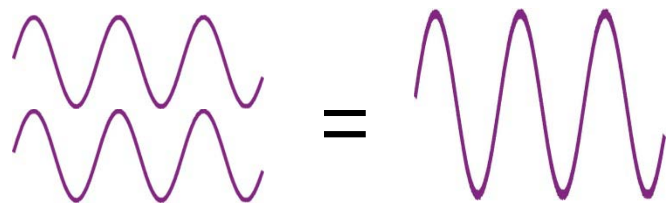
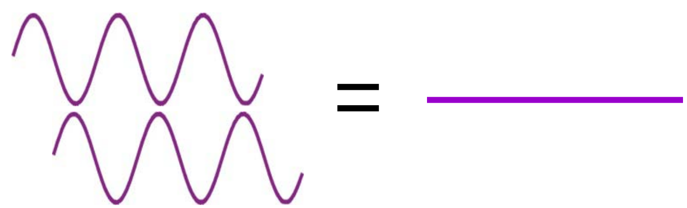
# Constructive vs. destructive interference; Coherent vs. incoherent interference

Waves that combine **in phase** add up to relatively high irradiance.



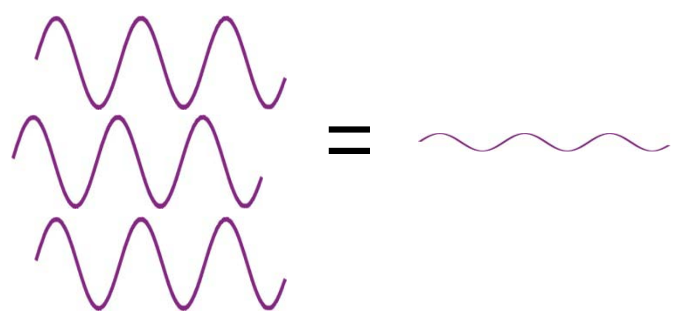
Constructive interference (coherent)

Waves that combine **180° out of phase** cancel out and yield zero irradiance.



Destructive interference (coherent)

Waves that combine with **lots of different phases** nearly cancel out and yield very low irradiance.

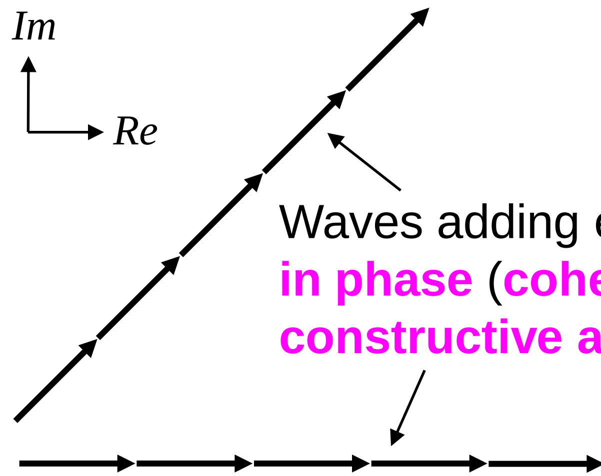


Incoherent addition

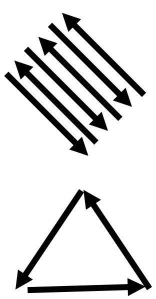
Source: Tribino, Georgia Tech

# Interfering many waves: in phase, out of phase, or with random phase…

If we plot the complex amplitudes:

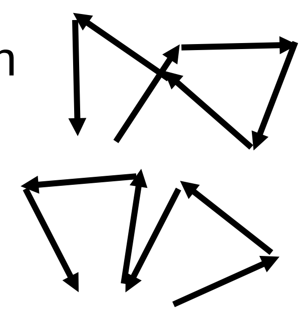



Waves adding exactly **in phase** (**coherent constructive addition**)



Waves adding exactly **out of phase**, adding to zero (**coherent destructive addition**)

Waves adding with **random phase**, partially canceling (**incoherent addition**)



# The Irradiance (intensity) of a light wave

The irradiance of a light wave is proportional to the square of the electric field:

or:

$$\boxed{I = \tfrac{1}{2} c\,\varepsilon \left| \vec{\underset{\sim}{E}}_0 \right|^2}$$

where:

$$\boxed{\left| \vec{\underset{\sim}{E}}_0 \right|^2 = \underset{\sim}{E}_{0x} \underset{\sim}{E}_{0x}^* + \underset{\sim}{E}_{0y} \underset{\sim}{E}_{0y}^* + \underset{\sim}{E}_{0z} \underset{\sim}{E}_{0z}^*}$$

This formula only works when the wave is of the form:

$$\vec{E}(\vec{r},t) = \operatorname{Re} \vec{\underset{\sim}{E}}_0 \exp\left[ i\left( \vec{k} \cdot \vec{r} - \omega t \right) \right]$$

# The relative phases are the key.

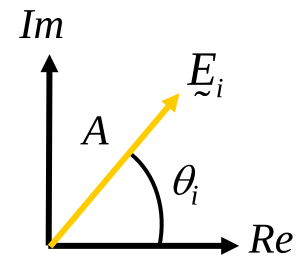
The irradiance (or intensity) of the sum of two waves is:

$$I = I_1 + I_2 + c\varepsilon \,\mathrm{Re}\left\{ \underset{\sim}{E}_1 \cdot \underset{\sim}{E}_2^{\,*} \right\}$$

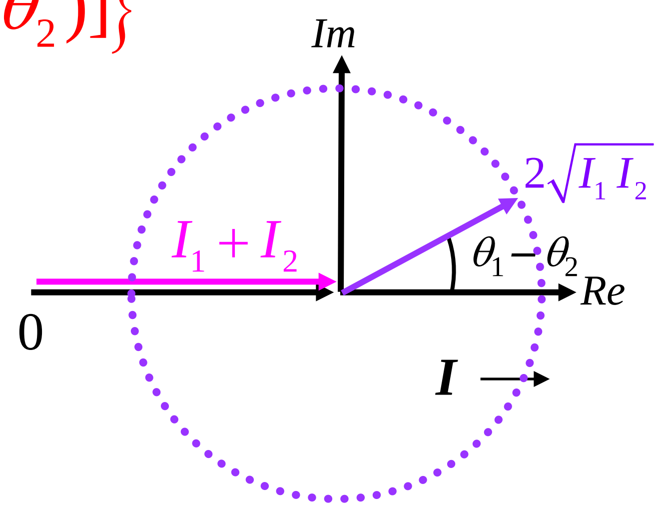
$\underset{\sim}{E}_1$ and $\underset{\sim}{E}_2$ are complex amplitudes.

If we write the amplitudes in terms of their intensities, $I_i$, and absolute phases, $\theta_i$,

$$\underset{\sim}{E}_i \propto \sqrt{I_i}\, \exp[-i\theta_i]$$




$$I = I_1 + I_2 + 2\sqrt{I_1\, I_2}\, \mathrm{Re}\left\{ \exp[-i(\theta_1 - \theta_2)] \right\}$$




Imagine adding many such fields.
In coherent interference, the $\theta_i - \theta_j$ will all be known.

In incoherent interference, the $\theta_i - \theta_j$ will all be random.

# Adding many fields with random phases

We find:

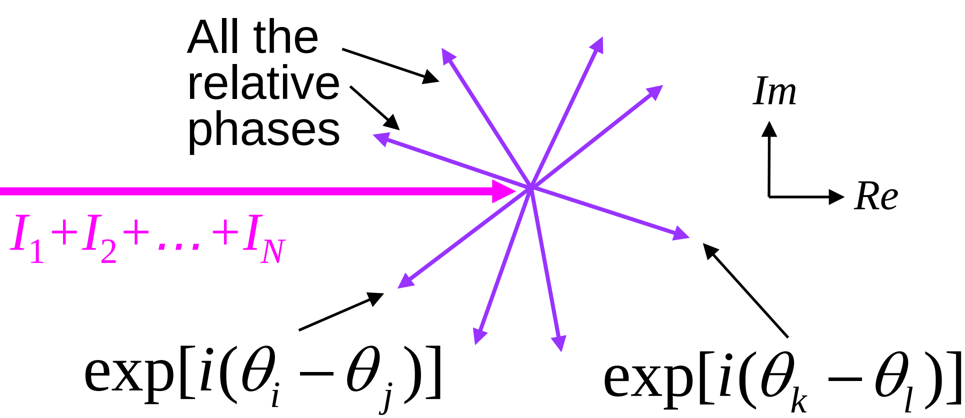
$$\underset{\sim}{E}_{total} = [\underset{\sim}{E}_1 + \underset{\sim}{E}_2 + \ldots + \underset{\sim}{E}_N]\,\exp[i(\vec{k}\cdot\vec{r} - \omega t)]$$

$$I_{total} = \underbrace{I_1 + I_2 + \ldots + I_N}\, + \underbrace{c\varepsilon\,\mathrm{Re}\left\{\underset{\sim}{E}_1\underset{\sim}{E}_2^* + \underset{\sim}{E}_1\underset{\sim}{E}_3^* + \ldots + \underset{\sim}{E}_{N-1}\underset{\sim}{E}_N^*\right\}}$$

$I_1, I_2, \ldots I_n$ are the irradiances of the various beamlets. They're all positive real numbers and they add.

$E_i\,E_j^*$ are cross terms, which have the phase factors: $\exp[i(\theta_i\text{-}\theta_j)]$. **When the $\theta$'s are random, they cancel out!**

$$\boxed{I_{total} = I_1 + I_2 + \ldots + I_n}$$

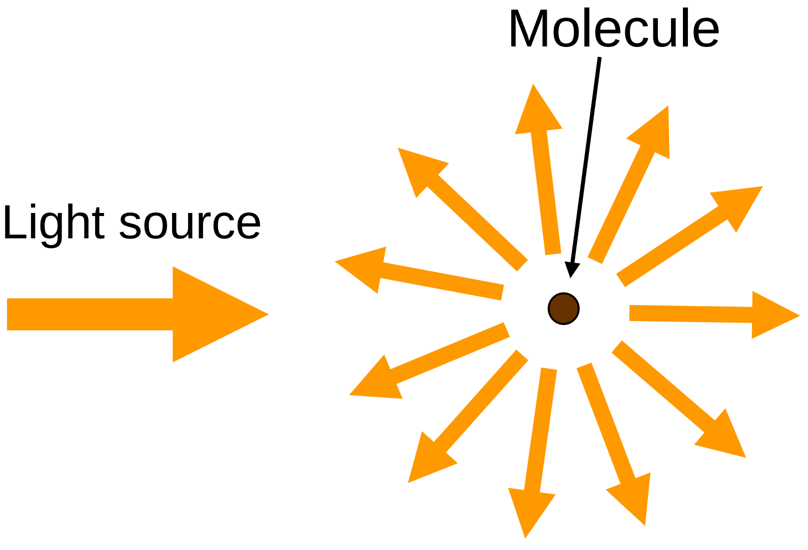



The intensities simply add!
Two 20W light bulbs yield 40W.

# Scattering

When a wave encounters a small object, it not only re-emits the wave in the forward direction, but it also re-emits the wave in all other directions.

This is called **scattering**.

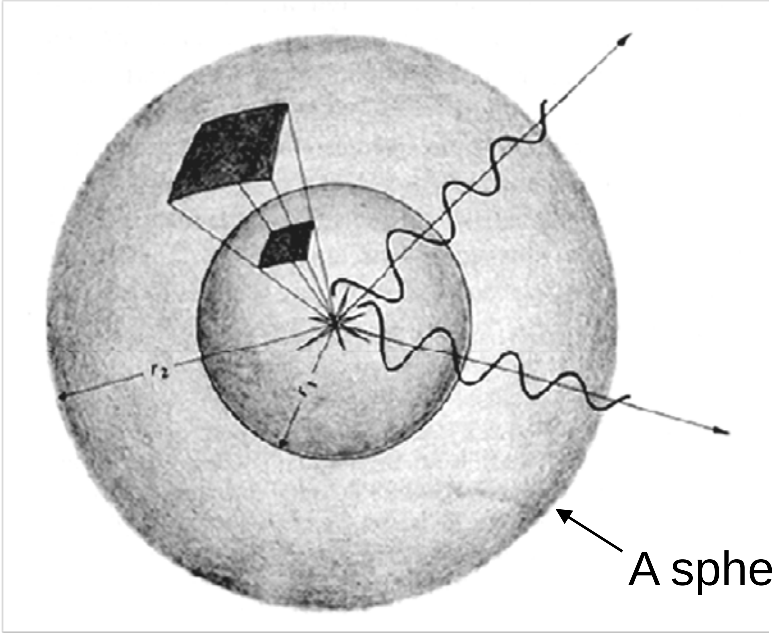



Scattering is everywhere. All molecules scatter light. Surfaces scatter light. Scattering causes milk and clouds to be white and water to be blue.  It is the basis of nearly all optical phenomena.

**Scattering can be coherent or incoherent.**

# Spherical waves

A spherical wave is also a solution to Maxwell's equations and is a good model for the light scattered by a molecule.




Note that $k$ and $r$ are **not** vectors here!

$$E(\vec{r},t) \propto \left(E_0 / r\right) \mathrm{Re}\{\exp[i(kr - \omega t)]\}$$

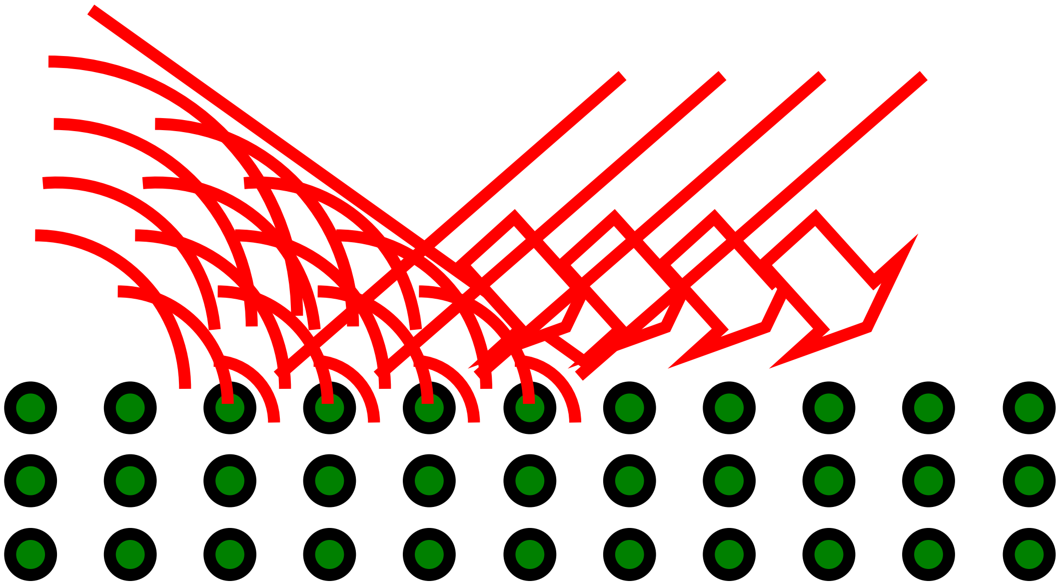
where *k* is a scalar, and *r* is the radial magnitude.

A spherical wave has spherical wave-fronts.

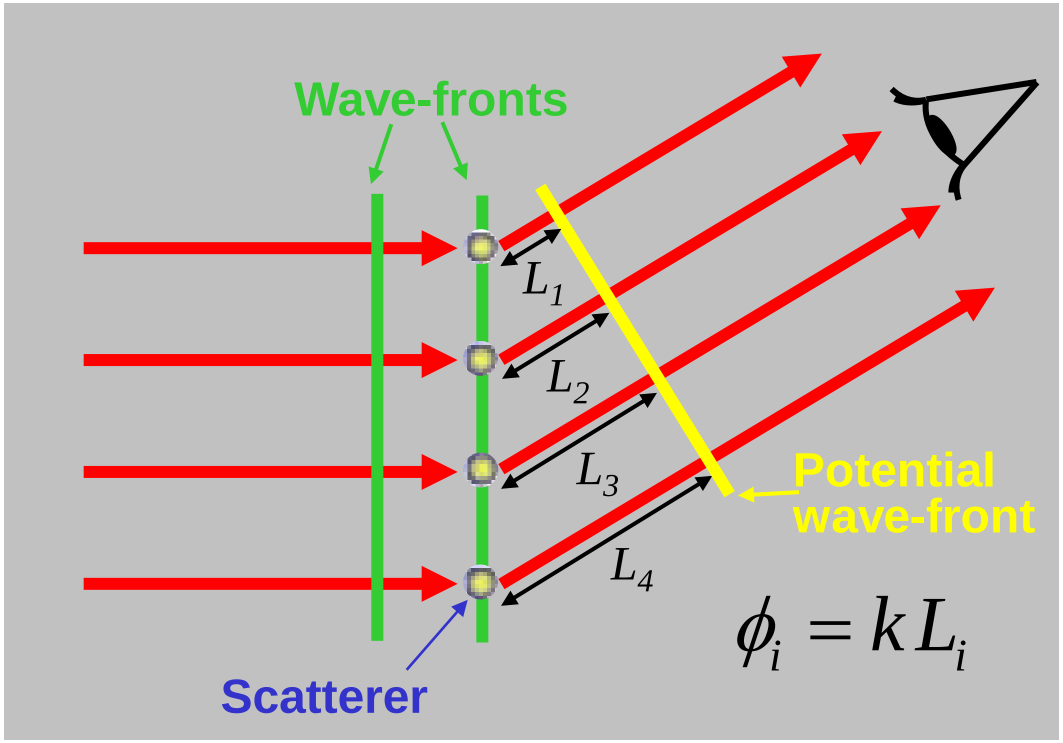
Unlike a plane wave, whose amplitude remains constant as it propagates, a spherical wave weakens. Its irradiance goes as $1/r^2$.

# Scattered spherical waves often combine to form plane waves.

A plane wave impinging on a surface (that is, lots of very small closely spaced scatterers!) will produce a reflected plane wave because all the spherical wavelets interfere constructively along a flat surface.



# To determine interference in a given situation, we compute *phase delays*.

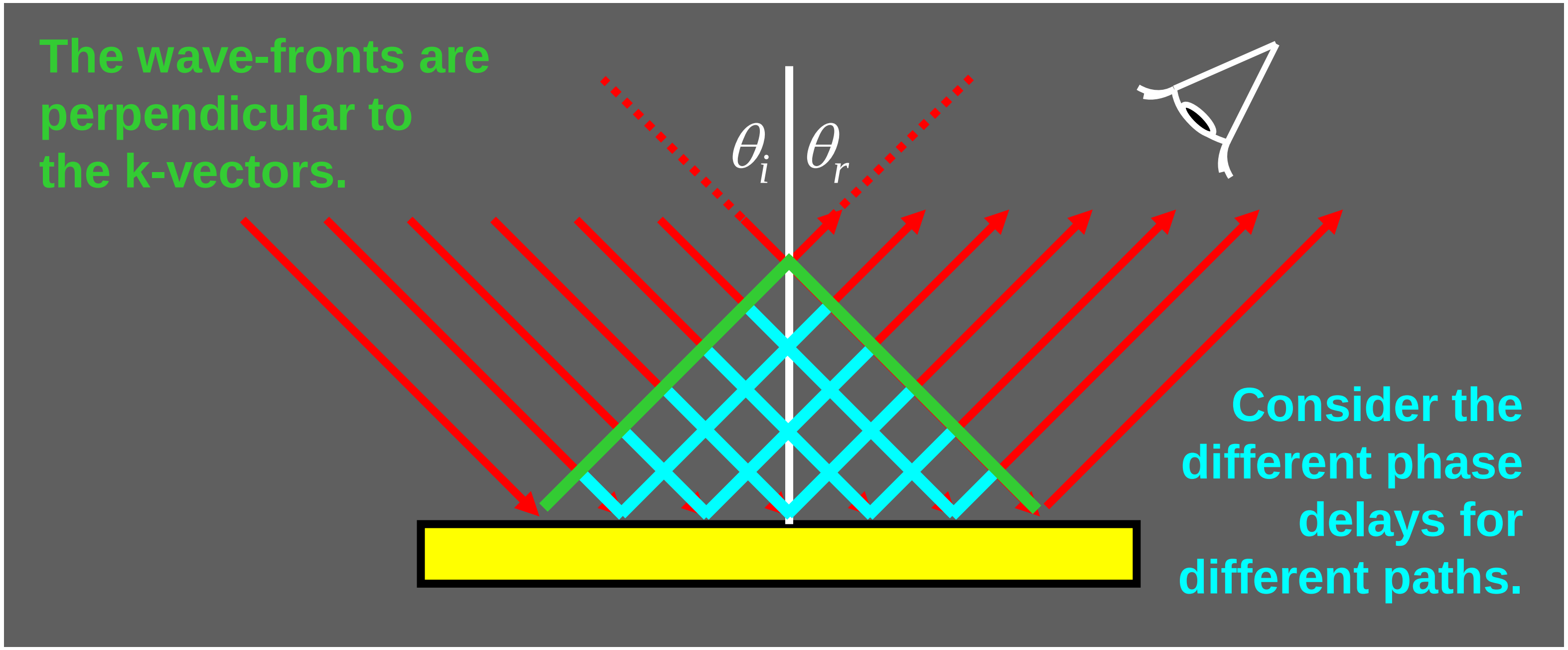



constructive coherent
destructive coherent

incoherent

# Coherent constructive scattering: Reflection from a smooth surface when angle of incidence equals angle of reflection

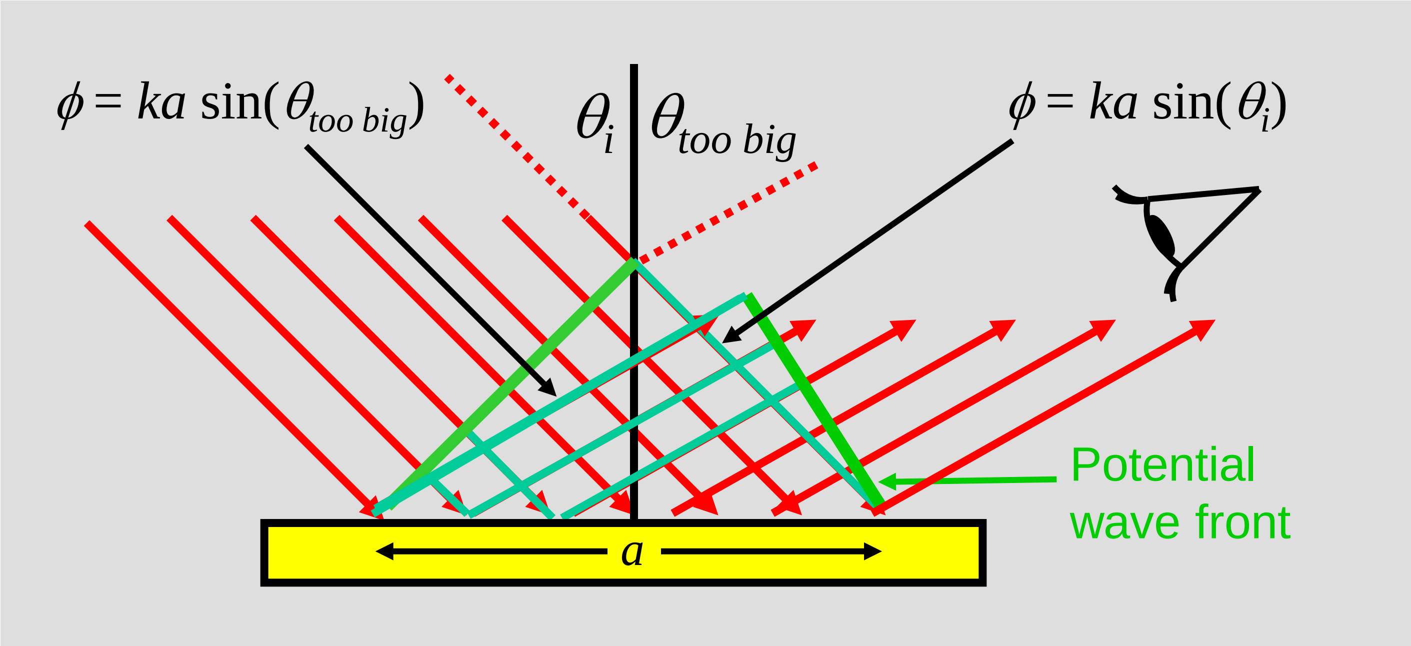
A beam can only remain a plane wave if there's a direction for which coherent constructive interference occurs.




Coherent constructive interference occurs for a reflected beam if the angle of incidence = the angle of reflection: $\theta_i = \theta_r$.

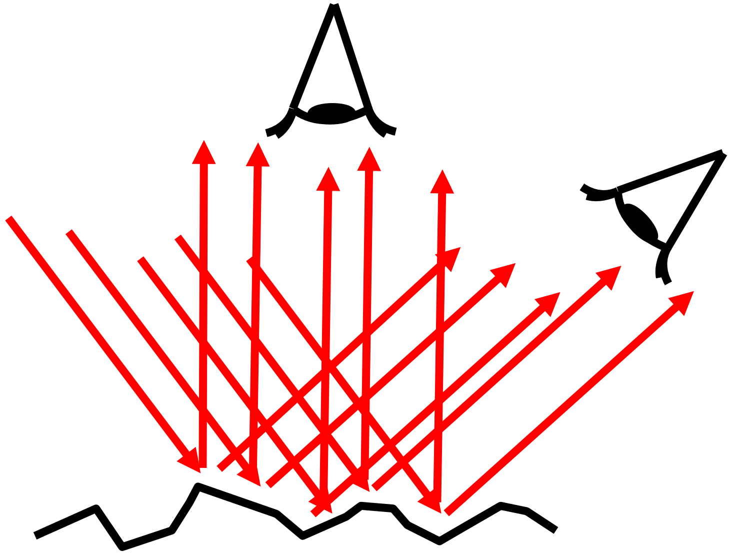
# Coherent destructive scattering: Reflection from a smooth surface when the angle of incidence is not the angle of reflection

Imagine that the reflection angle is too big.
The symmetry is now gone, and the phases are now all different.




Coherent destructive interference occurs for a reflected beam direction if the angle of incidence ≠ the angle of reflection: $\theta_i \neq \theta_r$.

# Incoherent scattering: reflection from a rough surface

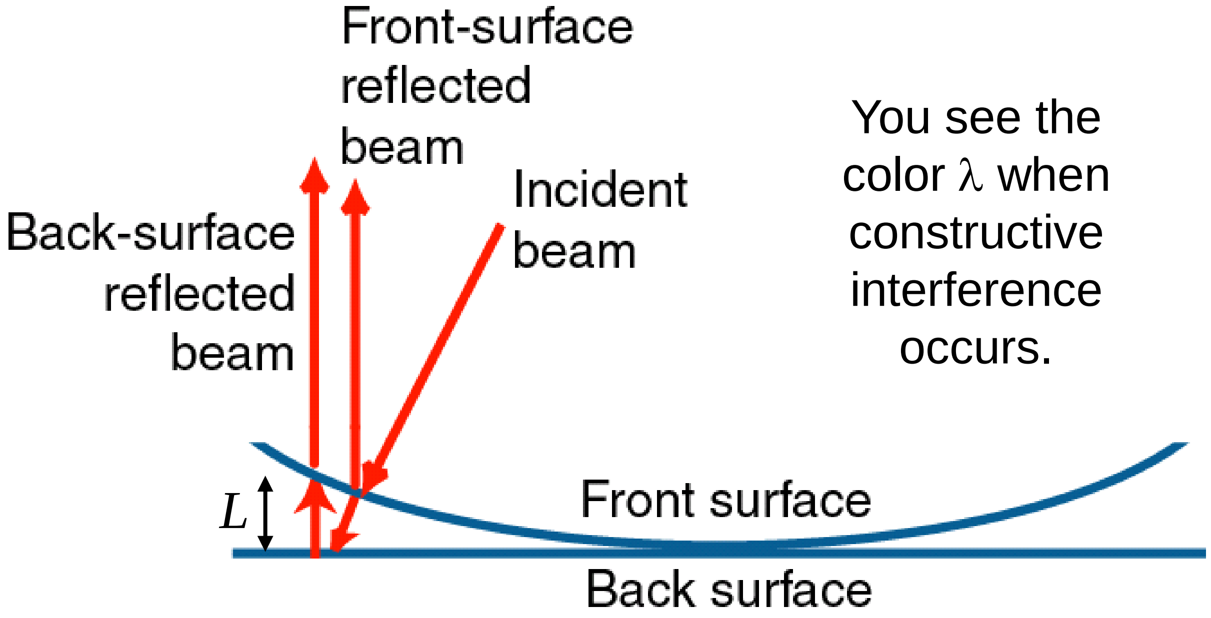


No matter which direction we look at it, each scattered wave from a rough surface has a different phase. So scattering is incoherent, and we'll see weak light in all directions.

This is why rough surfaces look different from smooth surfaces and mirrors.

# Newton's Rings

Get constructive interference when an integral number of half wavelengths occur between the two surfaces (that is, when an integral number of full wavelengths occur between the path of the transmitted beam and the twice reflected beam).

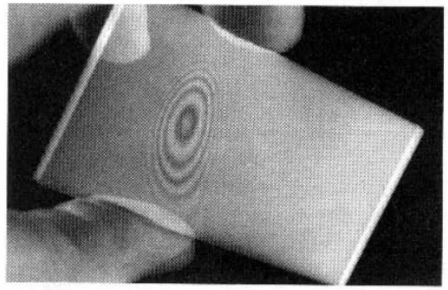



You see the color $\lambda$ when constructive interference occurs.

You only see bold colors when m = 1 (possibly 2). Otherwise the variation with $\lambda$ is too fast for the eye to resolve.

This effect also causes the colors in bubbles and oil films on puddles.

# Newton's Rings



Newton's rings with two microscope slides. The thin film of air between the slides creates the interference pattern. (Photo by E. H.)

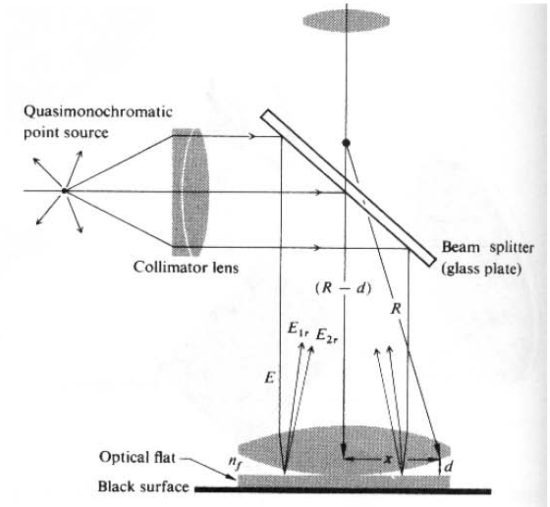



**Figure 9.23** A standard setup to observe Newton's rings

**Newton's Rings**

From the figure, if $R \gg d$, then

$$x^2+(R-d)^2=R^2 \Rightarrow x^2 \approx 2Rd$$

The interference maximum will occur if

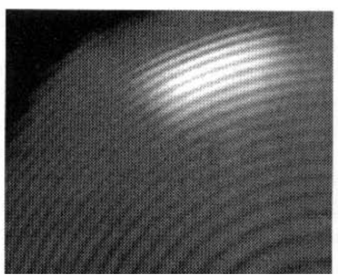
$$2n_f d_m=(m+\frac{1}{2})\lambda_0$$

Thus, the radius of the bring rings are

$$x_m=\sqrt{(m+\frac{1}{2})\lambda_f R}$$

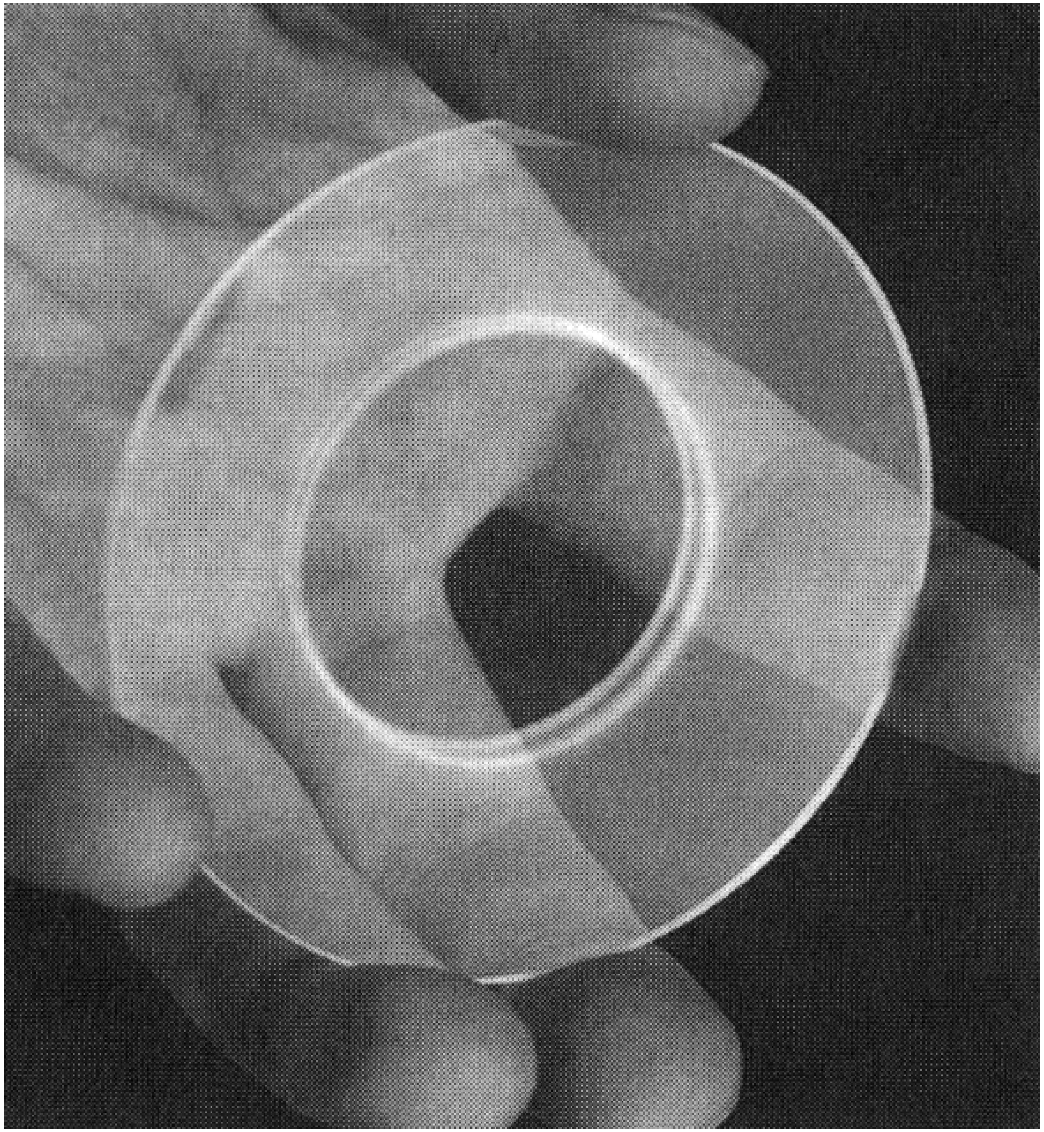
Similarly, the radius of dark rings are

$$x_m=\sqrt{m\lambda_f R}$$



Animation - [http://extraphysics.com/java/models/newtRings.html](http://extraphysics.com/java/models/newtRings.html)

# Anti-reflection Coatings



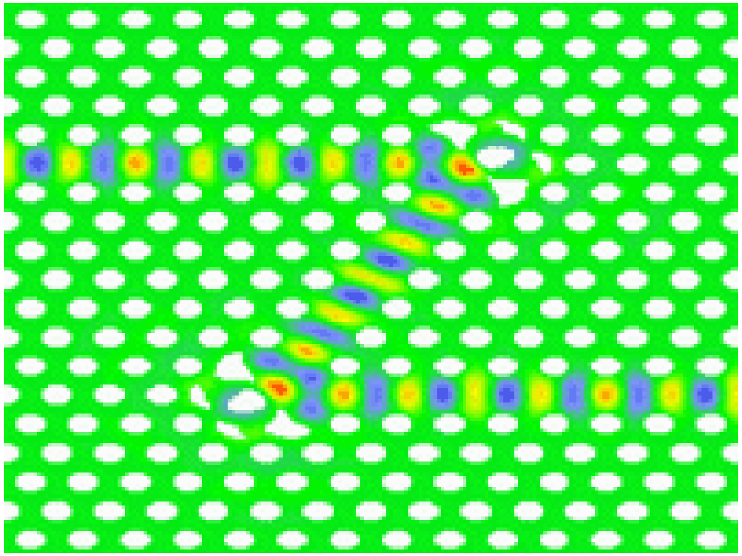
Notice that the center of the round glass plate looks like it's missing. It's not!  There's an **anti-reflection coating** there (on both the front and back of the glass).

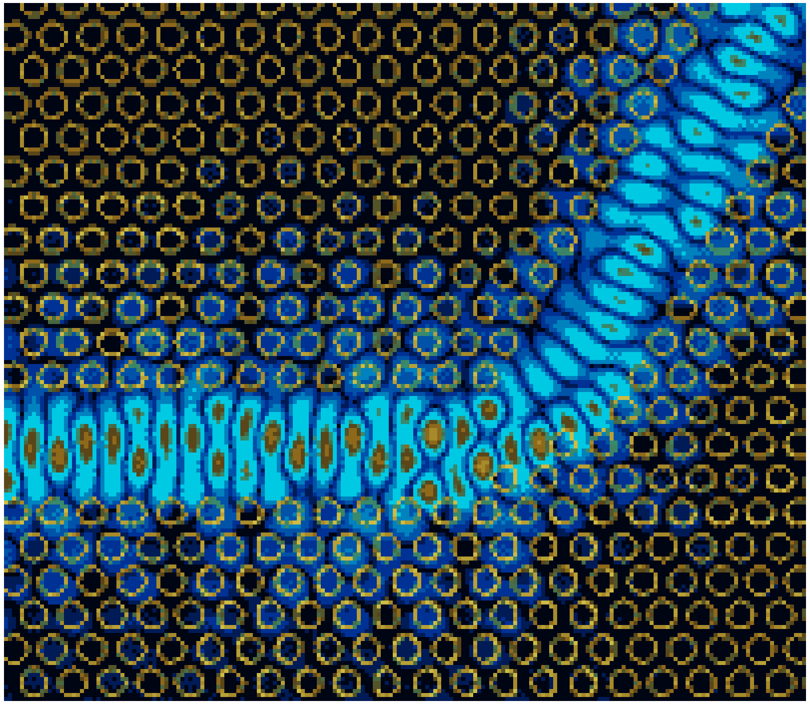
Such coatings have been common on photography lenses and are now common on eyeglasses. Even my new watch is AR-coated!

# Photonic crystals use interference to guide light—sometimes around corners!

Borel, et al., Opt. Expr. 12, 1996 (2004)

Yellow indicates peak field regions.





Augustin, et al., Opt. Expr., 11, 3284, 2003.

Interference controls the path of light. Constructive interference occurs along the desired path.

# The irradiance when combining a beam with a delayed replica of itself has fringes.

The irradiance is given by:

$$I = I_1 + c\varepsilon\,\mathrm{Re}\left\{\tilde{E}_1 \cdot \tilde{E}_2^{\,*}\right\} + I_2$$

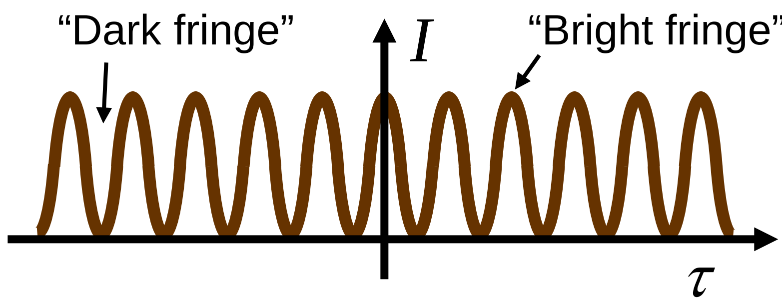
Suppose the two beams are $E_0 \exp(i\omega t)$ and $E_0 \exp[i\omega(t\text{-}\tau)]$, that is, a beam and itself delayed by some time $\tau$:

$$I = 2I_0 + c\varepsilon\,\mathrm{Re}\left\{\tilde{E}_0 \cancel{\exp[i\omega t]} \cdot \tilde{E}_0^* \exp[-i\omega(\cancel{t} - \tau)]\right\}$$

$$= 2I_0 + c\varepsilon\,\mathrm{Re}\left\{\left|\tilde{E}_0\right|^2 \exp[i\omega\tau]\right\}$$

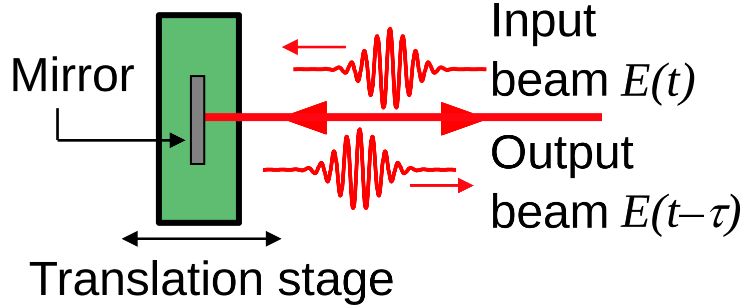
$$= 2I_0 + c\varepsilon\left|\tilde{E}_0\right|^2 \cos[\omega\tau]$$

$$I = 2I_0 + 2I_0\cos[\omega\tau]$$



# Varying the delay on purpose

Simply moving a mirror can vary the delay of a beam by many wavelengths.




Moving a mirror backward by a distance $L$ yields a delay of:

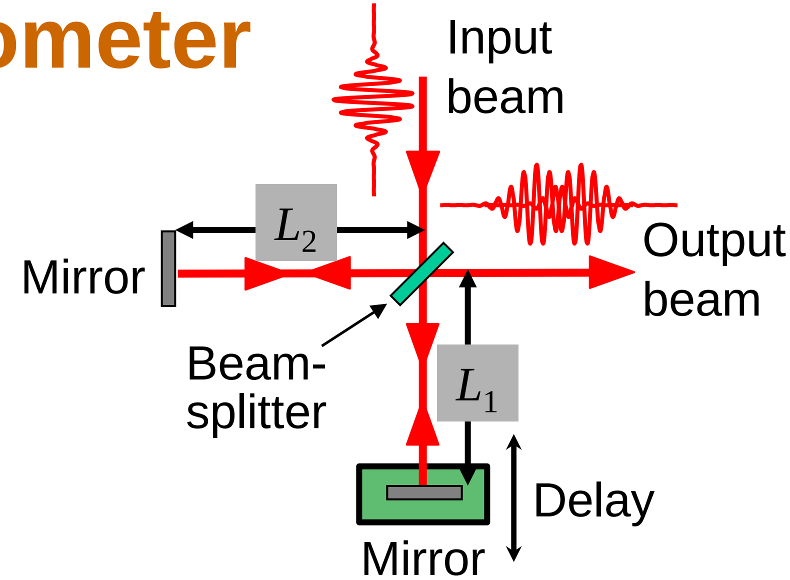
$$\boxed{\tau = 2L/c}$$

**Do not forget the factor of 2!**
**Light must travel the extra distance to the mirror—and back!**

$$\boxed{\omega\tau = 2\omega L/c = 2kL}$$

Since light travels 300 µm per ps, 300 µm of mirror displacement yields a delay of 2 ps.  Such delays can come about naturally, too.

# The Michelson Interferometer




The Michelson Interferometer splits a beam into two and then recombines them at the same beam splitter.

Suppose the input beam is a plane wave:

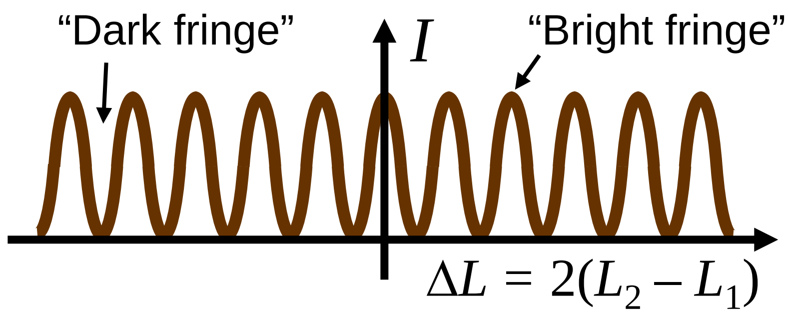
$$I_{out} = I_1 + I_2 + c\varepsilon \,\mathrm{Re}\left\{E_0 \exp\left[i(\cancel{\omega t - kz} - 2kL_1)\right] E_0^* \exp\left[-i(\cancel{\omega t - kz} - 2kL_2)\right]\right\}$$

$$= I + I + 2I \,\mathrm{Re}\left\{\exp\left[2ik(L_2 - L_1)\right]\right\} \qquad \text{since } I \equiv I_1 = I_2 = (c\varepsilon_0/2)\left|E_0\right|^2$$

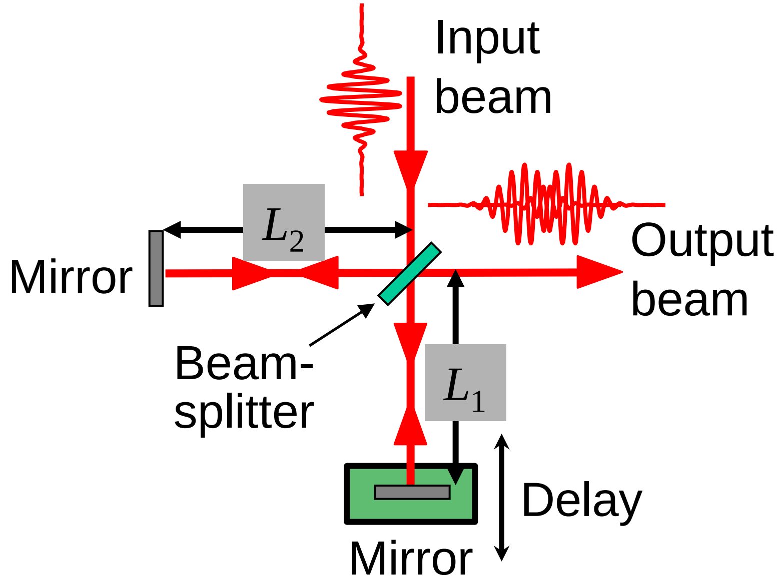
$$= 2I\left\{1 + \cos(k\Delta L)\right\}$$

where: $\Delta L = 2(L_2 - L_1)$

**Fringes (in delay):**



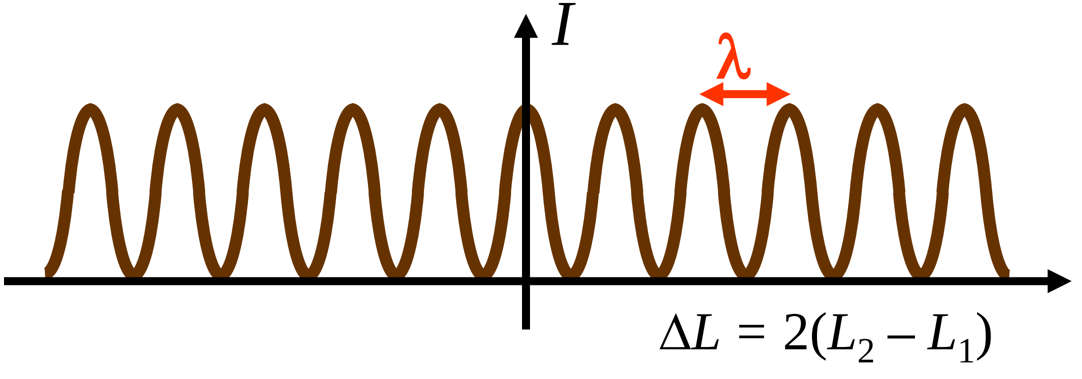
# The Michelson Interferometer




The most obvious application of the Michelson Interferometer is to measure the wavelength of monochromatic light.

$$I_{out} = 2I\left\{1+\cos(k\Delta L)\right\} = 2I\left\{1+\cos(2\pi\,\Delta L/\lambda)\right\}$$

**Fringes (in delay)**

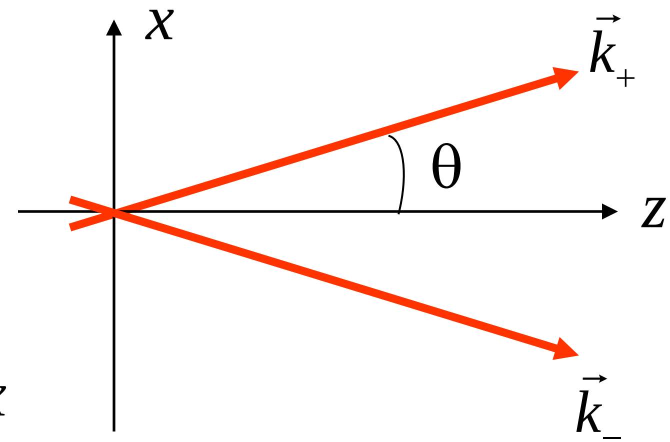


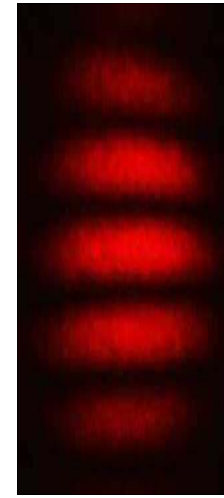
# Crossed Beams

$$\vec{k}_+ = k\cos\theta\,\hat{z} + k\sin\theta\,\hat{x}$$
$$\vec{k}_- = k\cos\theta\,\hat{z} - k\sin\theta\,\hat{x}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\Rightarrow \vec{k}_+ \cdot \vec{r} = k\cos\theta\, z + k\sin\theta\, x$$
$$\vec{k}_- \cdot \vec{r} = k\cos\theta\, z - k\sin\theta\, x$$



$$I = 2I_0 + c\varepsilon\,\mathrm{Re}\left\{E_0 \exp[i(\omega t - \vec{k}_+ \cdot \vec{r})]E_0^* \exp[-i(\omega t - \vec{k}_- \cdot \vec{r})]\right\}$$

Cross term is proportional to:

$$\mathrm{Re}\left\{E_0 \exp\left[i(\cancel{\omega t} - \cancel{kz\cos\theta} - kx\sin\theta\right]E_0^* \exp\left[-i(\cancel{\omega t} - \cancel{kz\cos\theta} + kx\sin\theta\right]\right\}$$

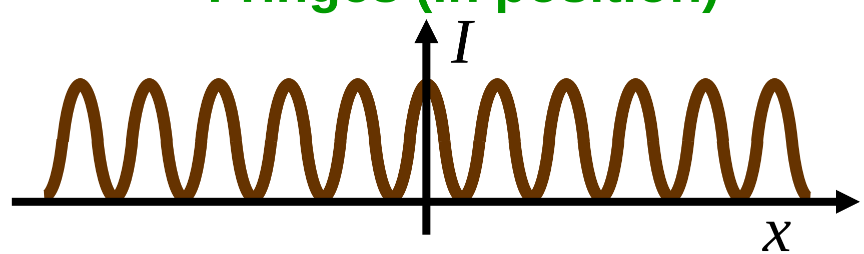
$$\propto \mathrm{Re}\left\{\left|E_0\right|^2 \exp\left[-2ikx\sin\theta\right]\right\}$$

$$\propto \left|E_0\right|^2 \cos(2kx\sin\theta)$$

Fringe spacing: $\Lambda = 2\pi/(2k\sin\theta)$

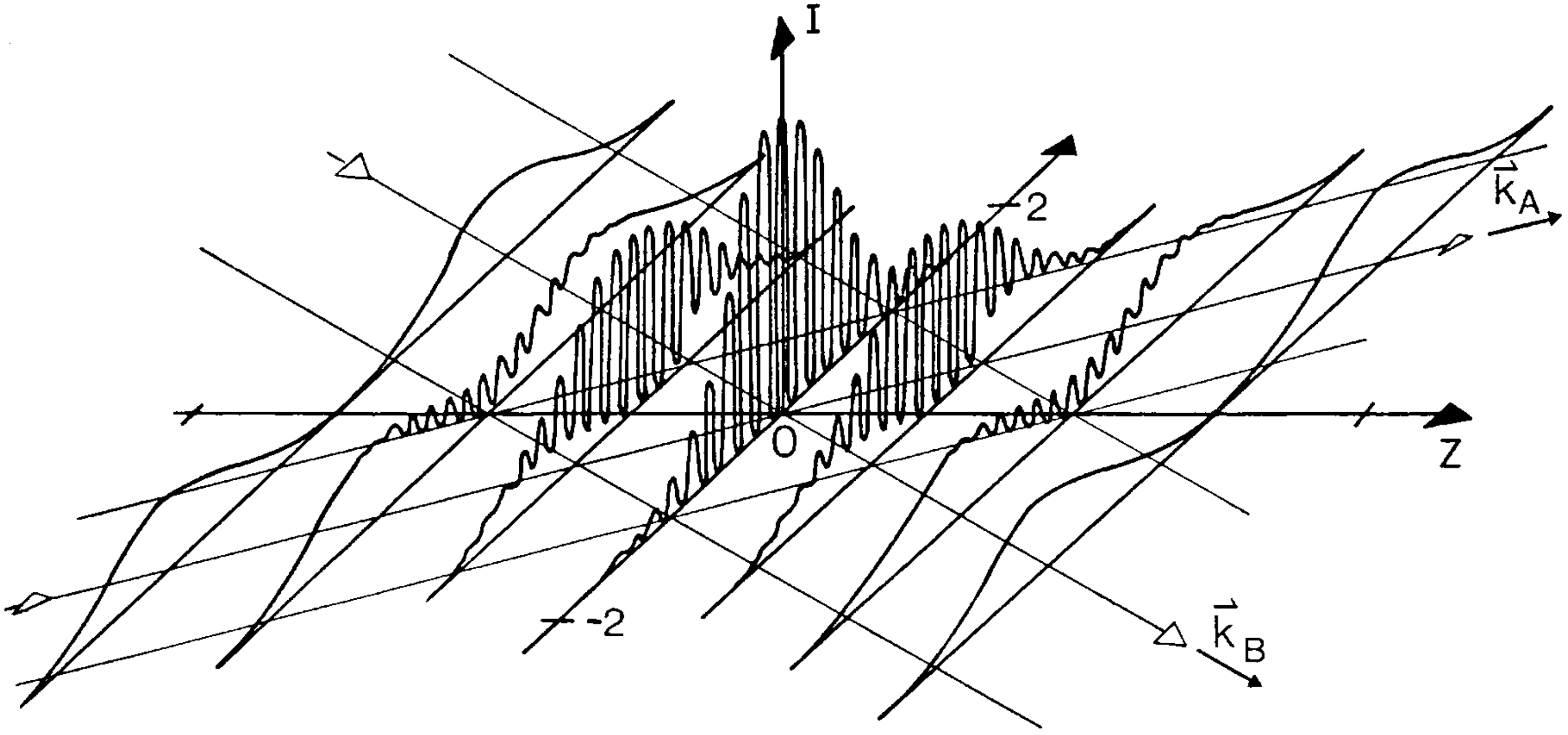
$= \lambda/(2\sin\theta)$

**Fringes (in position)**



# Irradiance vs. position for crossed beams

Fringes occur where the beams overlap in space and time.



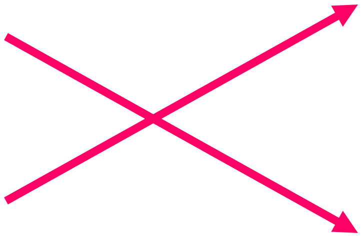
# Big angle: small fringes.
# Small angle: big fringes.

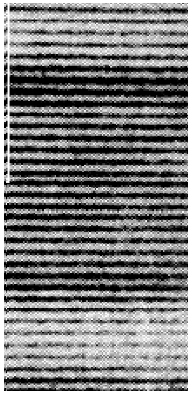
The fringe spacing, $\Lambda$:

$$\Lambda = \lambda / (2 \sin\theta)$$

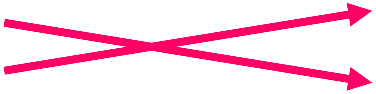
As the angle decreases to zero, the fringes become larger and larger, until finally, at $\theta = 0$, the intensity pattern becomes constant.

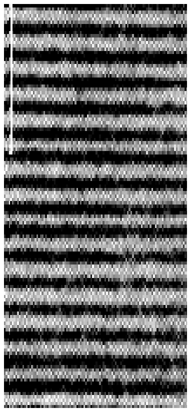
Large angle:





Small angle:





# You can't see the spatial fringes unless the beam angle is very small!

The fringe spacing is:

$$\Lambda = \lambda / (2 \sin \theta)$$

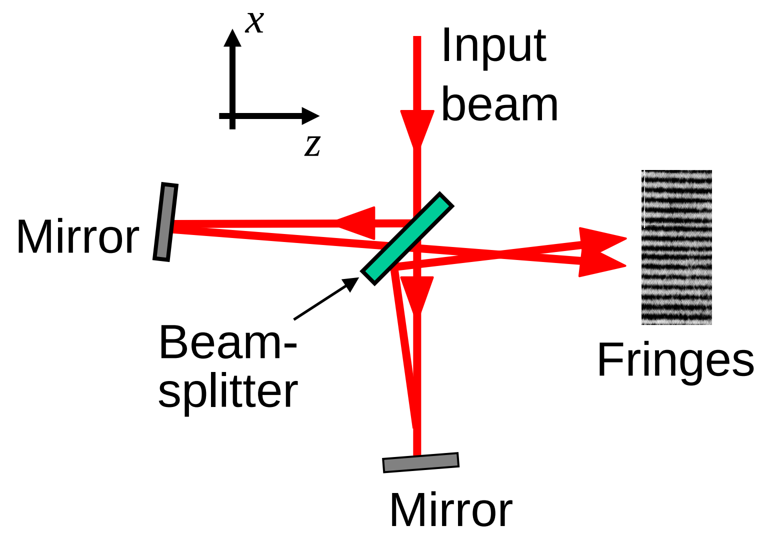
$\Lambda$ = 0.1 mm is about the minimum fringe spacing you can see:

$$\theta \approx \sin \theta = \lambda / (2\Lambda)$$

$$\Rightarrow \theta \approx 0.5 \mu m / 200 \mu m$$

$$\approx 1/400 \text{ rad} = 0.15^\circ$$

# The Michelson Interferometer and Spatial Fringes




Suppose we misalign the mirrors so the beams cross at an angle when they recombine at the beam splitter.  And we won't scan the delay.

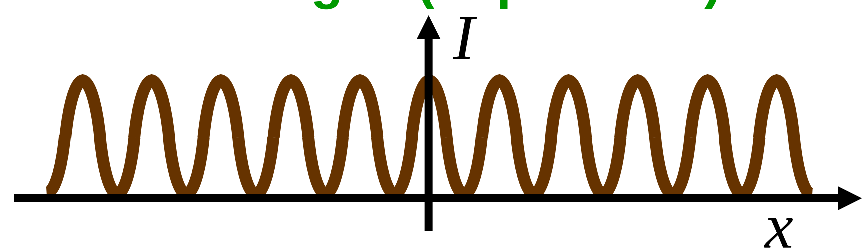
If the input beam is a plane wave, the cross term becomes:

$$\mathrm{Re}\left\{E_0 \exp\left[i(\cancel{\omega t - kz\cos\theta} - kx\sin\theta\right] E_0^* \exp\left[-i(\cancel{\omega t - kz\cos\theta} + kx\sin\theta\right]\right\}$$

$$\propto \mathrm{Re}\left\{\exp\left[-2ikx\sin\theta\right]\right\}$$

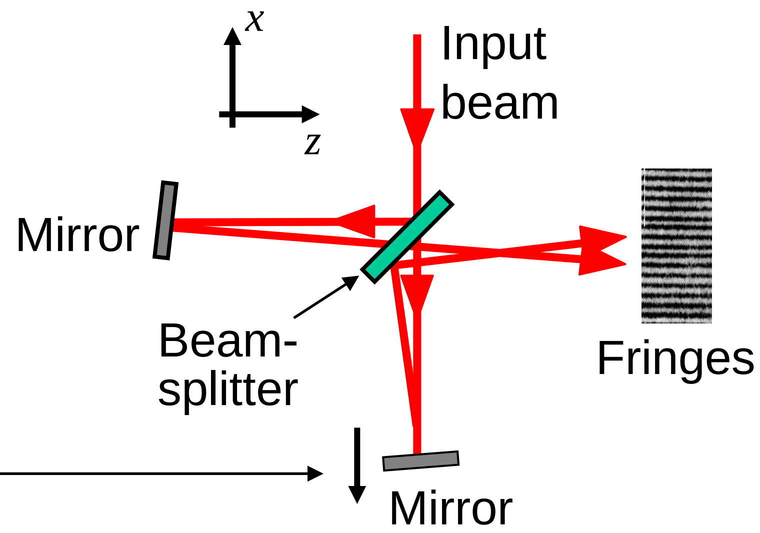
$$\propto \cos(2kx\sin\theta)$$

**Fringes (in position)**




Crossing beams maps delay onto position.

# The Michelson Interferometer and Spatial Fringes



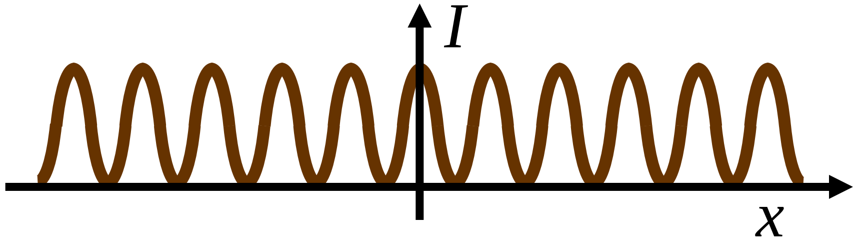

Suppose we change one arm's path length.

$$\mathrm{Re}\left\{E_0 \exp\left[i(\omega t - kz\cos\theta - kx\sin\theta + {\color{red}2kd}\right] E_0^* \exp\left[-i(\omega t - kz\cos\theta + kx\sin\theta\right]\right\}$$

$$\propto \mathrm{Re}\left\{\exp\left[-2ikx\sin\theta + 2kd\right]\right\}$$

$$\propto \cos(2kx\sin\theta + 2kd)$$

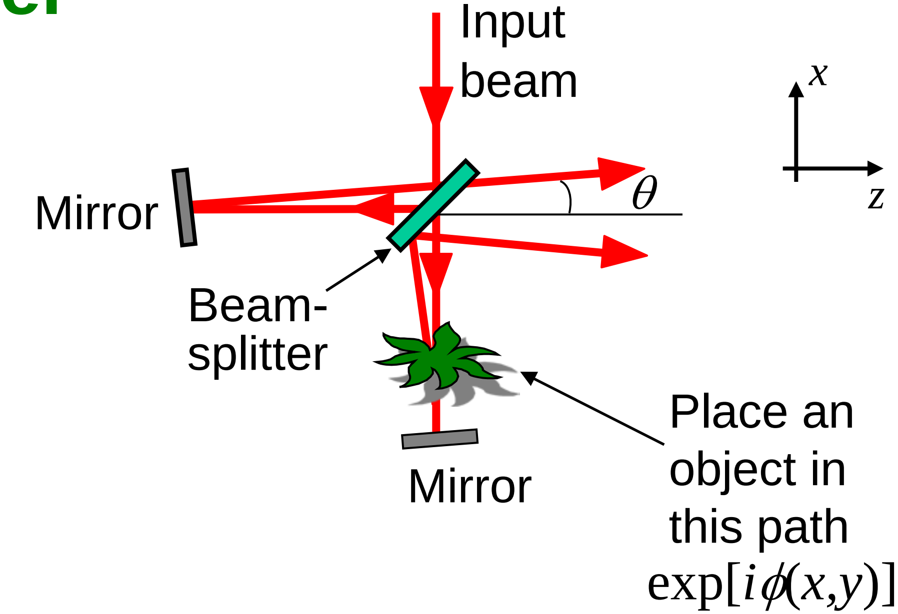
**Fringes (in position)**




The fringes will shift in phase by $2kd$.

# The Unbalanced Michelson Interferometer

Misalign mirrors, so beams cross at an angle.

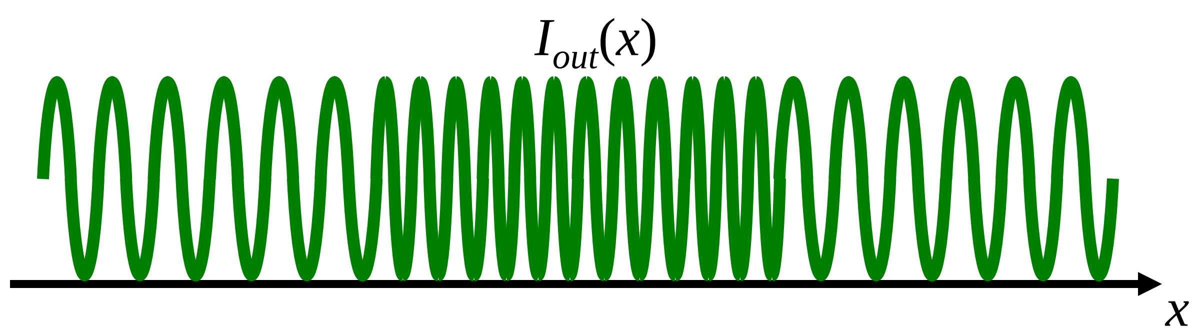



Now, suppose an object is placed in one arm. In addition to the usual spatial factor, one beam will have a spatially varying phase, $\exp[2i\phi(x,y)]$.

Now the cross term becomes:

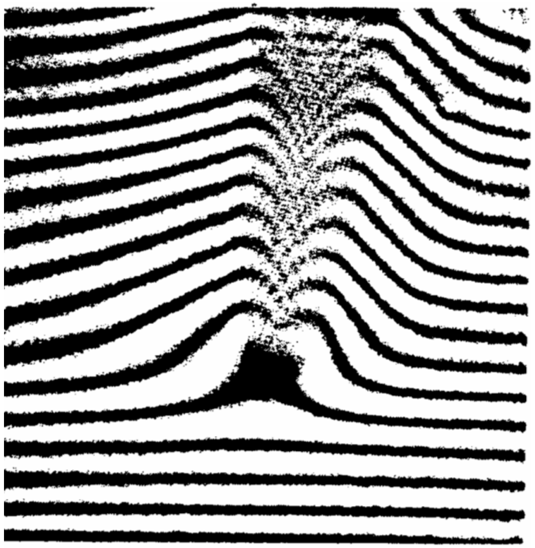
$$\mathrm{Re}\{\ \exp[2i\phi(x,y)]\ \exp[-2ikx\sin\theta]\ \}$$

Distorted fringes (in position)

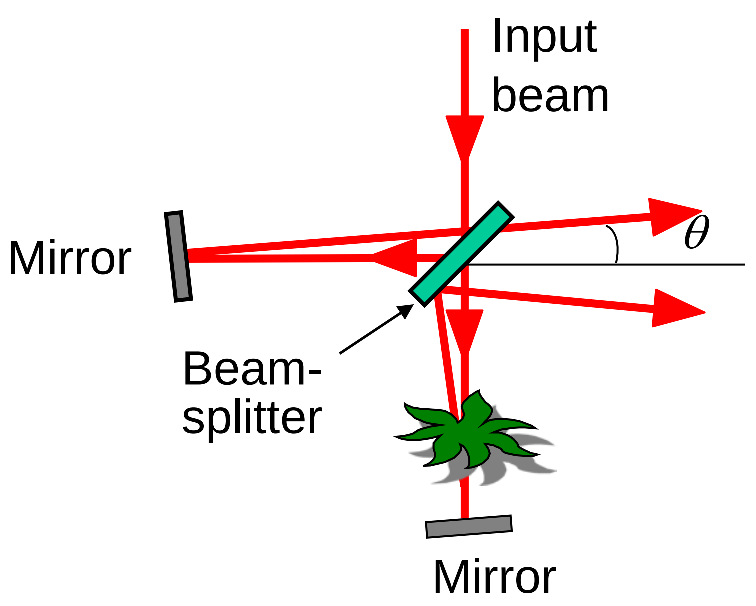


# The Unbalanced Michelson Interferometer can sensitively measure phase vs. position.

Spatial fringes distorted by a soldering iron tip in one path

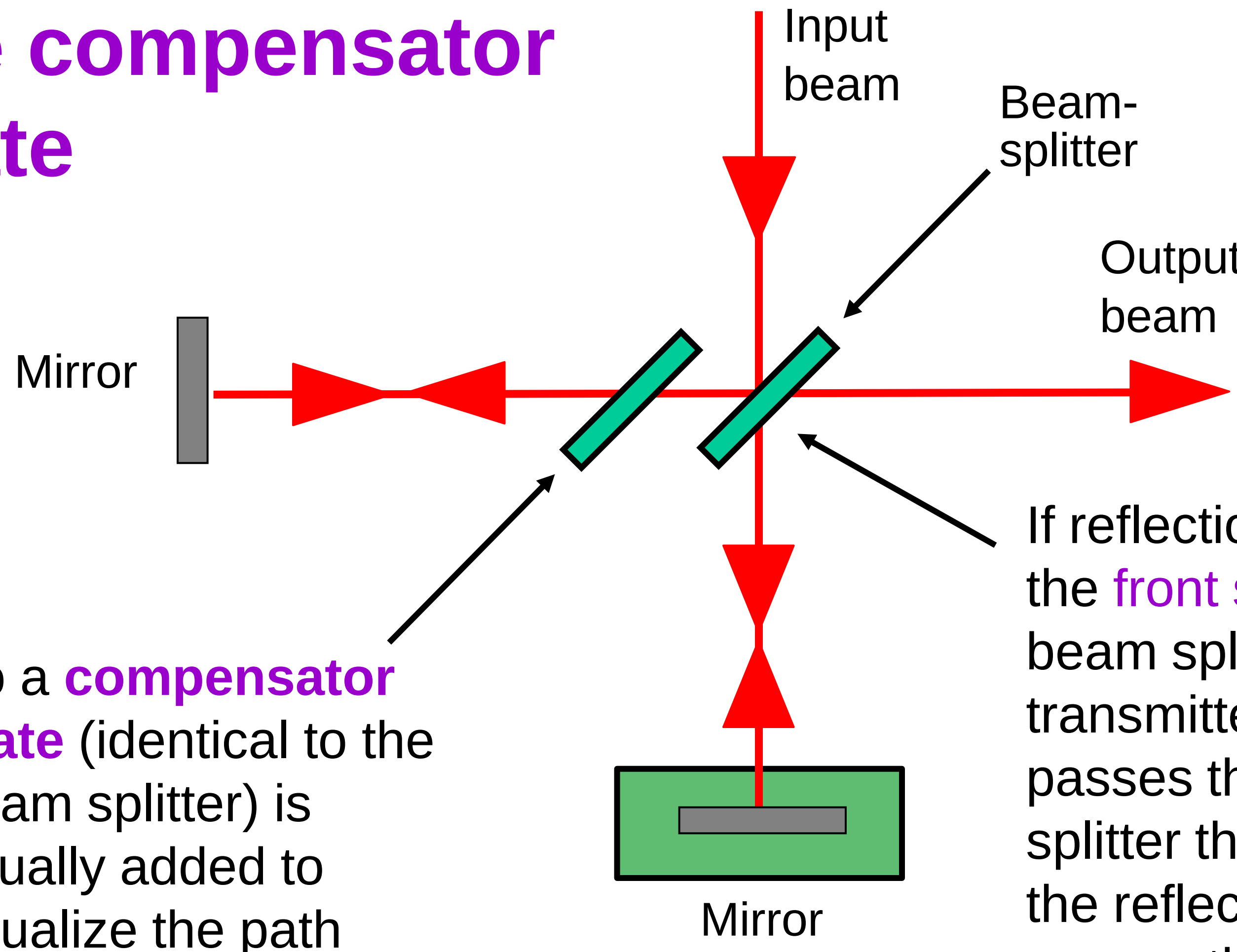


Placing an object in one arm of a misaligned Michelson interferometer will distort the spatial fringes.




Phase variations of a small fraction of a wavelength can be measured.

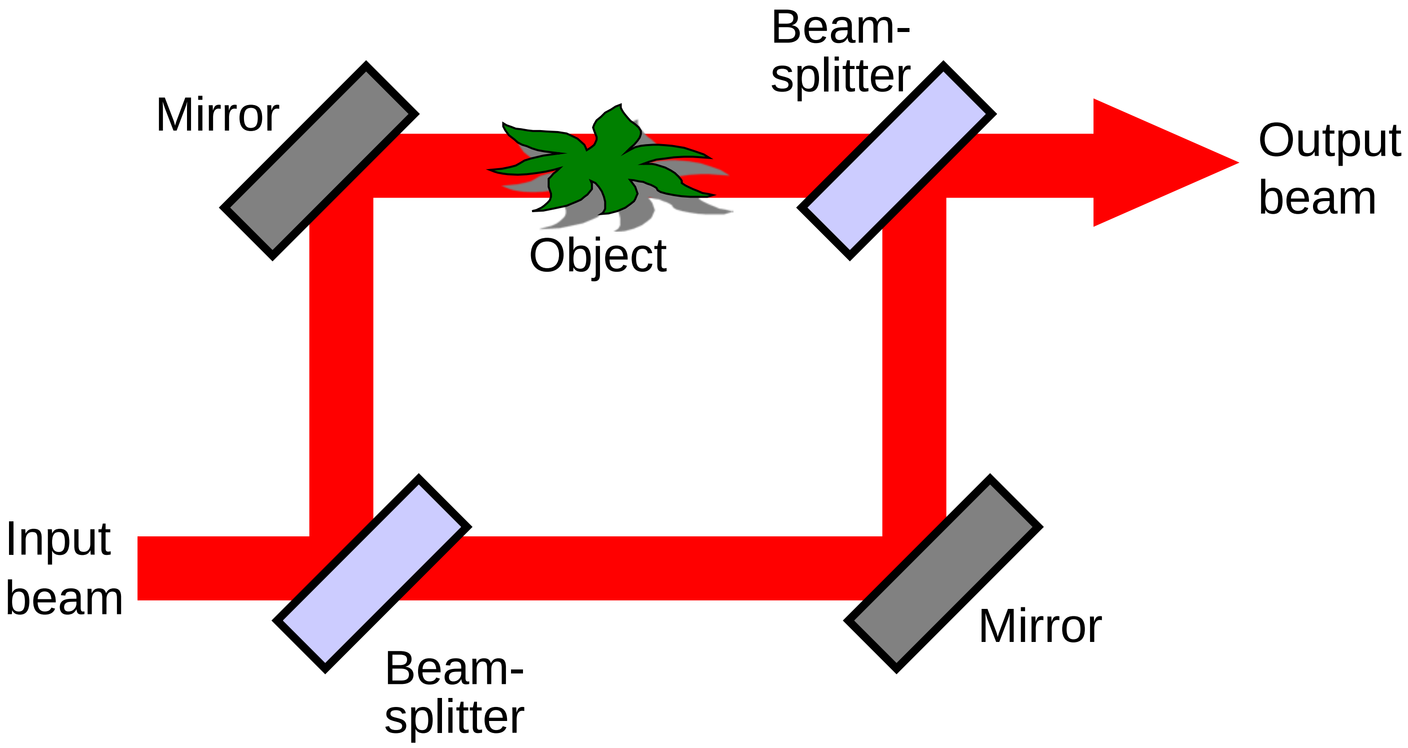
# Technical point about Michelson interferometers: the compensator plate




If reflection occurs off the front surface of beam splitter, the transmitted beam passes through beam splitter three times; the reflected beam passes through only once.

So a **compensator plate** (identical to the beam splitter) is usually added to equalize the path length through glass.

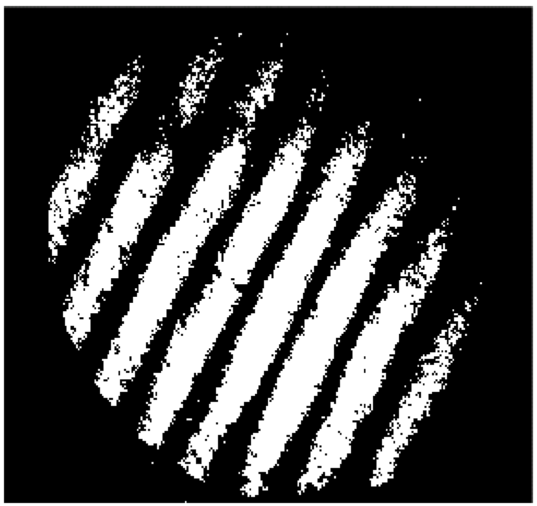
# The Mach-Zehnder Interferometer




The Mach-Zehnder interferometer is usually operated misaligned and with something of interest in one arm.

# Mach-Zehnder Interferogram

Nothing in either path



Plasma in one path

# Other applications of interferometers

To frequency filter a beam (this is often done inside a laser).

Money is now coated with interferometric inks to help foil counterfeiters.  Notice the shade of the “20,” which is shown from two different angles.