LECTURE #2

Note Title 1/13/2010

DRUDE THEORY

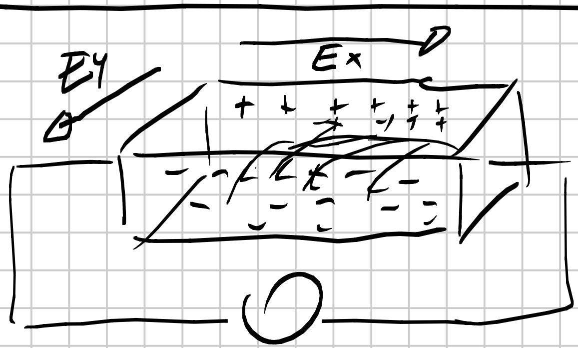
- CLASSICAL GAS OF ELECTRON
- COLLISIONS WITH FIXED POSITIVE IONS
- RELAXATION TIME τ

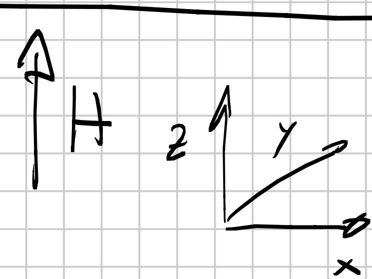
$\vec{J} = \sigma_0 \vec{E}$ $\qquad \sigma_0 = \dfrac{n e^2 \tau}{m}$

EQ. OF MOTION FOR e

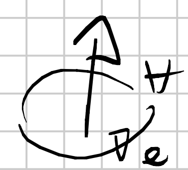
$$\boxed{\dot{\vec{p}}(t) = -\frac{\vec{p}(t)}{\tau} + \vec{F}}$$

$\dfrac{1}{\tau}$ DAMPING RATE





HALL



$$\dot{\vec{p}}(t) = -\frac{\vec{p}(t)}{\tau} - e\vec{E} - \frac{e}{cm}\left(\vec{p} \times \vec{H}\right)$$

$$\dot{p}_x = -\frac{p_x}{\tau} - eE_x - \left(\frac{eH}{mc}\right) p_y \qquad (I)$$

$$\dot{p}_y = -\frac{p_y}{\tau} - eE_y + \left(\frac{eH}{mc}\right) p_x \qquad (II)$$

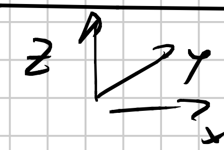
STEADY STATE $\Rightarrow$ $\dot{p}_x = 0 \quad \dot{p}_y = 0$

NO CURRENT IN y DIRECTION $\Rightarrow$ $p_y = 0$

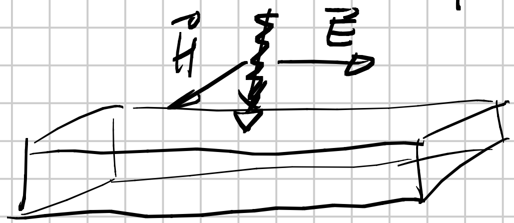
$(I) \Rightarrow p_x = -e\tau E_x \quad \Rightarrow \quad J_x = -\frac{en\, p_x}{m}$

$$E_y = -\left(\frac{eH}{mc}\right)\tau E_x \quad \Bigg| \quad R_H = \frac{E_y}{J_x H} = -\frac{1}{nec}$$

SENSITIVE TO SIGN OF CARRIERS



AC CONDUCTIVITY



MONOCHROMATIC

$$\vec{E}(t) = \mathrm{Re}\left[\vec{E}(\omega)\, e^{-i\omega t}\right]$$

$$\vec{p}(t) = \mathrm{Re}\left[\vec{p}(\omega)\, e^{-i\omega t}\right]$$

$\vec{E} \parallel \hat{x}$ $\quad$ $\vec{E} = E(\omega)\,\hat{x}$

$$\dot{p}(t) = -\frac{p}{\tau} - eE$$

$$-i\omega\, p(\omega) = -\frac{p(\omega)}{\tau} - eE(\omega)$$

• NEGLECT LORENTZ (H)

$$\vec{E}(t,\vec{r}) = E\, e^{i(k\cdot r - \omega t)}$$

$$|\vec{k}| = \frac{2\pi}{\lambda}$$

MEAN FREE PATH

$$\lambda \gg v_{\text{ELECTRONS}}\,\tau = \ell$$

$\Rightarrow \quad k = 0$

$$\left(-i\omega + \frac{1}{\tau}\right) p(\omega) = -eE(\omega)$$

AC CONDUCTIVITY

$$J(\omega) = -e\,n\,v(\omega) = -e\frac{n}{m}\,p(\omega)$$

$$J(\omega) = \boxed{\frac{\frac{ne^2\tau}{m}}{(1-i\omega\tau)}}\,E(\omega) \Rightarrow J(\omega) = \boxed{\sigma(\omega)}\,E(\omega)$$

$$\sigma(\omega) = \frac{\sigma_0}{(1-i\omega\tau)}$$

LINK $\sigma(\omega)$ TO $\varepsilon(\omega)$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t} \qquad \nabla \times \vec{H} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\vec{J}(\omega) = \sigma(\omega)\vec{E}(\omega)$$

$$\nabla \times \vec{E}(\omega) = i\frac{\omega}{c}\vec{H}(\omega)$$

$$\nabla \times \vec{H}(\omega) = \left[\frac{4\pi}{c}\sigma(\omega) - \frac{i\omega}{c}\right]\vec{E}(\omega)$$

$$\underline{\nabla \times \nabla \times \vec{E}} = i\frac{\omega}{c}\nabla \times \vec{H} = \frac{i\omega}{c}\left[\frac{4\pi}{c}\sigma(\omega) - \frac{i\omega}{c}\right]\underline{\vec{E}(\omega)}$$

$$-\nabla^2\vec{E} + \nabla(\cancel{\nabla \cdot E})^{=0} = $$

$$\vec{E} \sim E\hat{x}$$

$$\nabla^2 E + \frac{\omega^2}{c^2}\underbrace{\boxed{\left(1 + \frac{4\pi i\sigma(\omega)}{\omega}\right)}}_{\varepsilon(\omega)} E(\omega) = 0$$

$$\nabla^2 f + \frac{\omega^2}{v^2} f = 0$$

SPEED OF PROP IS v

$$\nabla^2 E + \left(\frac{\omega}{c/\sqrt{\varepsilon}}\right)^2 E = 0 \qquad \sqrt{\varepsilon} = n$$

INDEX OF REFRACTION

$$c' = \frac{c}{n} = \frac{c}{\sqrt{\varepsilon}}$$

$$\Downarrow$$

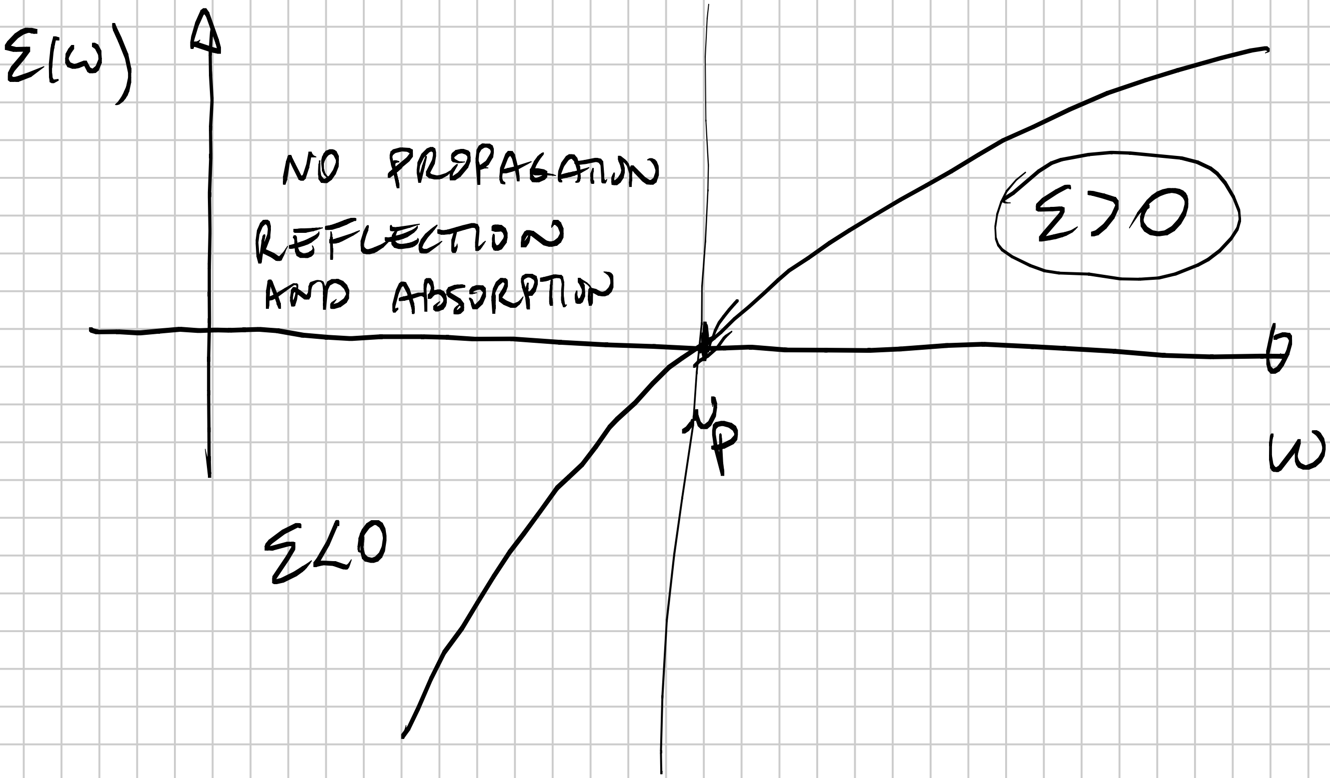
$$\boxed{\varepsilon(\omega) = 1 + \frac{4\pi i \sigma(\omega)}{\omega}} \qquad \sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$

OPTICAL RANGE $\quad \omega \gg \frac{1}{\tau} \Rightarrow \omega\tau \gg 1 \Rightarrow \sigma(\omega) = \frac{i\sigma_0}{\omega\tau}$

ω_p = PLASMA FREQUENCY

$$\varepsilon(\omega) = 1 - \frac{4\pi\sigma_0}{\omega^2\tau} = 1 - \frac{\frac{4\pi e^2 n}{m}}{\omega^2}$$

$$\varepsilon(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2$$



PROPAGATING WAVE $\Rightarrow$ TRANSPARENT MEDIUM

$n = \sqrt{\varepsilon}$

$\omega_p \sim$ IS IN THE UV

$$\omega_p^2 = \frac{4\pi e^2 n}{m}$$ IS RELATED TO PLASMONS

EM WAVES TRANSVERSAL MODES

$$\nabla \cdot \vec{E} = 0 \qquad \vec{k} \cdot \vec{E} = 0 \qquad \vec{E} \perp \vec{k}$$

DESCRIBE CHARGE OSCILLATIONS

$\rho(\omega) \neq 0$ OSCILLATION OF CHARGE PARALLEL TO $\vec{E}$

LONGITUDINAL EXCITATION $\Rightarrow$ PLASMONS

$$\nabla \cdot \vec{E} = 4\pi\rho(t)$$