# Physics 472 Midterm Exam #2 -- Wednesday, March 24, 2010

## Total points = 20.  Show all your work!

<u>Non-Degenerate Perturbation Theory:</u>  $H = H^0 + \lambda H'$    $H^0 \left| \psi_n^{(0)} \right\rangle = E_n^{(0)} \left| \psi_n^{(0)} \right\rangle$

$$E_n = E_n^{(0)} + \lambda \left\langle \psi_n^{(0)} \left| H' \right| \psi_n^{(0)} \right\rangle + \lambda^2 \sum_{m \neq n} \frac{\left| \left\langle \psi_m^{(0)} \left| H' \right| \psi_n^{(0)} \right\rangle \right|^2}{E_n^{(0)} - E_m^{(0)}} + ...$$

$$\left| \psi_n \right\rangle = \left| \psi_n^{(0)} \right\rangle + \lambda \sum_{m \neq n} \left| \psi_m^{(0)} \right\rangle \frac{\left\langle \psi_m^{(0)} \left| H' \right| \psi_n^{(0)} \right\rangle}{E_n^{(0)} - E_m^{(0)}} + ...$$

<u>Degenerate Perturbation Theory:</u>  If an energy level is $n$-fold degenerate with respect to $H^0$, then the first order energy shifts and corresponding eigenstates of $H$ are given by the eigenvalues and eigenvectors of the $n \times n$ matrix with elements $H'_{jk} = \left\langle \psi_j^{(0)} \left| H' \right| \psi_k^{(0)} \right\rangle$, where $j, k = 1, 2, ..., n$.

<u>Symmetries</u> are a powerful tool to help determine which matrix elements are zero.  For example, for hydrogen eigenstates $\left| nlm \right\rangle$, $[L_z, z] = 0$ implies $(m - m') \left\langle n'l'm' \left| z \right| nlm \right\rangle = 0$, and the parity transformation $\Pi z \Pi = -z$ implies $(-1)^{l+l'} \left\langle n'l'm' \left| z \right| nlm \right\rangle = -\left\langle n'l'm' \left| z \right| nlm \right\rangle$.

<u>Variational Method:</u> If the ground state energy of $H$ is $E_0$, then for any normalized state $\left| \psi \right\rangle$,

$$E_0 \leq \left\langle \psi \left| H \right| \psi \right\rangle$$

<u>Hydrogen Atom:</u>  $H = \dfrac{p^2}{2m} - \dfrac{e^2}{4\pi\varepsilon_0 r}$    $E_n = \dfrac{-Ry}{n^2}$  where  $Ry = \dfrac{1}{2} \dfrac{e^2}{4\pi\varepsilon_0 a_0} = \dfrac{\hbar^2}{2ma_0^2} = 13.6 \, \text{eV}$

$R_{10} = 2a^{-3/2} e^{-r/a}$        $R_{20} = \frac{1}{\sqrt{2}} a^{-3/2} \left(1 - \frac{1}{2}\frac{r}{a}\right) e^{-r/2a}$     $R_{21} = \frac{1}{\sqrt{24}} a^{-3/2} \frac{r}{a} e^{-r/2a}$

$Y_0^0 = \left(\frac{1}{4\pi}\right)^{1/2}$      $Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos\theta$      $Y_1^{\pm 1} = \mp \left(\frac{3}{8\pi}\right)^{1/2} \sin\theta \, e^{\pm i\phi}$

<u>Harmonic Oscillator:</u> $H = \dfrac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2$    $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

<u>Infinite Square Well for</u> $0 < x < a$:   $\psi_n(x) = \sqrt{\dfrac{2}{a}} \sin\left(\dfrac{n\pi x}{a}\right)$        $E_n = \dfrac{n^2 \pi^2 \hbar^2}{2ma^2}$

<u>Integrals:</u>

$$\int_0^\infty x^n e^{-x/a} \, dx = n! a^{n+1} \qquad \int_0^\infty x^{2n} e^{-x^2/a^2} \, dx = \sqrt{\pi} \frac{(2n)!}{n!} \left(\frac{a}{2}\right)^{2n+1} \qquad \int_0^\infty x^{2n+1} e^{-x^2/a^2} \, dx = \frac{n!}{2} a^{2n+2}$$