1. [7] On a previous problem set you found the matrices that represent the operators for the three component of spin, for a spin-1 particle. They are:

\[
\hat{S}_x = \begin{pmatrix}
0 & \hbar / \sqrt{2} & 0 \\
\hbar / \sqrt{2} & 0 & \hbar / \sqrt{2} \\
0 & \hbar / \sqrt{2} & 0
\end{pmatrix}, \quad \hat{S}_y = \begin{pmatrix}
0 & -i\hbar / \sqrt{2} & 0 \\
i\hbar / \sqrt{2} & 0 & -i\hbar / \sqrt{2} \\
0 & i\hbar / \sqrt{2} & 0
\end{pmatrix}, \quad \hat{S}_z = \begin{pmatrix}
\hbar & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -\hbar
\end{pmatrix}
\]

A beam of spin-1 atoms is prepared in the initial spin state: \( |\chi\rangle = \begin{pmatrix} 4 / 4 \\ 4 / 3 \\ i / 4 \end{pmatrix} \).

a) [1] If you send the beam into a SG apparatus with its spin axis oriented along the z-direction, calculate the probabilities associated with each of the three output ports of the apparatus, i.e. calculate the probabilities that measurement of \( S_z \) will produce \( h \), \( 0 \), and \( -\hbar \). (You should be able to do this just by looking at \( |\chi\rangle \), with little calculation.)

b) [2] Now change the spin orientation of your SG apparatus to the x-axis, and calculate the probabilities associated with each of the three output ports. This time you will need to do a real calculation.

c) [2] Do the same thing with the SG apparatus oriented along the y-axis.

d) [2] Calculate the expectation values, \( \langle S_x \rangle \), \( \langle S_y \rangle \), and \( \langle S_z \rangle \), using the probabilities you calculated in parts (a) – (c). Check your answers using direct matrix multiplication.


3. [4] Griffiths problem 6.2. The easiest way to do part (b) is to express the \( \hat{x} \) operator in terms of \( \hat{a} \) and \( \hat{a}^+ \), as we have done in class.

4. [5] Griffiths problem 6.4. You do not have to sum the series in part (a), but try to if you want a challenge.