1. (5 pts.) Prove $g_{\mu\nu}g^{\nu\sigma} = \delta^\sigma_\mu$, where $g_{\mu\nu}$ is the metric for contravarient vectors, $g^{\mu\nu}$ is the metric for covarient vectors. $\delta^\sigma_\mu = 1$ if $\mu = \sigma$, and $\delta^\sigma_\mu = 0$ if $\mu \neq \sigma$. You must write the reasons for the steps in your proof.

2. (6 pts.) Consider spherical coordinates $(x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)$ in flat space. Compute Γ^1_{22}, Γ^1_{33}, and Γ^3_{23}.

3. (5 pts.) The figure shows the distance of two light rays that were emitted soon after the Big Bang and reach us at the present time. The model assumes a universe with the critical mass density, no radiation, and no vacuum energy density. Why do the paths of the light pulses appear to change directions?

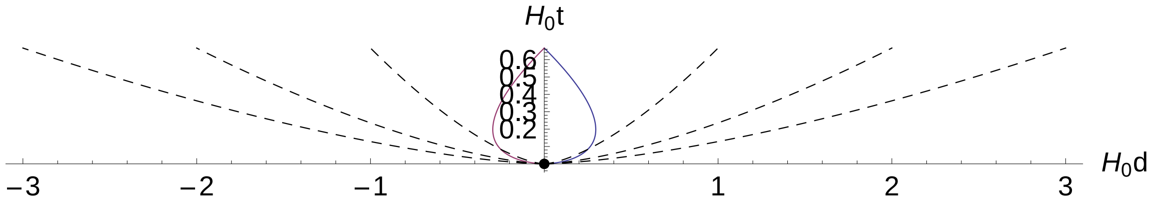

Figure 1: Spacetime diagram. Two solid lines show the paths of light pulses (solid) that reach us. Dashed lines show the world lines of "galaxies."

4. (6 pts.) Hartle 18-15.

5. Hartle 19-7. Define $N_{\rm gal}(z)$ more precisely: $N_{\rm gal}(z)$ is the number of galaxies *observed* to have a redshift $z' < z$ and to be within a solid angle $d\omega$ of a certain direction. Assume the number density of galaxies $n_{\rm gal}(a) = n_0 a^{-3}$, where a is the expansion parameter and n_0 is the present number density. Recall the redshift z and expansion parameter are related by $a = (1+z)^{-1}$. Interpret how $N_{\rm gal}(z)$ depends on z for $z \ll 1$.