1. *The angle subtended by a yardstick.* Suppose there is a length l that does not change with time—it is a "yardstick," and further suppose that one can observe it at cosmological distances. In class we found that the angle subtended by this length $\theta = l/(ra)$, where a is expansion parameter of the epoch when the light was emitted and r is the comoving coordinate. Let $\Omega = 2$.

   (a) Show that $\theta \to l/d$ for $a \to 1$, where d is the distance.

   (b) Give a physical argument why $\theta \to \infty$ as $a \to 0$.

2. *Can I see the back of my head?* Let $\Omega_0 = 2$. The universe will stop expanding and collapse again for this value of Ω_0.

   (a) (5 pts.) What is the expansion parameter a_{m} at which the expansion stops?

   (b) What is the comoving coordinate of a galaxy that emitted light that we see at the epoch a_{m}?

   (c) What is the comoving coordinate of a galaxy that emitted light that we see at the epoch for which $a = 0$ for the second time?

   (d) Can I see the back of my head?

3. Simplicio reasons, "For $\Omega_0 = 0$, the comoving coordinate of a source that we see now at the telescope is $rH_0 = \frac{1}{2}(a^{-1} - a)$. If a is very small, r approaches ∞. The light travels more than c times the age of the universe. Einstein or Friedmann must be wrong, since the universe has a finite age."

   (a) Explain how we can see so far if $\Omega \ll 1$.

4. The form of the Robertson-Walker metric that we have used is

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{(1 - r^2/r_0^2)} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right].$$

   An alternative metric is

$$ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r_0^2 \sin^2(r/r_0)(d\theta^2 + \sin^2\theta d\phi^2) \right].$$

   (a) Interpret the coordinates (t, r, θ, ϕ) for the standard and alternative metrics. Which coordinates measure true time and true distance? Does a galaxy stay at the same spatial coordinate for both metrics? Does the universe expand for both metrics?

(b) Consider a 4-vector $x^\mu = (dt, dr, d\theta, d\phi)$. Find x_μ for each metric. Why are some components of x^μ and x_μ the same and some different?

5. Problem 18.2 in Hartle.

6. Problem 18.3 in Hartle.

7. Problem 18.20 in Hartle.