1. Make certain you are able to do problem 5.1 in the textbook. The answers are at the end.

2. (5 pts.) Do problem 5.2 in the textbook.

3. (5 pts.) Derive the addition law of velocities by observing the 4-momentum of a particle moving in the x-direction with speed v_1 in a frame moving at speed v_2 in the negative x-direction.

4. The coordinate time for a light pulse to go from $(-x_1, 0)$ to $(x_2, 0)$ is

$$t = (x_1^2 + \delta^2)^{1/2} + (x_2^2 + \delta^2)^{1/2} + 2M \log \frac{4x_1 x_2}{(y_0 + \delta)^2}.$$

The sun is at $(x, y) = (0, -y_0)$, and the light ray passes nearest the sun at $(0, \delta)$. In class we found δ_0 for which the time is an extremum.

$$\delta_0/x_1 + \delta_0/x_2 = 4M/(y_0 + \delta_0),$$

and we considered only the case $y_0 \gg \delta_0$.

   (a) (5 pts.) Find the difference in time between the actual path and the path for which $\delta = 0$. Let $y_0 = 700\,\mathrm{Mm}$, which means the undeflected ray grazes the sun's surface. Let $x_1 = x_2 = 1\,\mathrm{AU} = 1.5 \times 10^8\,\mathrm{km}$.

   (b) (5 pts.) Now pretend that the sun is a point mass. Find the second path for which the time is minimal. Sketch the path. Let $x_1 = x_2 = 1\,\mathrm{AU}$. Let $y_0 = 700\,\mathrm{Mm}$.

5. Light from a lamp at $(x, y) = (1, -1)$ reaches my eye at $(x, y) = (-1, 1)$. The lamp is in water, which has an index of refraction n. Water fills the region at $y < 0$.

   (a) (5 pts.) Show that the straight line between the lamp and my eye is not an extremum, *i., e.,* that the path length changes linearly with ϵ, where the path goes through the point $(\epsilon, 0)$.

   (b) (5 pts.) Show that the path given by Snell's Law is an extremum.

6. Gravitational slowing of clocks. An atomic clock, shot vertically on a rocket, rises ballistically to an altitude $h = 10\,\mathrm{Mm}$, and returns to earth. (Vessot, R., *et al.*, 1980, *PRL*, 45, 2081) This experiment measures the frequency difference between a 1.42-GHz signal emitted by the rocket f_r and $\frac{1}{2}(f_1 - f_0)$, where f_1 is

a 1.42-GHz signal sent by the ground station and returned by the rocket, which you can consider a reflection. The 1.42-GHz signals are generated by clocks. The signal from the on-board clock is affected by the gravitational redshift and the Doppler effect. The signal f_1 has twice the Doppler effect, once on reception and once on reemission.

$$\frac{\Delta f}{f_0} = \frac{\phi_s - \phi_e}{c^2} - \frac{|\vec{v_e} - \vec{v_s}|^2}{2c^2}.$$

(To simplify the problem, a term due to the acceleration of the earth has been left off.) The first term is the gravitational redshift and the second term is the second-order Doppler shift. Useful numbers: The mass of the earth is 4.5 mm. The radius of the earth is 6.4 Mm.

(a) (3 pts.) A key point in the experiment is that the first-order Doppler effect of the reflected signal is removed. How large is the first-order Doppler effect?

(b) (3 pts.) At what height in the flight is the first term the largest? Calculate its maximum.

(c) (3 pts.) At what height in the flight is the second term the largest? Calculate its maximum.

(d) (3 pts.) At what height is $\Delta f = 0$?

Answers

1. 5.1

   - 4-vector a is time-like; b is null.
   - $a - 5b = (-27, 0, -15, -19)$.
   - $a \cdot b = 14$.