac{1}{r_1{}''} - \frac{1}{r_2{}''}\right)$$

Chromatic Aberration [Reading Hecht 6.3.2]

1. Due to the dependency of the refraction index on frequency. Lights with different wavelengths have different focuses.
2. A·CA: axial chromatic aberration.
3. L·CA: lateral chromatic aberration.

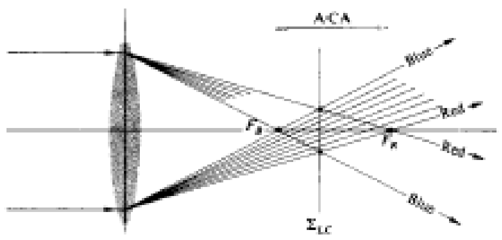


Figure 6.36 Axial chromatic aberration.

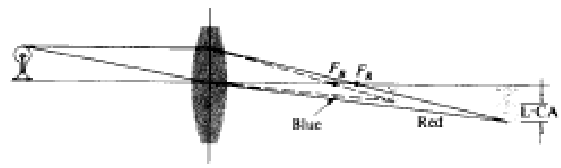


Figure 6.37 Lateral chromatic aberration.

Thin Achromatic Doublets

Purpose: to bring the focus of the red and blue lights together by a combination of two thin lens separated by a distance d.

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

Let

$$\frac{1}{f_1}=(n_1-1)\left(\frac{1}{R_{11}}-\frac{1}{R_{12}}\right)=(n_1-1)\rho_1,\ \frac{1}{f_2}=(n_2-1)\left(\frac{1}{R_{21}}-\frac{1}{R_{22}}\right)=(n_2-1)\rho_2.$$

then

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}=(n_1-1)\rho_1+(n_2-1)\rho_2-d(n_1-1)\rho_1(n_2-1)\rho_2.$$

Let the focus of red light be f_R and blue light f_B. What we want is $\frac{1}{f_R}=\frac{1}{f_B}$.

This leads to

$$(n_{1R}-1)\rho_1+(n_{2R}-1)\rho_2-d(n_{1R}-1)\rho_1(n_{2R}-1)\rho_2=(n_{1B}-1)\rho_1+(n_{2B}-1)\rho_2-d(n_{1B}-1)\rho_1(n_{2B}-1)\rho_2$$

Case 1: Select $d=0$, we have

$$\frac{\rho_1}{\rho_2}=-\frac{n_{2B}-n_{2R}}{n_{1B}-n_{1R}}.$$

Consider yellow as the center of the spectrum from blue to red.

Let the focus of yellow light be f_Y, then

$$\frac{1}{f_{1Y}}=(n_{1Y}-1)\rho_1,\ \frac{1}{f_{2Y}}=(n_{2Y}-1)\rho_2 \rightarrow \frac{\rho_1}{\rho_2}=\frac{(n_{2Y}-1)f_{2Y}}{(n_{1Y}-1)f_{1Y}}.$$

Therefore,

$$\frac{f_{2Y}}{f_{1Y}}=-\frac{(n_{2B}-n_{2R})/(n_{2Y}-1)}{(n_{1B}-n_{1R})/(n_{1Y}-1)}.$$

Definition:

1. Dispersive power: $\frac{n_B-n_R}{n_Y-1}$.
2. Dispersive index, or V-number, or Abbe number: $V=\frac{n_Y-1}{n_B-n_R}$.

Thus,

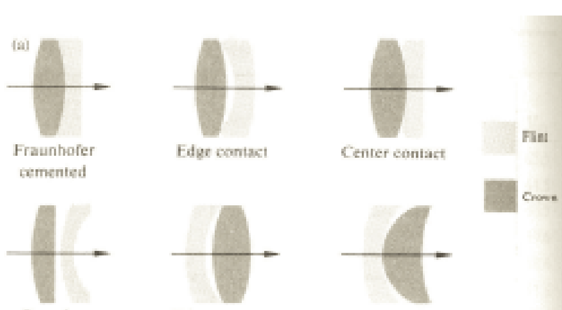
$$\frac{f_{2Y}}{f_{1Y}}=-\frac{V_1}{V_2} \rightarrow f_{1Y}V_1+f_{2Y}V_2=0$$

Case 2: Select $n_1=n_2$, then

$$d=\frac{1}{n_B+n_R-2}\left(\frac{1}{\rho_1}+\frac{1}{\rho_2}\right)=\frac{(f_{1Y}+f_{2Y})(n_Y-1)}{n_B+n_R-2}.$$

If $n_Y=\frac{n_B+n_R}{2}$, we have

$$d=\frac{f_{1Y}+f_{2Y}}{2}$$



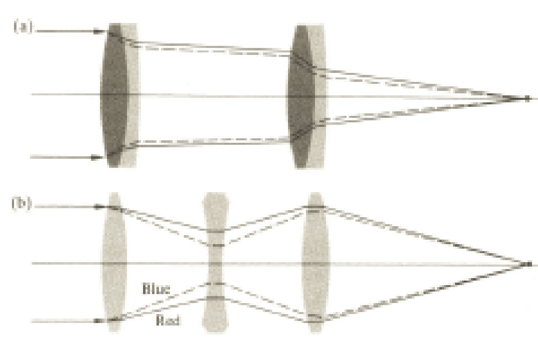


Figure 6.41 Achromatized lenses.

Aberrations (Additional online video resources)

Spherical Aberration: Youtube Clip http://www.youtube.com/watch?v=E85FZ7WLvao&NR=1

Chromatic Aberration: Youtube Clip http://www.youtube.com/watch?v=yOR4WHgRfvI&NR=1

Coma Aberration: Youtube Clip http://www.youtube.com/watch?v=EXmaY2txEBo&NR=1

MiniTutorial of Geometrical Aberrations (Seidel Aberrations):
Youtube clips 1. http://www.youtube.com/watch?v=wzEQX1tMLdY
2. http://www.youtube.com/watch?v=TWGXKj_RIs0