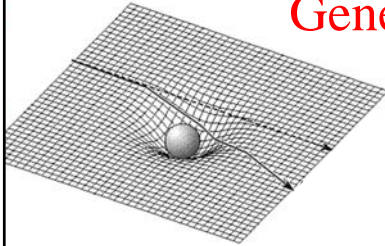


Midterm 3

Homework 6 due late tonight.

- Sit in assigned rows, as for previous exams.
- Bring a photo-ID.
- Closed book, closed notes, no calculators etc.
- 34 multiple-choice questions *about the material covered in the lectures.*
- Suggestions for how to prepare:
 - Read study guide (on course web site).
 - Review lecture notes.
 - Then use textbook to reinforce what you know.
 - Use a few *relevant* sample questions to test your knowledge.

General Relativity



$$R_{\psi\psi} = \frac{2a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta + 2ac \frac{\partial^2}{\partial \psi^2} \cot \theta + a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta} - \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta}$$

$$= \frac{2a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{4ac \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{2ac \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta} - \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta}$$

$$= \frac{2a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{4ac \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{2ac \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta} - \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta}$$

Newton:

$$F_{\text{gravity}} = \frac{Gm_1 m_2}{r^2}$$

$$R_{\psi\psi} = \frac{2ac \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{2ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\psi} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta}$$

$$= \frac{2ac \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{2ac \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{4ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\psi^2} - \frac{ac \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi}$$

$$= \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{2a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{3 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{2a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{2a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{2a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2}$$

$$+ \frac{2abc \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} - \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} - \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} - \frac{2bc \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi}$$

$$+ \frac{4ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{6 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\psi} - \frac{2bc \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{bc \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi}$$

$$+ \frac{2ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{3 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{2abc \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} - \frac{2a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2}$$

$$+ \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi}$$

$$+ \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta d} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta d} - \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta d} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{2d} + \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta^2} - \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta d}$$

$$+ \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta d} + \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta d} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta}$$

$$= \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta^2} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta^2} - \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta^2} - \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta^2} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta^2} - \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta^2}$$

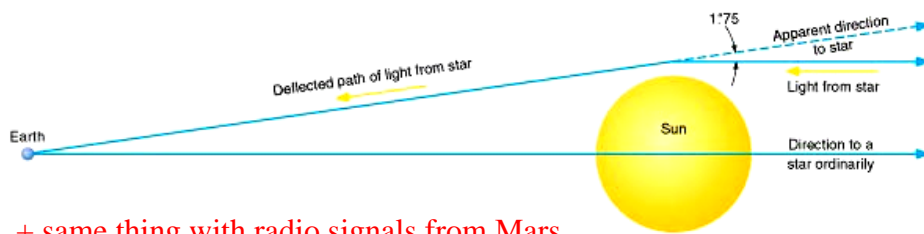
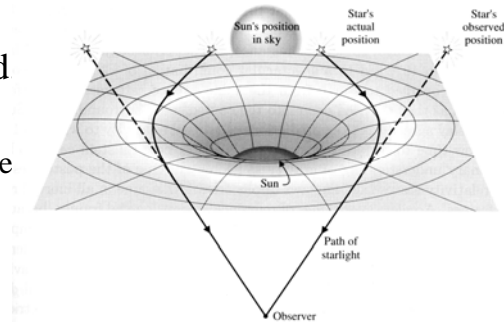
$$+ \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta^2} + \frac{a^2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta^2} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta^2} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{4\delta^2}$$

$$R_{\psi\psi} = -\frac{2ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} + \frac{2bc \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{d} - \frac{c \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta} + \frac{\frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta} + \frac{a \frac{\partial^2}{\partial \psi^2} \cot \theta}{2\delta}$$

$$= \frac{2ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi} - \frac{2 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\psi} - \frac{2ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} + \frac{6 \frac{\partial^2}{\partial \psi^2} \cot \theta}{\psi^2} + \frac{4bc \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi^2} - \frac{ab \frac{\partial^2}{\partial \psi^2} \cot \theta}{\delta \psi}$$

Tests (Proofs) of General Relativity

- Bending of starlight in Sun's gravitational field
 - Seen during 1919 eclipse



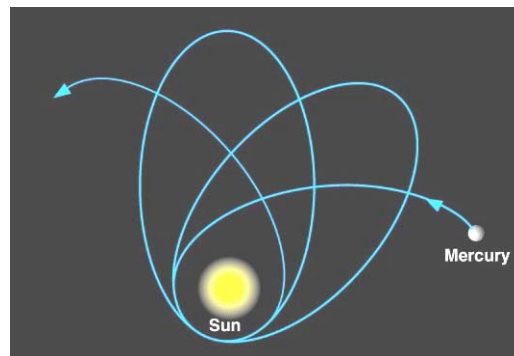
+ same thing with radio signals from Mars.

Tests (Proofs) of General Relativity

- Precession of Mercury
 - 1/90 degree per century in excess of amount expected from Newton's laws.
 - Easy to observe because of long time span of observations.
 - GR predicts this.
 - Need extra planet to explain it with Newton's laws.

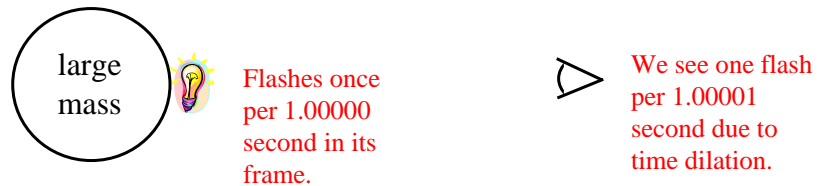
[Orbits in strongly curved spacetime](#)

[GR vs Newtonian orbits](#)

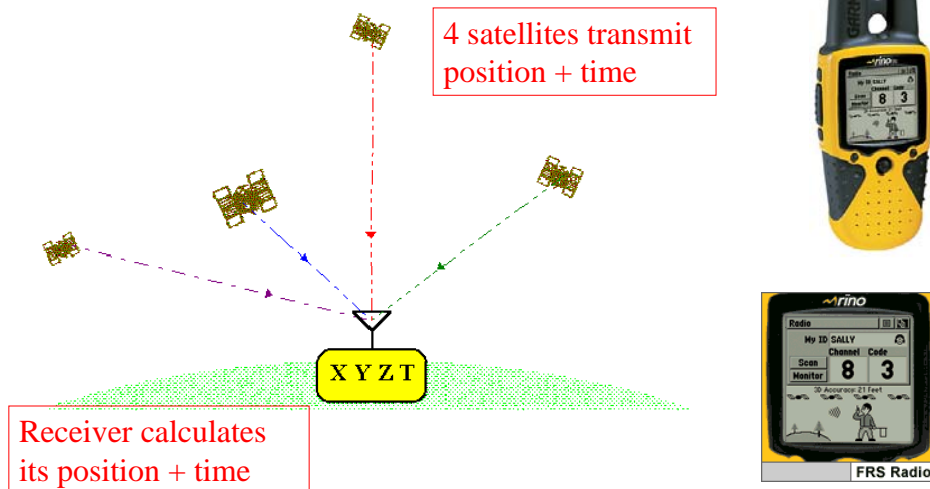


Tests (Proofs) of General Relativity

- Time runs slower in stronger gravitational field
 - *General relativistic time dilation* .
- → **gravitational redshift**: light waves emitted at different frequency than we receive them.
 - Observed from surface of white dwarfs.



GPS Navigation



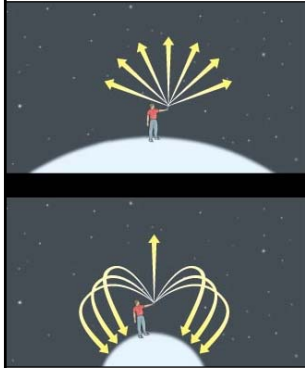
- General Relativity → time runs **faster** in the satellites.
- If no correction applied, 6 mile position error after only 1 day!

Stars - Possible ending #3: a black hole

For $M > 3M_{\odot}$: Further collapse

→ black hole

Mass is so concentrated that light cannot escape.



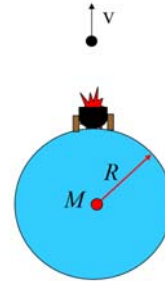
$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

becomes greater than speed of light.

→ photons can't escape.

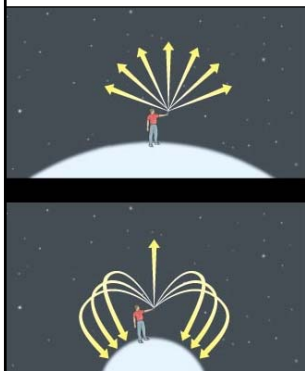
$$R = \frac{2GM}{v_{\text{escape}}^2}$$

$$R_s = \frac{2GM}{c^2} = \textit{Schwarzschild radius}$$

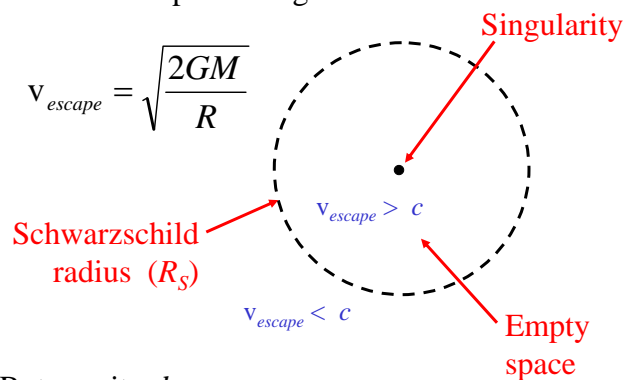


Schwarzschild Radius = R_s

- Light follows distorted paths slightly outside R_s .
- No wave phenomenon (*i.e.* no light) escapes from inside Schwarzschild radius.
 - Would have to go faster than speed of light



$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$



But gravity *does* escape.

X-ray binary:
 • Luminous star orbiting a black hole.
 • Cygnus X-1

[Fig 13.15]

Diagram illustrating the escape velocity equation:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

Labels in the diagram:

- Schwarzschild radius (R_S)
- Singularity
- Empty space
- $v_{\text{escape}} > c$
- $v_{\text{escape}} < c$

But gravity *does* escape.

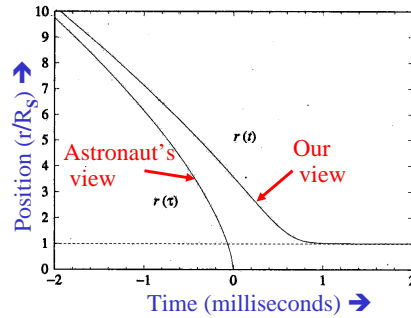
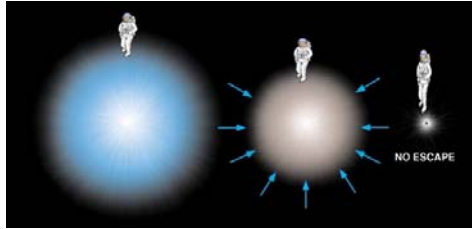
What's Inside?

- Only known mathematical solutions are for free space.
- All mass concentrated at *singularity*.
 - Radius = 0
 - \rightarrow density = ∞ (infinity)
- We don't know what really happens.

Labels in the diagram:

- Schwarzschild radius (R_S)
- Singularity
- Empty space

Falling into a Black Hole



Astronaut's view:

- Tidal forces become stronger & stronger.
- Time runs normally.
- Nothing special happens at Schwarzschild radius.
- But singularity is real. **splat!**

Our view:

- Tidal forces become stronger & stronger.
- Time runs slower & slower.
- Astronaut never quite reaches Schwarzschild radius.
- No knowledge about interior.