Diffraction

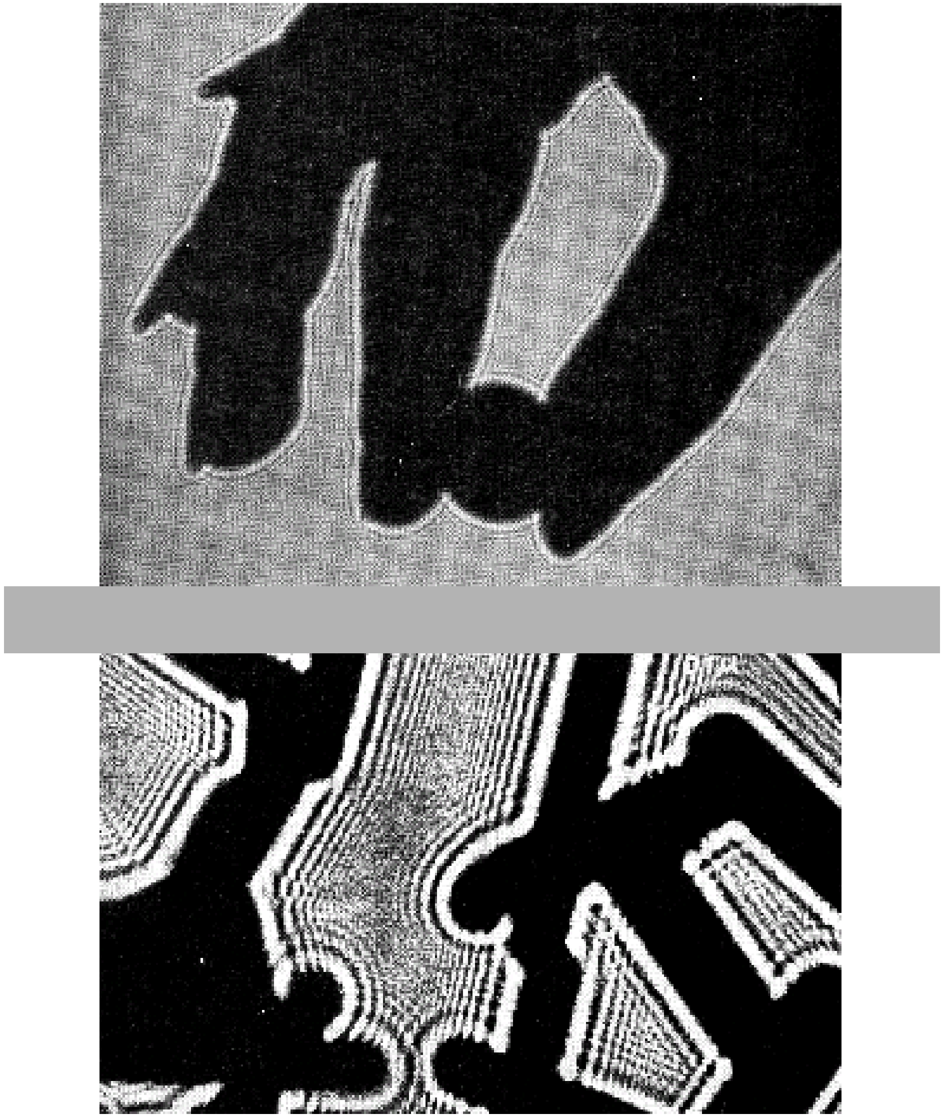
Light bends!

Diffraction assumptions

Solution to Maxwell's Equations

The far-field

- Fraunhofer Diffraction
- Some examples



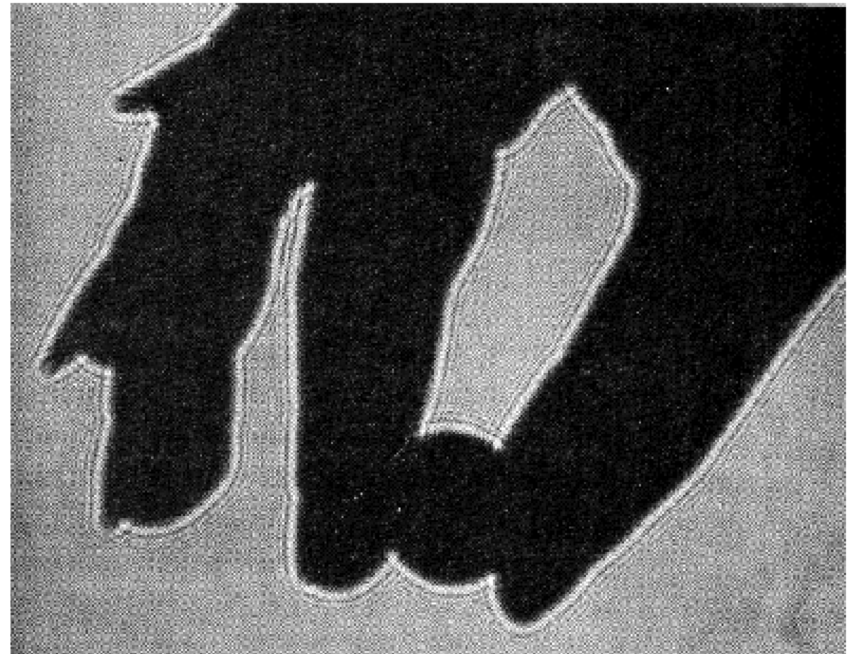
Diffraction

Light does not always travel in a straight line.

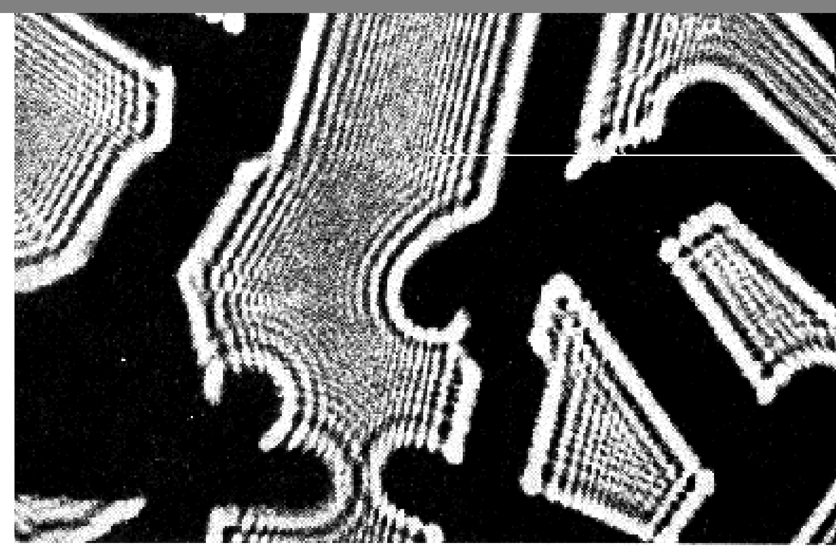
It tends to bend around objects. This tendency is called **diffraction**.

Any wave will do this, including matter waves and acoustic waves.

Shadow of a hand illuminated by a Helium-Neon laser



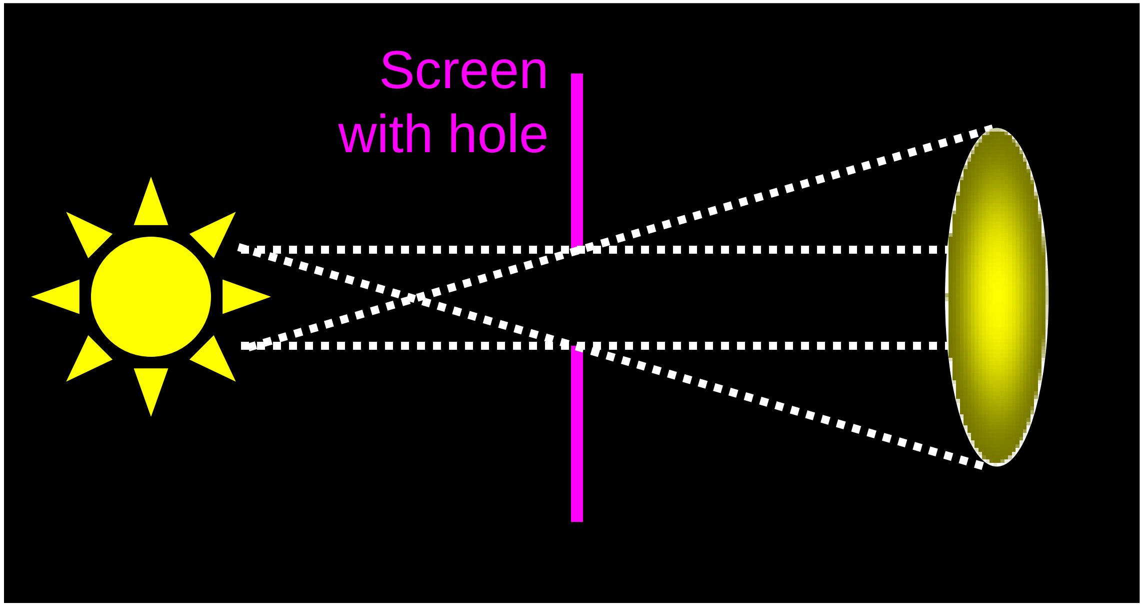
Shadow of a zinc oxide crystal illuminated by a electrons



Why it's hard to see diffraction

Diffraction tends to cause ripples at edges. But poor source temporal or spatial coherence masks them.

Example: a large spatially incoherent source (like the sun) casts blurry shadows, masking the diffraction ripples.

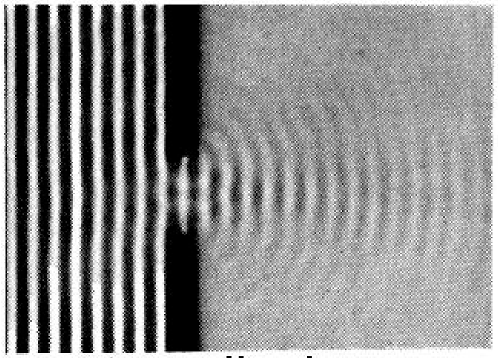


Untilted rays yield a perfect shadow of the hole, but off-axis rays blur the shadow.

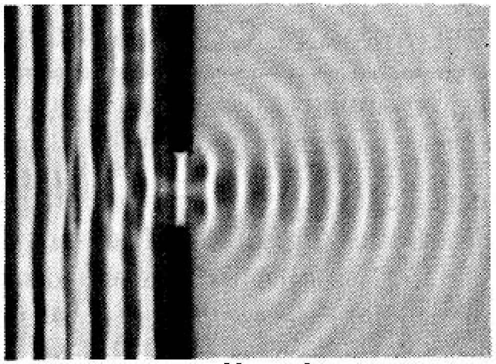
A point source is required.

Diffraction of a wave by a slit

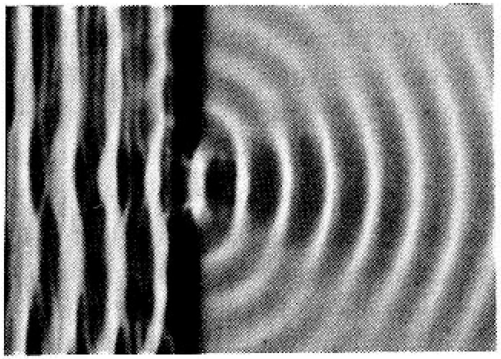
Whether waves in water or electromagnetic radiation in air, passage through a slit yields a diffraction pattern that will appear more dramatic as the size of the slit approaches the wavelength of the wave.



$\lambda \ll$ slit size



$\lambda <$ slit size



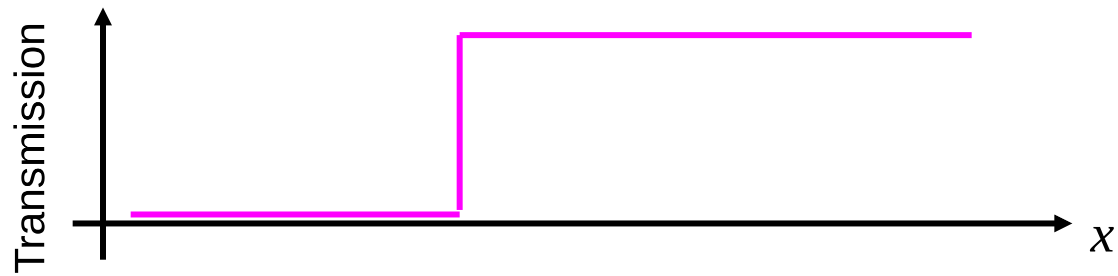
$\lambda \approx$ slit size

Diffraction of ocean water waves

Ocean waves passing through slits in Tel Aviv, Israel

Diffraction occurs for all waves, whatever the phenomenon.

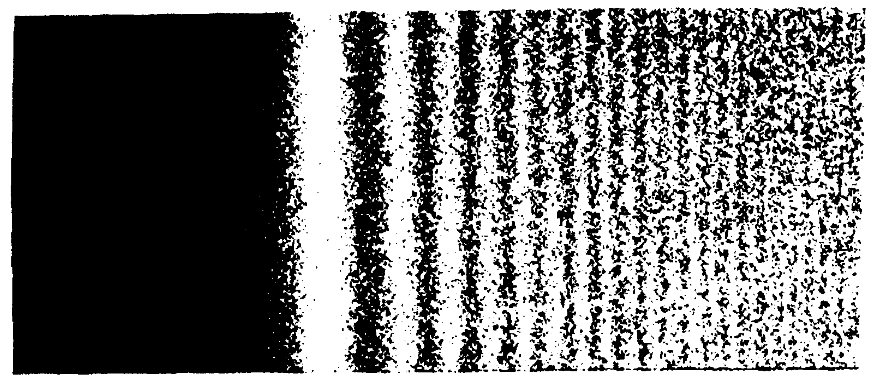
Diffraction by an Edge



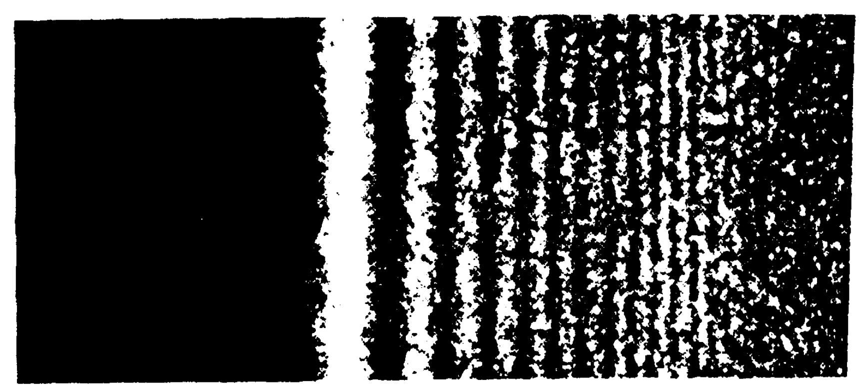
Even without a small slit, diffraction can be strong.

Simple propagation past an edge yields an unintuitive irradiance pattern.

Light passing by edge

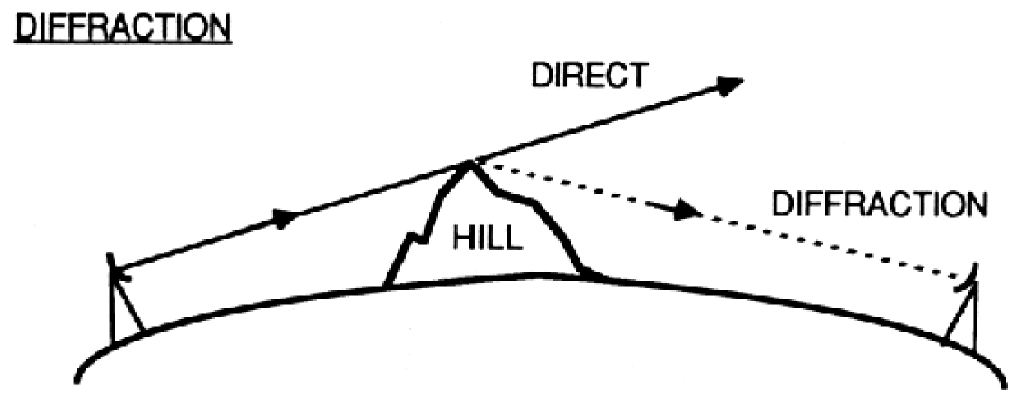


Electrons passing by an edge (Mg0 crystal)

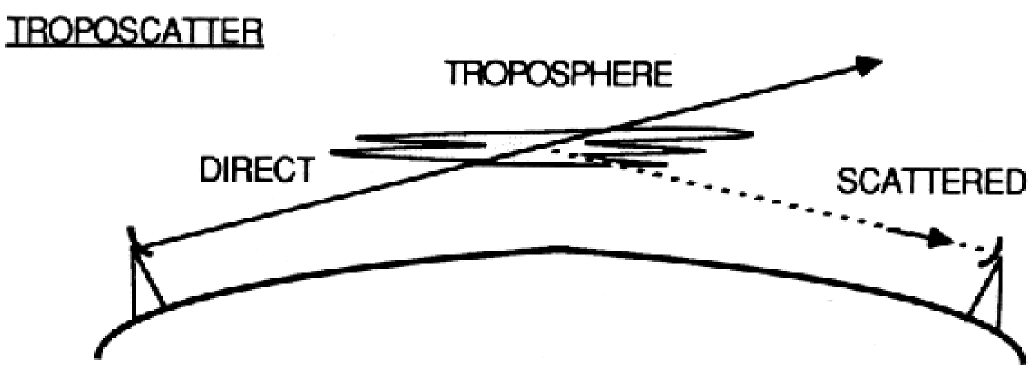


Radio waves diffract around mountains.

When the wavelength is km long, a mountain peak is a very sharp edge!

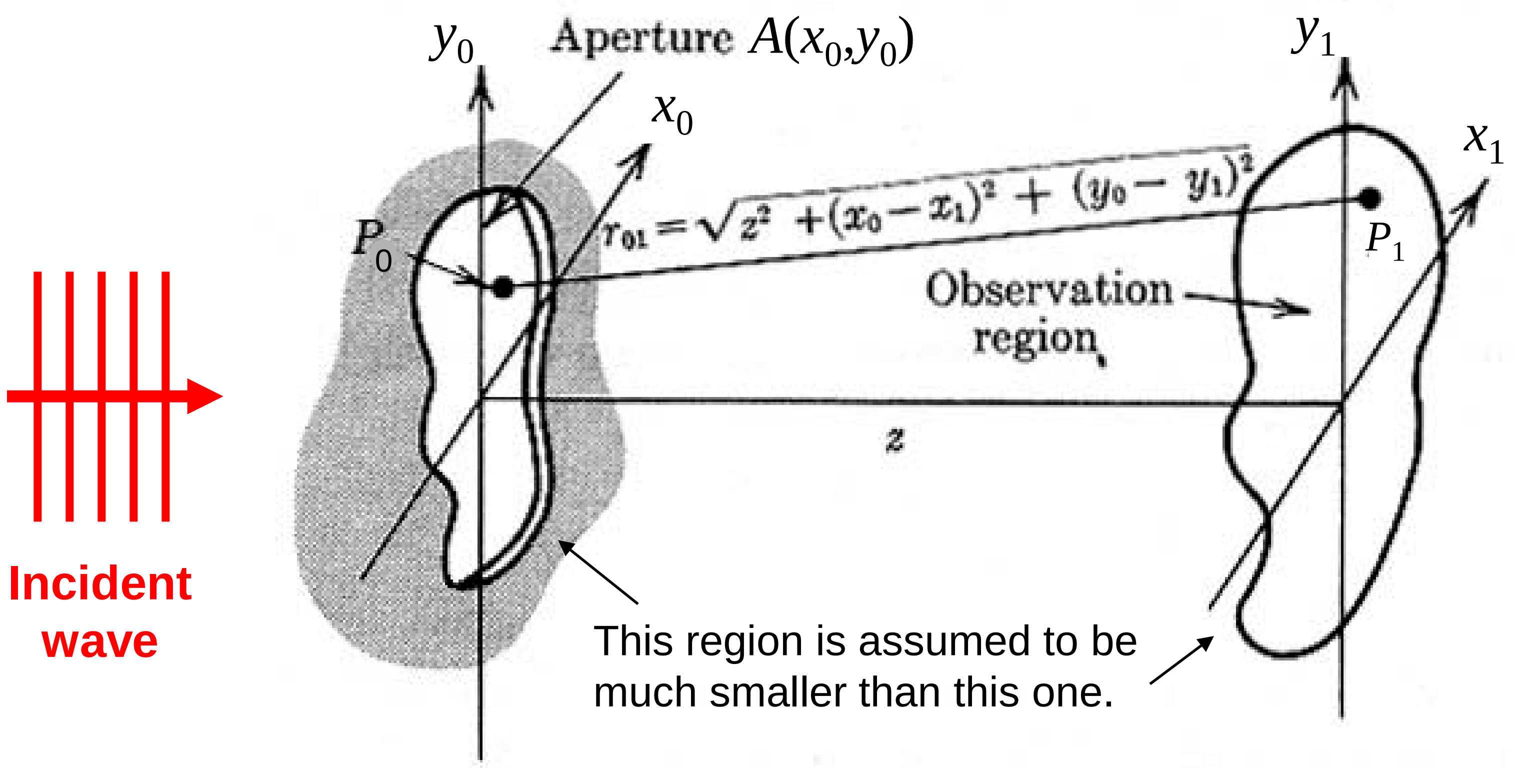


Another effect that occurs is scattering, so diffraction's role is not obvious.



Diffraction Geometry

We wish to find the light electric field after a screen with a hole in it. This is a very general problem with far-reaching applications.



What is $E(x_1,y_1)$ at a distance z from the plane of the aperture?

Diffraction Solution

The field in the observation plane, $E(x_1,y_1)$, at a distance z from the aperture plane is given by:

$$E(x_1, y_1, z) = \iint_{A(x_0, y_0)} h(x_1 - x_0, y_1 - y_0, z) E(x_0, y_0)\, dx_0\, dy_0$$

where :

$$h(x_1 - x_0, y_1 - y_0, z) = \frac{1}{i\lambda} \frac{\exp(ikr_{01})}{r_{01}}$$

Spherical wave

and :

$$r_{01} = \sqrt{z^2 + (x_0 - x_1)^2 + (y_0 - y_1)^2}$$

A very complicated result! And we cannot approximate r_{01} in the exp by z because it gets multiplied by k, which is big, so relatively small changes in r_{01} can make a big difference!

Fraunhofer Diffraction: The Far Field

We can approximate r_{01} in the denominator by z, and if D is the size of the aperture, $D^2 \geq x_0^2 + y_0^2$, so when $k\,D^2/\,2z << 1$, the quadratic terms << 1, so we can neglect them:

$$r_{01} = \sqrt{z^2 + (x_0 - x_1)^2 + (y_0 - y_1)^2} \approx z\left[1 + (x_0 - x_1)^2 / 2z^2 + (y_0 - y_1)^2 / 2z^2\right]$$

$$kr_{01} \approx kz + k\left(x_0^2 - 2x_0 x_1 + x_1^2\right)/\,2z + k\left(y_0^2 - 2y_0 y_1 + y_1^2\right)/\,2z$$

Small, so neglect these terms.

Independent of x_0 and y_0, so factor these out.

$$E(x_1, y_1) = \frac{\exp(ikz)}{i\lambda z} \exp\left[ik\frac{x_1^2 + y_1^2}{2z}\right] \iint\limits_{A(x_0, y_0)} \exp\left\{-\frac{ik}{z}(x_0 x_1 + y_0 y_1)\right\} E(x_0, y_0)\, dx_0\, dy_0$$

This condition means going a distance away: $z >> kD^2/2 = \pi D^2/\lambda$

If $D = 1$ mm and λ = 1 micron, then $z >> 3$ m.

Fraunhofer Diffraction

We'll neglect the phase factors, and we'll explicitly write the aperture function in the integral:

$$E(x_1, y_1) \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left\{-\frac{ik}{z}\left(x_0 x_1 + y_0 y_1\right)\right\} A(x_0, y_0)\, E(x_0, y_0)\, dx_0\, dy_0$$

$E(x_0,y_0)$ = constant if a plane wave

This is just a Fourier Transform!

Interestingly, it's a Fourier Transform from position, x_0, to another position variable, x_1 (in another plane). Usually, the Fourier "conjugate variables" have reciprocal units (e.g., t & ω, or x & k). The conjugate variables here are really x_0 and $k_x = kx_1/z$, which have reciprocal units.

So the far-field light field is the Fourier Transform of the apertured field!

The Fraunhofer Diffraction formula

We can write this result in terms of the off-axis k-vector components:

$E(x,y)$ = const if a plane wave

$$E\left(k_x,k_y\right) \propto \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-i\left(k_x x + k_y y\right)\right] A(x,y)\, E(x,y)\, dx\, dy$$

Aperture function

where we've dropped the subscripts, 0 and 1,

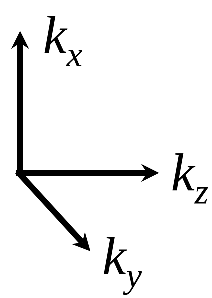
$$E\left(k_x,k_y\right) \propto \mathscr{F}\left\{A(x,y)\, E(x,y)\right\}$$

and:

$$k_x = kx_1/z \text{ and } k_y = ky_1/z$$

or:

$$\theta_x = k_x/k = x_1/z \text{ and } \theta_y = k_y/k = y_1/z$$



The Uncertainty Principle in Diffraction!

$$E\left(k_x, k_y\right) \propto \mathscr{F}\left\{A(x,y)\,E(x,y)\right\} \qquad k_x = kx_1/z$$

Because the diffraction pattern is the **Fourier transform** of the slit, there's an uncertainty principle between the slit width and diffraction pattern width!

If the input field is a plane wave and $\Delta x = \Delta x_0$ is the slit width,

$$\Delta x \, \Delta k_x > 1$$

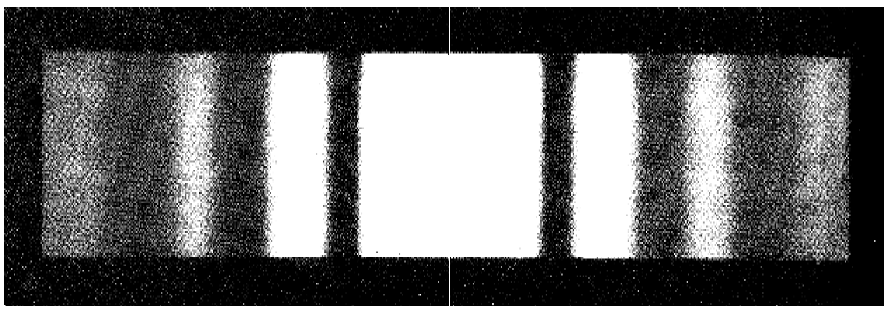
Or:

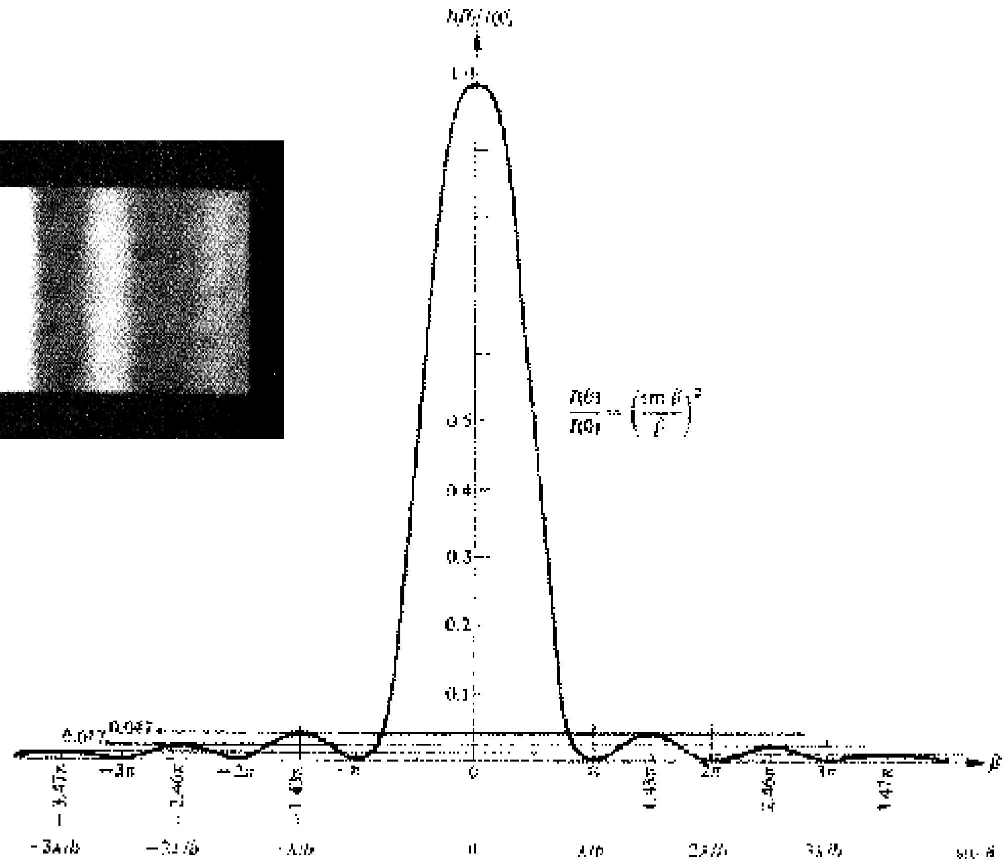
$$\Delta x_0 \, \Delta x_1 > z \,/\, k$$

The smaller the slit, the larger the diffraction angle and the bigger the diffraction pattern!

Fraunhofer Diffraction from a slit

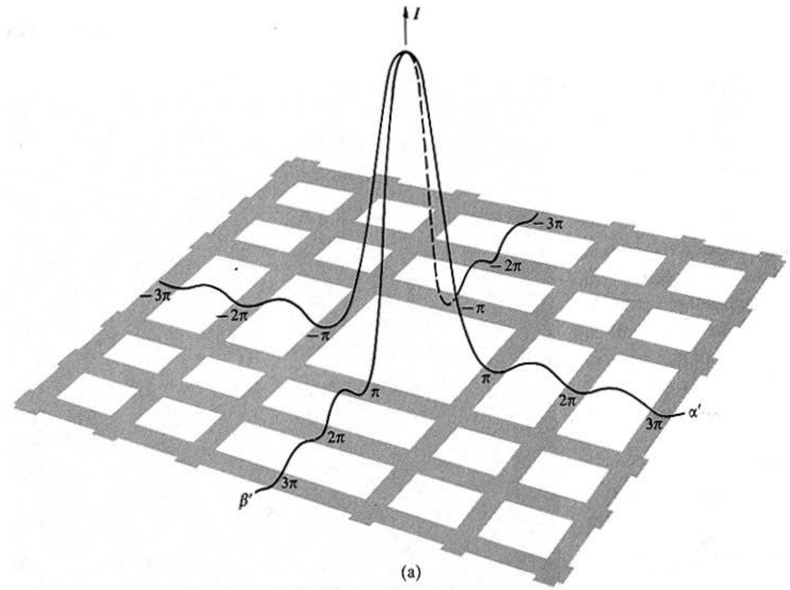
Fraunhofer Diffraction from a slit is simply the Fourier Transform of a rect function, which is a sinc function. The irradiance is then sinc^2 .



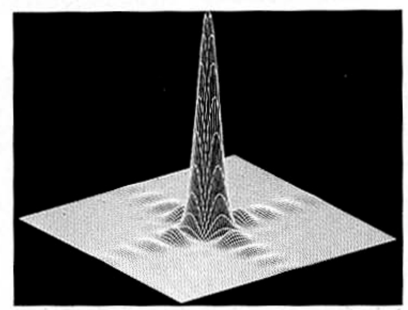


Fraunhofer Diffraction from a Square Aperture

The diffracted field is a sinc function in both x_1 and y_1 because the Fourier transform of a rect function is sinc.

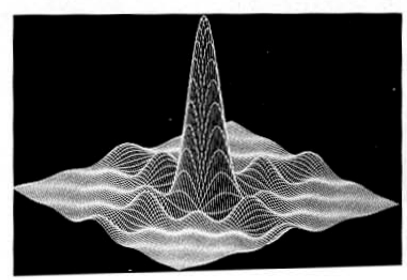


(a)



(b)

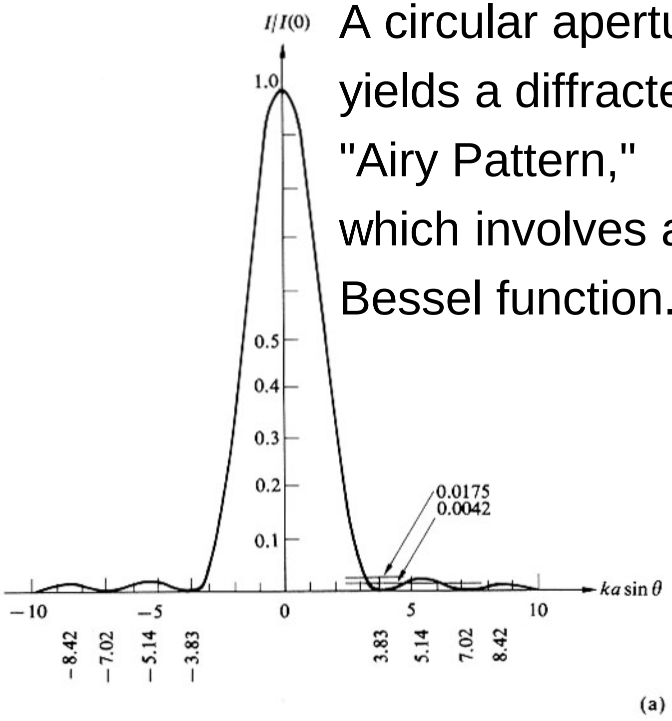
Diffracted irradiance

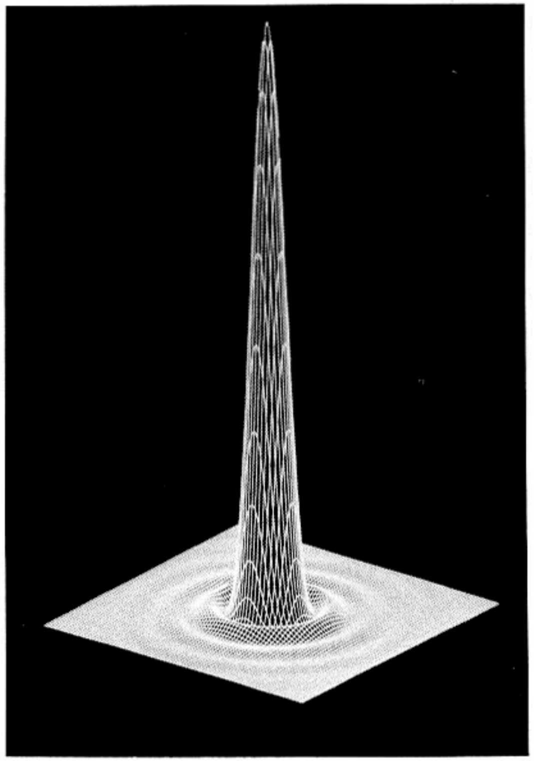


Diffracted field

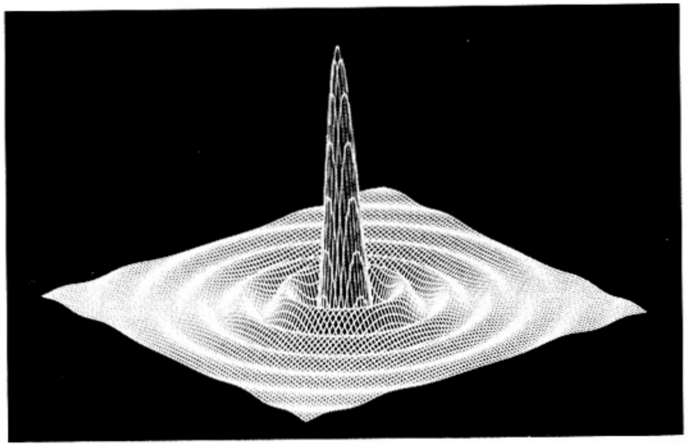
Diffraction from a Circular Aperture

A circular aperture yields a diffracted "Airy Pattern," which involves a Bessel function.





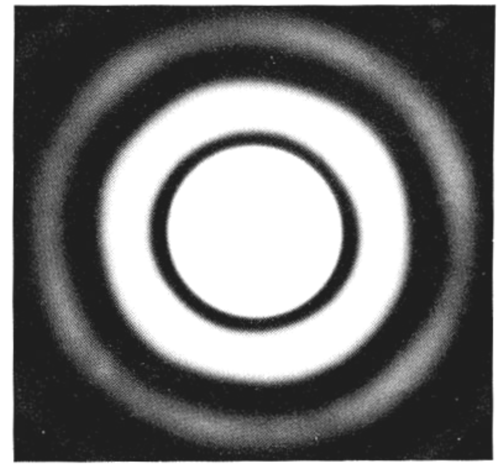
Diffracted Irradiance



Diffracted field

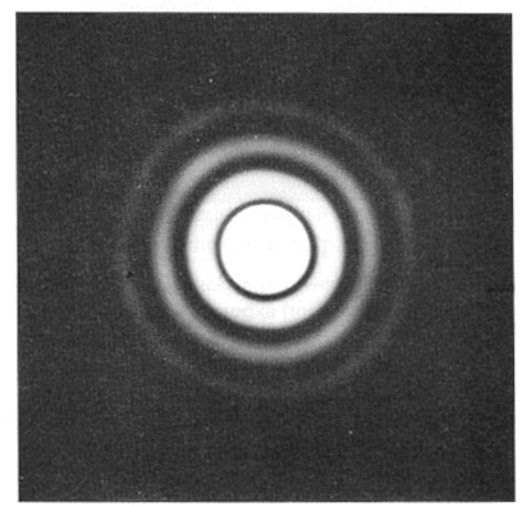
Diffraction from small and large circular apertures

Far-field intensity pattern from a small aperture

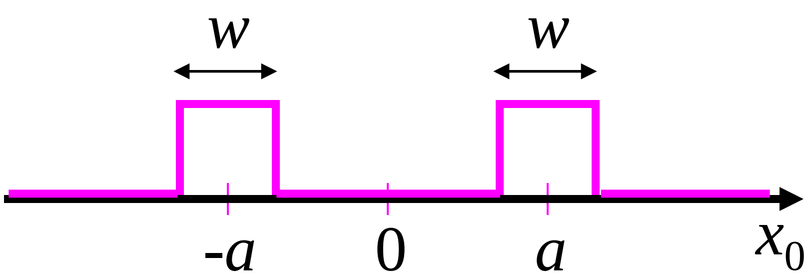


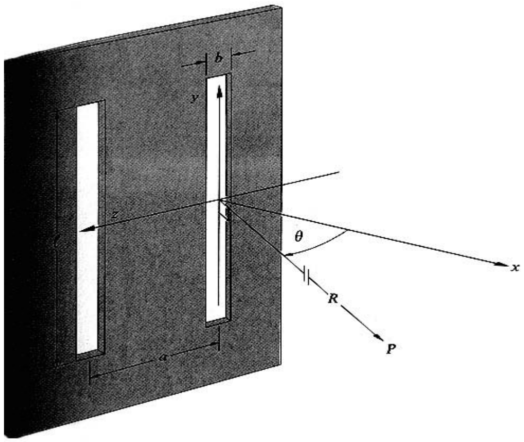
Recall the Scale Theorem! This is the Uncertainty Principle for diffraction.

Far-field intensity pattern from a large aperture



Fraunhofer diffraction from two slits



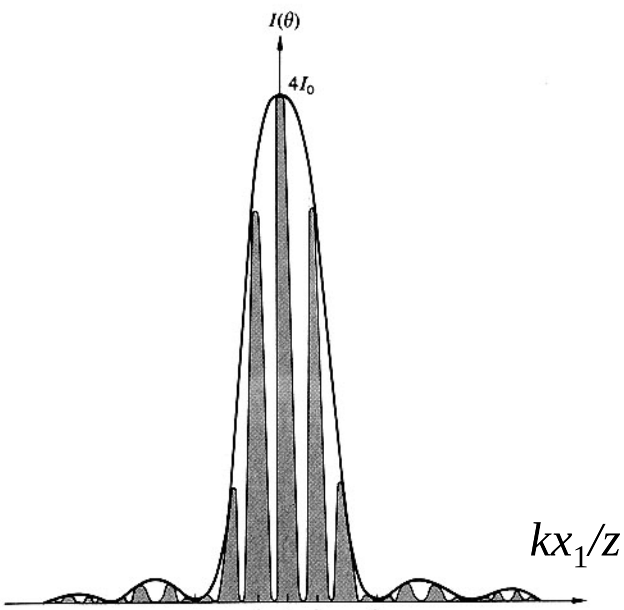


$$A(x_0) = \mathrm{rect}[(x_0+a)/w] + \mathrm{rect}[(x_0-a)/w]$$

$$E(x_1) \propto \mathscr{F}\{A(x_0)\}$$

$$\propto \mathrm{sinc}[w(kx_1/z)/2]\exp[+ia(kx_1/z)]+ \mathrm{sinc}[w(kx_1/z)/2]\exp[-ia(kx_1/z)]$$

$$E(x_1) \propto \mathrm{sinc}(wkx_1/2z)\cos(akx_1/z)$$

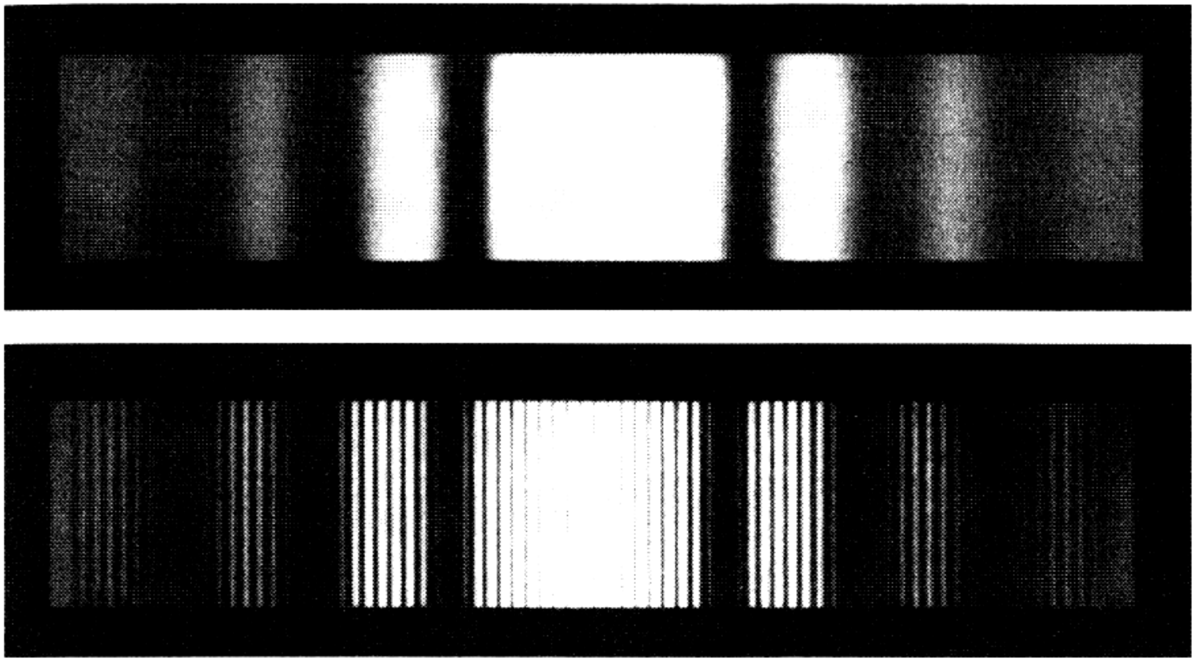


Diffraction from one- and two-slit screens

Fraunhofer diffraction patterns

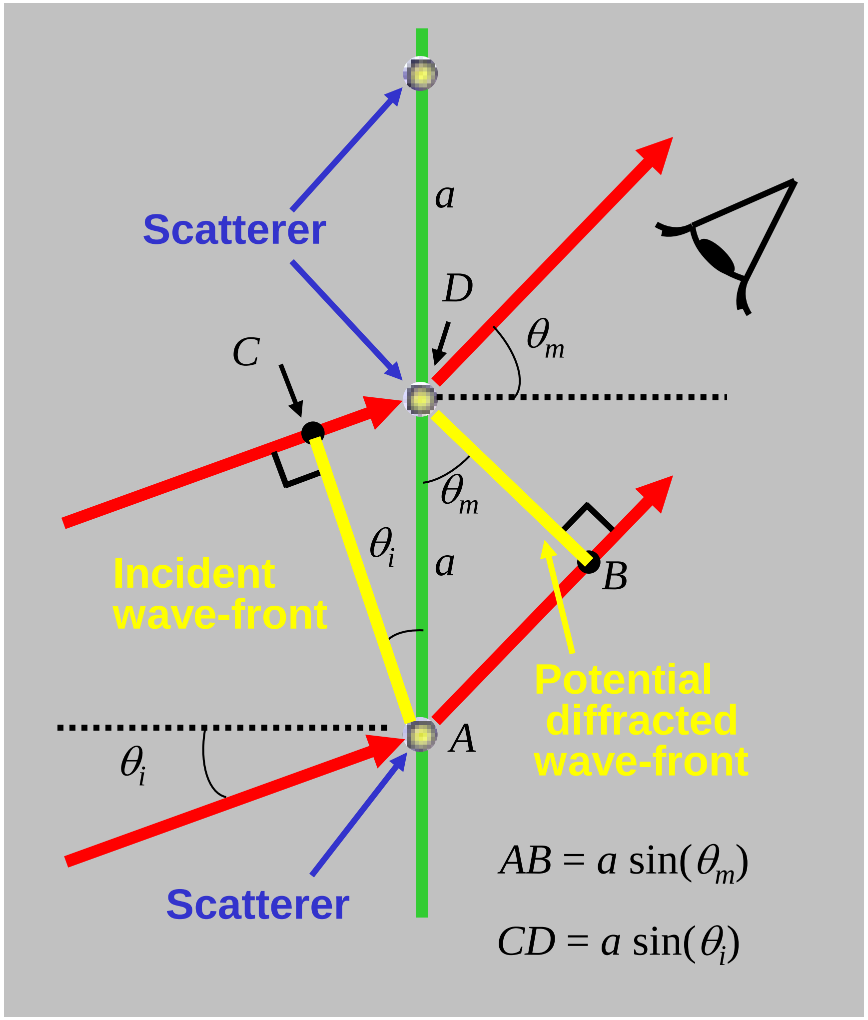
One slit

Two slits



Diffraction Gratings

•Scattering ideas explain what happens when light impinges on a periodic array of grooves. Constructive interference occurs if the delay between adjacent beamlets is an integral number, m, of wavelengths.



Path difference: $AB - CD = m\lambda$

$$a\left[\sin(\theta_m) - \sin(\theta_i)\right] = m\lambda$$

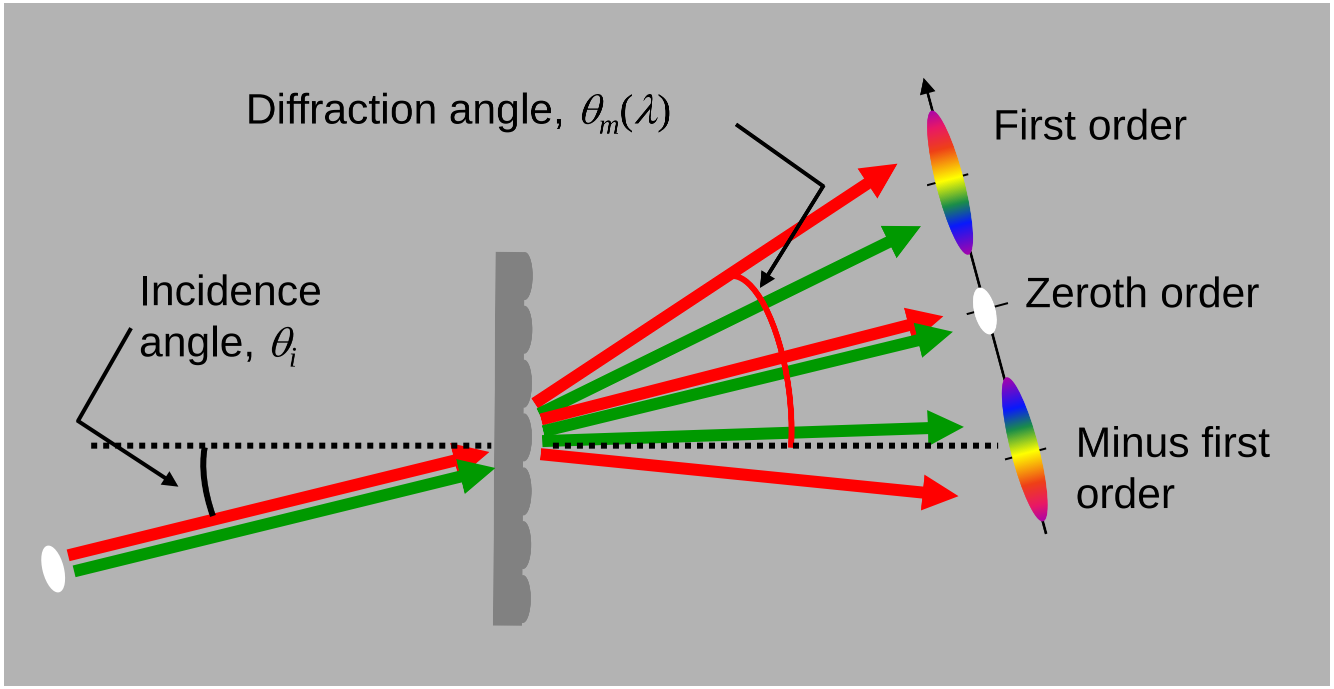
where m is any integer.

A grating has solutions of zero, one, or many values of m, or **orders**.

Remember that m and θ_m can be negative, too.

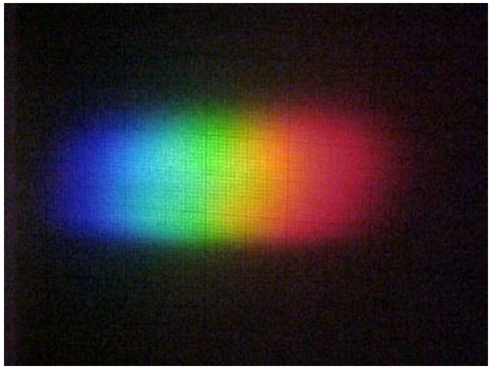
Diffraction orders

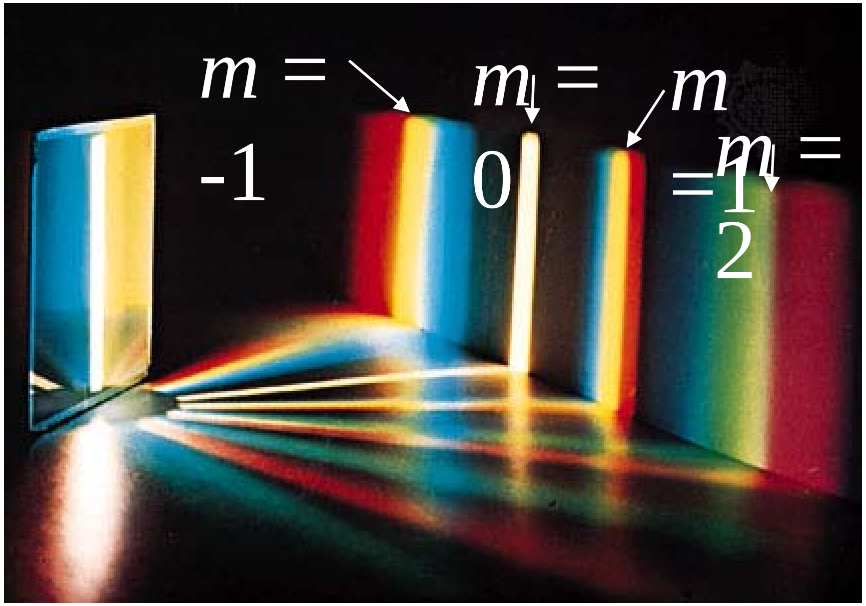
Because the diffraction angle depends on λ, different wavelengths are separated in the nonzero orders.



No wavelength dependence occurs in zero order.

The longer the wavelength, the larger its deflection in each nonzero order.




m = -1
m = 0
m = 1
m = 2

World's largest diffraction grating

Lawrence Livermore National Lab