Name (please print): _____ **PID:** _____

Practice Midterm Exam #2

Total points = 25. Show all of your work!

1. [12 points] A damped oscillator consists of a spring (with a spring constant k = 8.00 N/m), a block of mass m = 1.50 kg, and a damping force given by $-b\dot{x}$ (where b = 0.230 kg/s). Suppose that the block is initially pulled so that the spring is extended a distance of x = 0.12 m and then released from rest.

(a) [3 points] What is the equation of motion for the block?

(b) [3 points] What is the solution to the equation of motion i.e. what is x(t)?

(c) [3 points] Calculate the time required for the amplitude of the oscillations to fall to one-third of its initial value.

(d) [3 points] How many oscillations are made by the block in this time?

Note: There is another question on the next page!

2. [13 points] Consider a particle of mass m constrained by a weightless, extensionless rod to move in a vertical circle of radius l (i.e. a plane pendulum). The particle can be thought of as moving in a periodic potential $U(\theta) = mgl(1-\cos\theta)$.

(a) [2 points] If the total energy of the particle is $E_1 = mgl$, what is the maximum value of the angular velocity, $\dot{\theta}$?

(b) [1 point] At what angle does this maximum occur?

(c) [1 point] At what angle is $\dot{\theta} = 0$?

(d) [4 points] If the total energy = $E_2 = 3mgl$, what are the maximum and minimum values of the angular velocity, $\dot{\theta}$?

(e) [2 points] At what two angles do these extreme values occur?

(f) [3 points] Sketch the phase paths ($\dot{\theta}$ vs. θ) for both the energies E_1 and E_2.

Useful Formulae

Oscillations:

SHM $\ddot{x} + \omega_o^2 x = 0$ $\qquad x(t) = A\sin(\omega_o t - \delta)$

Damped SHM $\ddot{x} + 2\beta\dot{x} + \omega_o^2 x = 0$ The general solution is:-

$x(t) = \exp(-\beta t)[A_1\exp(\sqrt{(\beta^2 - \omega_o^2)}t) + A_2\exp(-\sqrt{(\beta^2 - \omega_o^2)}t)]$
depending on the relative value of ω_o^2 and β^2.