# LECTURE # 28

Note Title 3/29/2010

CHAP 21-22-23

① CONDUCTIVITY

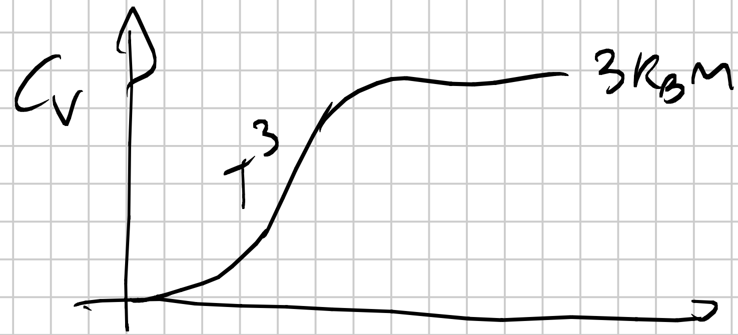
BLOCH ELECTRONS $\tau \to \infty \implies$ ELECTRON-PHONON SCATTERING

② SPECIFIC HEAT

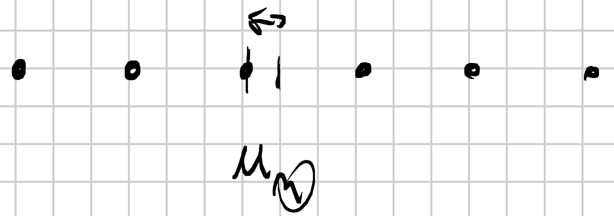
SOMMERFELD MODEL $C_V \sim \underline{\underline{T}}$

METAL

INSULATOR



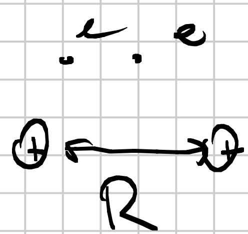

DEBYE MODEL




$\dot{U}(\{u_m\})$ TOTAL ENERGY

BORN – OPPENHEMER APPROXIMATIN

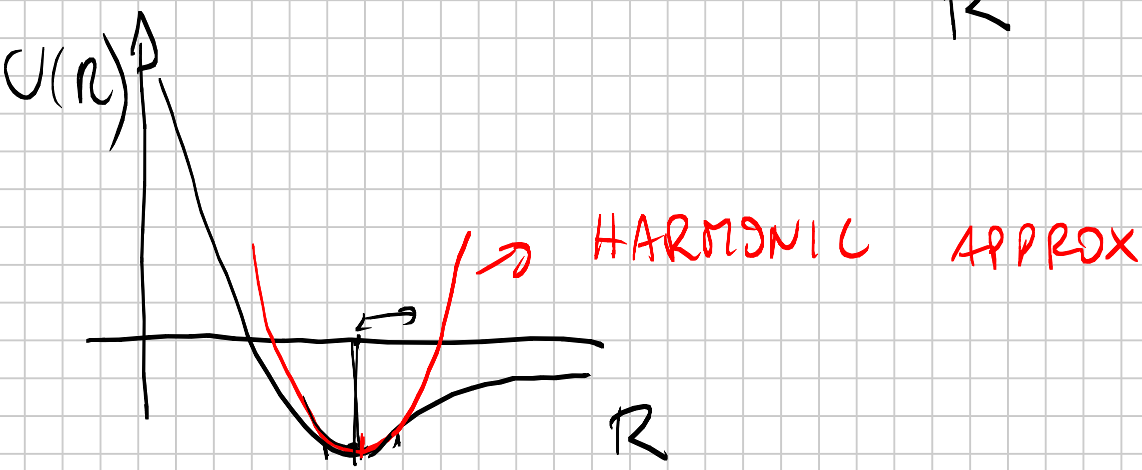
$H_2$




① FIX POSITION OF PROTONS AT R

② SOLVE PROBLEM FOR 2 ELECTRONS AS A FUNCTION OF R

$$U\{R\} = V_{eL}(R) + \frac{e^2}{R}$$

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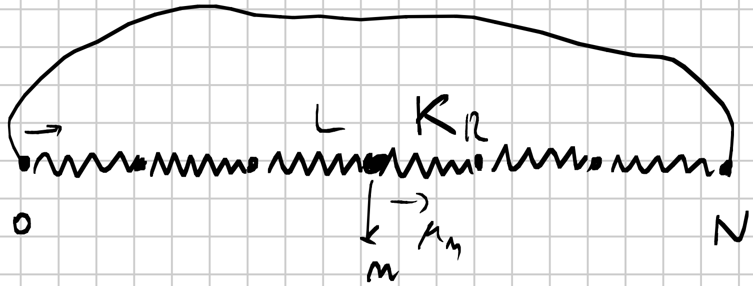
$$U = U_0 + \frac{1}{2}\sum_{n,m'} \frac{\partial^2 U}{\partial u_n \partial u_m}\bigg|_{\{u_n\}=0} u_n u_m + O(u_n^3)$$

$$U = \sum_n \phi(u_n - u_{n-1} + a) \to \text{ONLY FIRST NEIGHBORS}$$




$$K = \frac{d^2}{dx^2}\phi(x)\Big|_{x=a}$$

$u_0 = u_N$ PERIODIC BOUNDARY CONDITIONS

$$M\ddot{u}_m = -K\left(\overset{R}{u_m - u_{m+1}} + \overset{L}{(u_m - u_{m-1})}\right)$$

N NORMAL MODES

$$u_m = e^{i(qma - \omega t)}$$

$$q \to q + \frac{2\pi}{a} \qquad e^{iqma + 2\pi m} \Rightarrow$$ q ARE IN THE FIRST BRILLOUIN ZONE

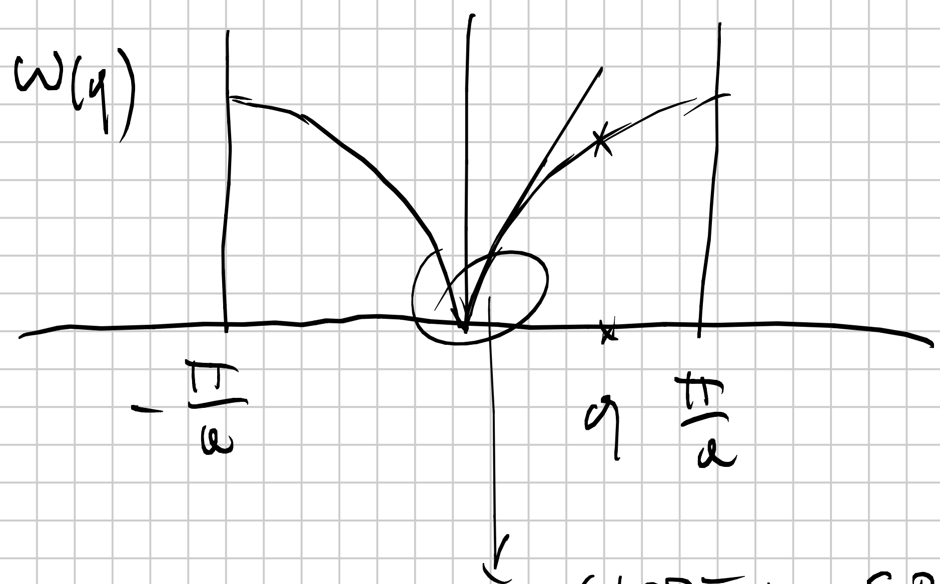
$$u_{m+1}(q) = e^{iqa}\, u_m(q)$$

$$-M\omega^2 \cancel{u_n(q)} = -K\left(2u_n(q) - u_{n+1}(q) - u_{n-1}(q)\right) =$$

$$-K\left(2 - e^{iqa} - e^{-iqa}\right)\cancel{u_n(q)}$$

$$\omega^2(q) = \frac{K}{M}\,2\left(1-\cos qa\right) = \frac{4K}{M}\sin^2\frac{qa}{2}$$

$$\Rightarrow \omega(q) = \sqrt{\frac{4K}{M}}\left|\sin\frac{qa}{2}\right|$$



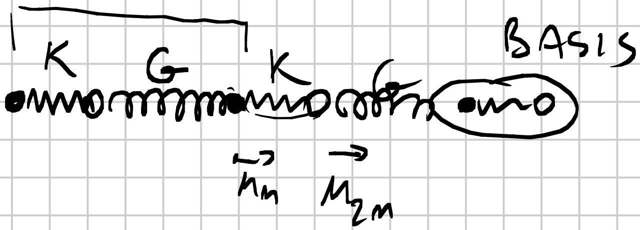

$$V_G = \frac{d\,\omega(q)}{dq}$$

$qa \ll 1$

$$V_G = \frac{d}{dq}\left(\sqrt{\frac{4K}{M}}\,\frac{qa}{2}\right) = \sqrt{\frac{K}{M}}\,a$$

SLOPE = SPEED OF SOUND $\quad v_s$

# 1D LATTICE WITH BASIS (2 ATOMS)




MASS • = MASS o

$u_{1m}$ $\qquad$ $u_{2m}$

$$M\ddot{u}_{1m} = -K(u_{1m} - u_{2m}) - G(u_{1m} - u_{2(m-1)})$$

$$M\ddot{u}_{2m} = -G(u_{2m} - u_{1m+1}) - K(u_{2m} - u_{1m})$$

$$\boxed{\begin{aligned} u_{1m}(q) &= \varepsilon_1 e^{i(qma-\omega t)} \\ u_{2m}(q) &= \varepsilon_2 e^{i(qma-\omega t)} \end{aligned}}$$

$$u_{2m-1} = e^{-iqa} u_{2m}$$

$$\left[M\omega^2 - (K+G)\right]\varepsilon_1 + \left(K+Ge^{-iqa}\right)\varepsilon_2 = 0$$

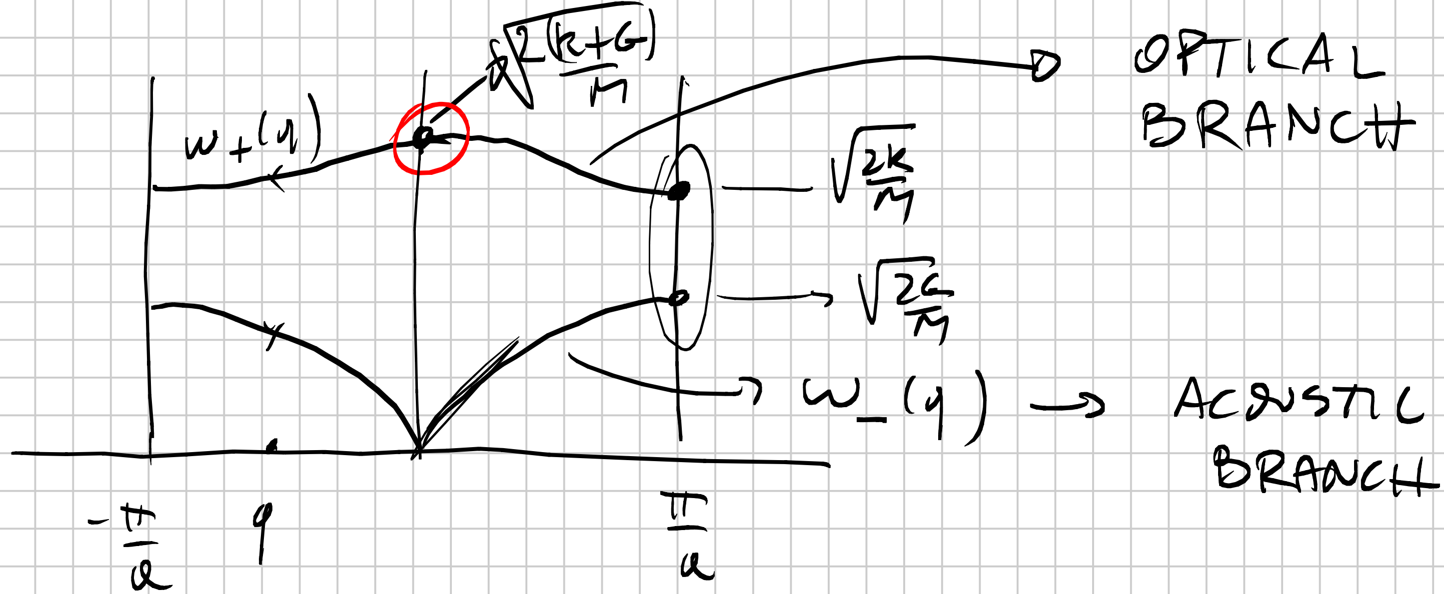
$$\left(K+Ge^{iqa}\right)\varepsilon_1 + \left[M\omega^2 - (K+G)\right]\varepsilon_2 = 0$$

$$M(\omega)\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix} = 0$$

$$\text{DET } M = 0 \Rightarrow \omega(q)$$

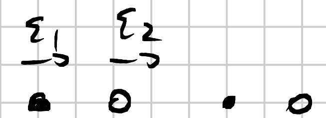
$$\omega_\pm^2(q) = \left(\frac{K+G}{M}\right) \pm \frac{1}{M}\sqrt{K^2+G^2+2KG\cos qa}$$

$$\frac{\varepsilon_2}{\varepsilon_1} = \mp \frac{K+G\,e^{iqa}}{\left|K+G\,e^{iqa}\right|}$$



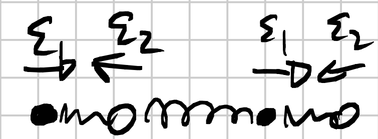

## FOR $q = 0$

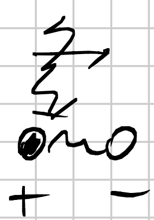
$\omega_-(q) \to 0 \qquad \frac{\varepsilon_2}{\varepsilon_1} \to 1$




ATOMS IN PHASE

$\omega_+(q) \to \sqrt{\frac{2(G+K)}{M}} \qquad \frac{\varepsilon_2}{\varepsilon_1} \to -1$



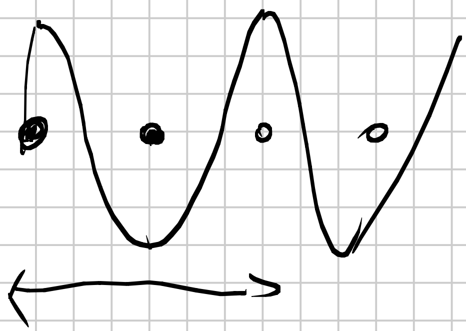




→ DIPOLE CAN BE EXCITED WITH LIGHT

→ OPTICAL MODES

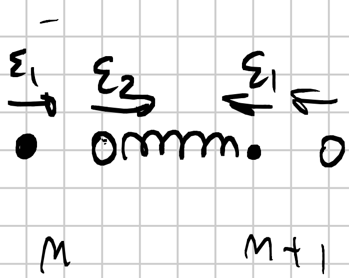
MODES AT $q = \frac{\pi}{a}$ ZONE BOUNDARY

$q = \frac{\pi}{a} \Rightarrow \lambda = \frac{2\pi}{q} \Rightarrow \lambda = 2a$



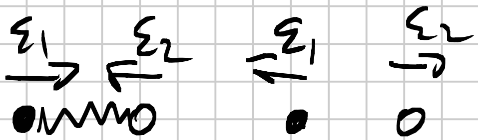
$\lambda = 2a$

ACOUSTIC MODE AT $q = \frac{\pi}{a}$; $\omega_-\left(\frac{\pi}{a}\right)$




$$\omega_-\left(\frac{\pi}{a}\right) = \sqrt{\frac{G}{M}}$$

OPTICAL MODE AT $q = \frac{\pi}{a}$




$$\omega_+\left(\frac{\pi}{a}\right) = \sqrt{\frac{K}{m}}$$

IN 3D

$\mu_{\parallel}(q)$ LONGITUDINAL

$2\ \mu_{\perp}(q)$ TRANSVERSAL