QMII-1. Consider two kets and their corresponding column vectors:

$$|\Psi\rangle = \begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix} \qquad |\phi\rangle = \begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix}$$

Are these two state orthogonal? Is $\langle\psi|\phi\rangle = 0$?

A) Yes B) No

Answer: A

Are these states normalized? A) Yes B) No

Answer: B (each state has a norm of 2)

QMII-2. In spin space, the basis states (eigenstates of S^2 , S_z) are orthogonal: $\langle \uparrow | \downarrow \rangle = 0$.

Are the following matrix elements zero or non-zero?

$\langle \uparrow | S^2 | \downarrow \rangle$ $\quad$ $\langle \uparrow | S_z | \downarrow \rangle$

A) Both are zero $\quad$ B) Neither are zero

C) The first is zero; second is non-zero

D) The first is non-zero; second is zero

Answer: A. Since the kets on the right are eigenstates of both operators, the eigenvalues can be pulled outside, and one is left with $\langle \uparrow | \downarrow \rangle = 0$.

QMII-3. A spin ½ particle in the spin state $|\chi\rangle = a|\uparrow\rangle + b|\downarrow\rangle$. A measurement of S_z is made. What is the probability that the value of S_z will be $+\hbar/2$?

A) $\left|\langle\uparrow|S_z|\chi\rangle\right|^2$ B) $\left|\langle\uparrow|\chi\rangle\right|^2$ C) $\left|\langle\chi|S_z|\chi\rangle\right|^2$

D) $\left|\langle\uparrow|S_z|\uparrow\rangle\right|^2$ E) None of these

Answer: B. Of course, this is also equal to $|a|^2$.

QMII-4. The raising operator operating on the up and down spin states:
$S_+\left|\downarrow\right\rangle = \hbar\left|\uparrow\right\rangle$, $S_+\left|\uparrow\right\rangle = 0$ What is the matrix form of the S_+ ?

A) $\hbar\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ B) $\hbar\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ C) $\hbar\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ D) $\hbar\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$
E) None of these.

Answer: B.

QMII-5. Is the raising operator S_+ Hermitian?
A) Yes, always B) No, never C) sometimes

Answer: B. The Hermitian conjugate of S_+ is S_-

QMII-6. Consider the matrix equation $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \lambda \begin{pmatrix} a \\ b \end{pmatrix}$.

This is equivalent to

A) $\begin{pmatrix} -\lambda & 1 \\ 1 & -\lambda \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$

B) $\begin{pmatrix} 0 & 1-\lambda \\ 1-\lambda & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$

C) $\begin{pmatrix} 1-\lambda & 0 \\ 0 & 1-\lambda \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$

D) $\begin{pmatrix} -\lambda & 1-\lambda \\ 1-\lambda & -\lambda \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0$

E) None of these

Answer: A. (If you have trouble seeing this, insert the identity matrix between λ and the column vector on the right-hand side of the equation.)

QMII-7. Suppose a spin ½ particle is in the spin state $|\chi\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$,

the $+\hbar/2$ eigenstate of $\hat{S}_z$. Suppose we measure S_x and then immediately measure S_z. What is the probability that the second measurement (S_z) will leave the particle in the S_z = down state:

$|\chi\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$?

A) zero B) non-zero

Answer: B. S_x and S_z are incompatible, so the measurement of S_x will change the state to something that is not an eigenstate of S_z. The new state (which is an eigenstate of S_x) will have nonzero components of both $|\uparrow\rangle_z$ and $|\downarrow\rangle_z$.

QMII-8. A quantum system consists of two particles, one of spin ½, and the other with spin 3/2. What is the dimension of the spin Hilbert space for this system?

A) 3/4 B) 15/4 C) 2 D) 8 E) I don't know

Answer: D. Each spin Hilbert space has dimension (2s+1). So the tensor product space has dimension 2 x 4 = 8.

QMII-9. Scandium has one electron in the 3d shell. If we measure the z-component of that electron's total angular momentum, how many possible values might we get?

A) 2 B) 5 C) 6 D) 10 E) 12

Answer: C. This is a bit tricky. In the d shell, l=2, and the electron has s=1/2. So the total j can be 5/2 or 3/2. Hence m_j can be any of the six values with integer spacing between -5/2 and 5/2.

QMII-10. Consider Scandium again. If we measure S^2 of the 3d electron, what possible values might we get?

A) $\pm\frac{1}{2}\hbar$ B) $\frac{1}{2}\hbar$ only C) $\frac{3}{4}\hbar^2$ only D) $\frac{3}{4}\hbar^2$ or $\frac{15}{4}\hbar^2$
E) None of these

Answer: C. s=1/2 for an electron.

QMII-11. Consider Scandium again. If we measure J^2 (J is the total angular momentum) of the 3d electron, what possible values might we get?

A) $\frac{3}{4}\hbar^2$ B) $\frac{15}{4}\hbar^2$ C) $\frac{35}{4}\hbar^2$ D) All of these E) B and C only

Answer: E. In the answer to problem 9, we said that j can be 5/2 or 3/2.

QMII-12. Consider Scandium again. If we don't know anything about the outermost electron other than that it is in a 3d orbital, what is the probability that a measurement of J^2 will produce the result $\frac{15}{4}\hbar^2$?

A) 1/2 B) 2/5 C) 3/7 D) 2/3
E) Impossible to compute without table of Clebsch-Gordan coefficients.

Answer: B. There are 4 states with j=3/2, and 6 states with j=5/2. If they are all equally probable, then the answer is 4/(4+6) = 2/5.

QMII-13. Which of the following two-electron quantum states satisfies the requirements of the Spin-Statistics Theorem? (For this problem, my notation for the spin states will be $\left|\chi_s^{m_s}\right\rangle$ rather than $\left|s, m_s\right\rangle$.

A) $\left|\Phi_A\right\rangle^{(1)}\left|\Phi_B\right\rangle^{(2)}\left|\chi_0^0\right\rangle$

B) $\frac{1}{\sqrt{2}}\left(\left|\Phi_A\right\rangle^{(1)}\left|\Phi_B\right\rangle^{(2)} - \left|\Phi_B\right\rangle^{(1)}\left|\Phi_A\right\rangle^{(2)}\right)\left|\chi_0^0\right\rangle$

C) $\frac{1}{\sqrt{2}}\left(\left|\Phi_A\right\rangle^{(1)}\left|\Phi_B\right\rangle^{(2)} + \left|\Phi_B\right\rangle^{(1)}\left|\Phi_A\right\rangle^{(2)}\right)\left|\chi_0^0\right\rangle$

D) $\frac{1}{\sqrt{2}}\left(\left|\Phi_A\right\rangle^{(1)}\left|\Phi_B\right\rangle^{(2)} + \left|\Phi_B\right\rangle^{(1)}\left|\Phi_A\right\rangle^{(2)}\right)\left|\chi_1^0\right\rangle$

E) Both B and D

Answer: C. Electrons are Fermions, hence the state must be antisymmetric under exchange of the two particles. The s=0 spin state is antisymmetric under exchange, while the s=1 spin state is symmetric.

QMII-14. Consider two identical spin-1 particles. We want to find eigenstates of the total spin $\vec{S} = \vec{S}_1 + \vec{S}_2$. Which one of the following statements is correct? (I have omitted all tensor product symbols.)

A) $|s=1, m_s=1\rangle = \frac{1}{\sqrt{2}}\left(|1,1\rangle^{(1)}|1,0\rangle^{(2)} + |1,0\rangle^{(1)}|1,1\rangle^{(2)}\right)$

B) $|s=1, m_s=1\rangle = \frac{1}{\sqrt{2}}\left(|1,1\rangle^{(1)}|1,0\rangle^{(2)} - |1,0\rangle^{(1)}|1,1\rangle^{(2)}\right)$

C) $|s=1, m_s=1\rangle = \frac{1}{\sqrt{2}}\left(|1,1\rangle^{(1)}|0,0\rangle^{(2)} + |0,0\rangle^{(1)}|1,1\rangle^{(2)}\right)$

D) $|s=1, m_s=1\rangle = \frac{1}{\sqrt{2}}\left(|1,1\rangle^{(1)}|1,-1\rangle^{(2)} - |1,-1\rangle^{(1)}|1,1\rangle^{(2)}\right)$

E) Help, I need my Table of Clebsch-Gordan coefficients!

Answer: B. You can rule out C because one particle there has spin 0. You can rule out D because it would give m_s=0. The state shown on the right-hand side in A is actually $|s=2, m_s=1\rangle$, which you can show by applying the lowering operator to the state $|s=2, m_s=2\rangle$.

QMII-15. Consider two identical spin-1 bosons. Which of the following two-electron quantum states satisfies the requirements of the Spin-Statistics Theorem? (For this problem, my notation for the spin states will be $\left|\chi_s^{m_s}\right\rangle$ rather than $\left|s,m_s\right\rangle$.

A) $\frac{1}{\sqrt{2}}\left(\left|\Phi_A\right\rangle^{(1)}\left|\Phi_B\right\rangle^{(2)}+\left|\Phi_B\right\rangle^{(1)}\left|\Phi_A\right\rangle^{(2)}\right)\left|\chi_2^1\right\rangle$

B) $\frac{1}{\sqrt{2}}\left(\left|\Phi_A\right\rangle^{(1)}\left|\Phi_B\right\rangle^{(2)}-\left|\Phi_B\right\rangle^{(1)}\left|\Phi_A\right\rangle^{(2)}\right)\left|\chi_2^1\right\rangle$

C) $\frac{1}{\sqrt{2}}\left(\left|\Phi_A\right\rangle^{(1)}\left|\Phi_B\right\rangle^{(2)}+\left|\Phi_B\right\rangle^{(1)}\left|\Phi_A\right\rangle^{(2)}\right)\left|\chi_1^0\right\rangle$

D) $\frac{1}{\sqrt{2}}\left(\left|\Phi_A\right\rangle^{(1)}\left|\Phi_B\right\rangle^{(2)}-\left|\Phi_B\right\rangle^{(1)}\left|\Phi_A\right\rangle^{(2)}\right)\left|\chi_1^0\right\rangle$

E) Both A and D

Answer: E. Since the particles are bosons, the state must be symmetric under exchange of the two particles. The states in the highest ladder (s=2 for this problem) are always symmetric under exchange. From the previous problem, you know that the states in the s=1 ladder are antisymmetric under exchange. All the states in a given spin ladder have the same symmetry under exchange.