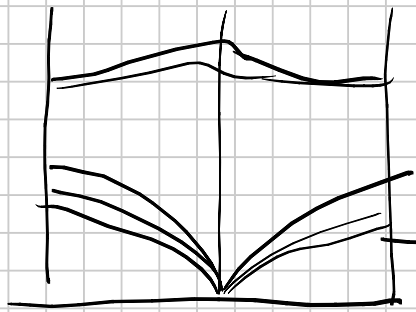
LECTURE #31

Note Title 4/5/2010



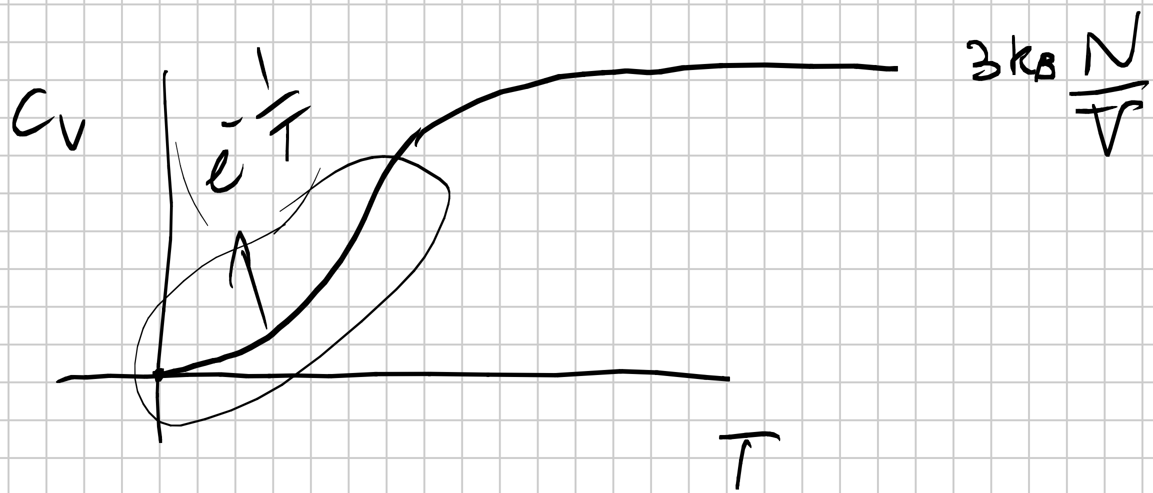
$$H^{TOT} = \sum_{q,m} \hbar \omega_m(q) \left(a^{+}_{q,m} a_{q,m} + \frac{1}{2} \right)$$

$$[a_{qm}\ a^{+}_{q'm'}] = \delta_{qq'}\, \delta_{m,m'} \qquad \langle a^{+}_{qm} a_{qm} \rangle = \frac{1}{e^{\frac{\hbar \omega_m(q)}{k_B T}} - 1}$$

EINSTEIN MODEL

$$\hbar \omega(q) \sim \hbar \omega_E$$

$$\langle E\rangle = 3NK_BT\,\frac{x}{e^x-1} \qquad x = \frac{\hbar\omega_E}{K_BT}$$



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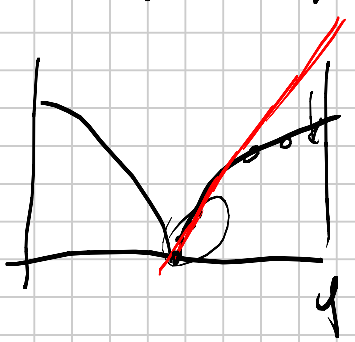
(2) DEBYE MODEL

$$\langle E\rangle = \sum_q \hbar\omega(q)\,\frac{1}{e^{\frac{\hbar\omega(q)}{K_BT}}-1}$$

(A) ACOUSTIC MODES ONLY



$$\hbar\omega(q) \simeq \hbar v|q|$$

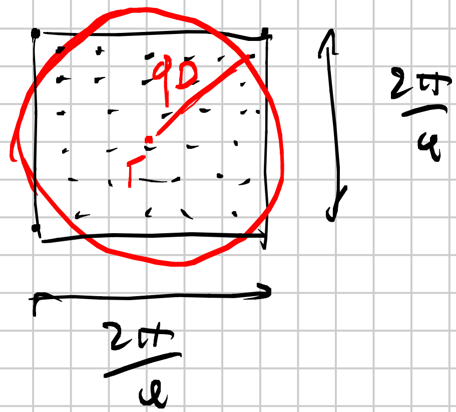
$\sin\frac{qa}{2}$

(B) $\sum_{q \in BZ} \to$ SPHERICAL APPROX

$\sum_{q \in BZ} =$

2D SQUARE

SIDE a



$$\boxed{\pi q_D^2 = \left(\frac{2\pi}{a}\right)^2} \to q_D^2 \sim \frac{1}{a^2} \sim n$$

$\hbar v q_D = \hbar \omega_D =$ DEBYE ENERGY

$k_B T_D = \hbar \omega_D$ $\qquad T_D\left(\frac{N}{V}\right)$

3D

$$\frac{4\pi}{3} q_D^3 = \frac{(2\pi)^3}{a^3}$$

$$\langle E \rangle = \sum_q \hbar\omega(q)\, n_q \to V \int_{BZ} \frac{d^3q}{(2\pi)^3} \frac{\hbar v |q|}{e^{\frac{\hbar v |q|}{k_B T}} - 1} \to$$

$$V \frac{4\pi}{(2\pi)^3} \int_0^{q_D} q^2\, dq \frac{\hbar v q}{e^{\frac{\hbar v q}{k_B T}} - 1}$$

$$\hbar v q = \hbar \omega$$

$$\langle E \rangle = V \int_0^{\omega_D} \frac{\omega^2}{2\pi^2 v^3} \frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1} d\omega$$

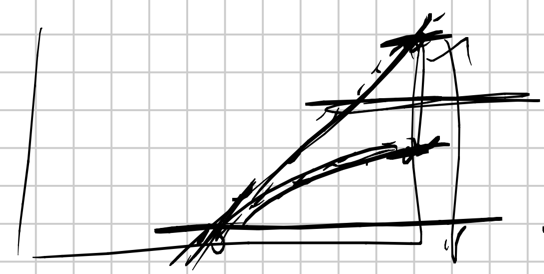
$$\boxed{\frac{\hbar\omega}{k_B T} = x}$$

$$\langle E \rangle = 9 N k_B T \left(\frac{T}{T_D}\right)^3 \int_0^{T_D/T} \frac{x^3}{e^x - 1} dx$$

LARGE $T \gg T_D \Rightarrow x \ll 1$

$$\frac{x^3}{e^x-1} \sim \frac{x^3}{\cancel{1}+x-\cancel{1}} \sim x^2 \Rightarrow \int_0^{T_D/T} x^2\,dx \sim \frac{1}{3}\left(\frac{T_D}{T}\right)^3$$

$$\langle E \rangle_{T \gg T_D} = \cancel{9}^{\,3} N k_B T \cancel{\left(\frac{T}{T_D}\right)^3} \cdot \frac{1}{\cancel{3}} \cancel{\left(\frac{T_D}{T}\right)^3} = \underline{3 N k_B T}$$

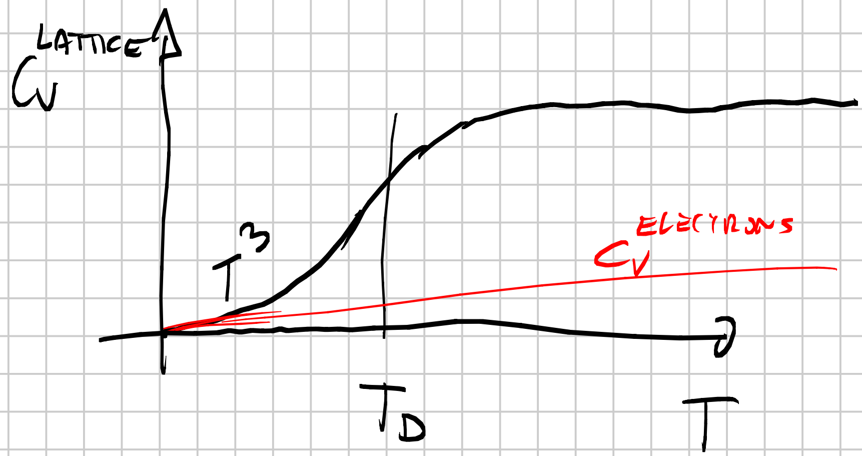


$T \ll T_D$

$$\int_0^{T_D/T} \longrightarrow \int_0^{\infty} \frac{x^3}{e^x-1}\,dx$$

$$\langle E \rangle = 9 N k_B T \left(\frac{T}{T_D}\right)^3 \boxed{\int_0^\infty \frac{x^3}{e^x - 1}\,dx} \rightarrow \frac{\pi^4}{15}$$

$$\langle E \rangle \propto T^4 \Rightarrow c_V = \frac{1}{V}\frac{\partial \langle E \rangle}{\partial T} \propto T^3$$



$T_D \sim 100\,K$

LATTICE CONTRIBUTION TO THERMAL CONDUCTIVITY

$$J_Q = -\varkappa_L \nabla T$$

$\propto \tau$

HARMONIC APPROXIMATION

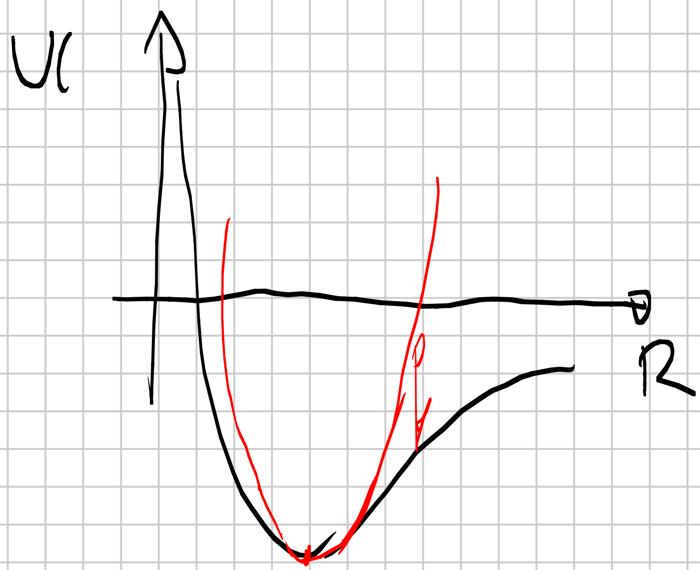
$\kappa_L \longrightarrow \infty$

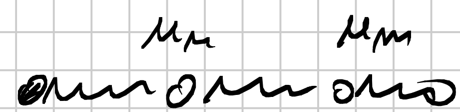
IN REALITY $\kappa_L \neq \infty$ DUE TO 3 CONTRIBUTIONS

1. IMPURITIES
2. SURFACE EFFECTS

] SAMPLE DEPENDENT

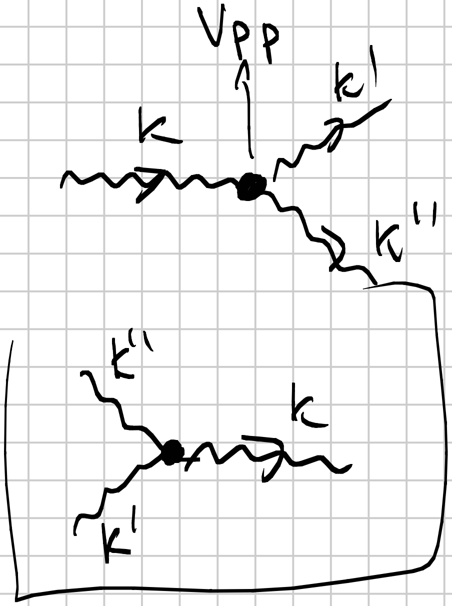
3. ANHARMONIC EFFECTS

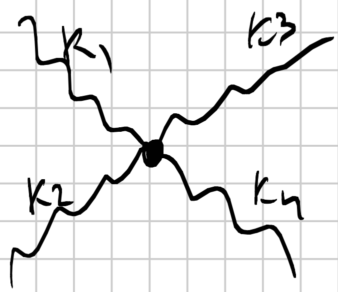




$$U = U_0 + \sum_{nm} \left(\frac{\partial^2 U}{\partial u_n \partial u_m} \right)_{\vec{u}=0} u_n u_m +$$

V_{pp} INTERACTION $\longrightarrow$ $O(u_m^3) + O(u_m^4)$





CUBIC TERMS

① ENERGY IS CONSERVED

② QUASI MOMENTUM IS CONSERVED

$$K = K' + K'' + G$$

G RECIPROCAL LATTICE VECTORS

I

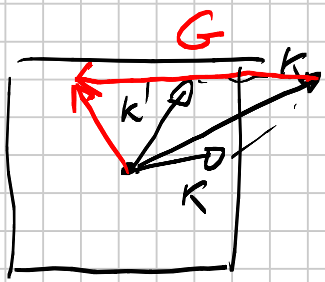
NORMAL PROCESSES

$G = 0$

II

UMKLAPP PROCESSES

$G \neq 0$



UMKLAPP IMPORTANT FOR κ_L

UMKLAPP → "FLIPPING"

(GERMAN)