LECTURE # 4

Note Title 1/20/2010

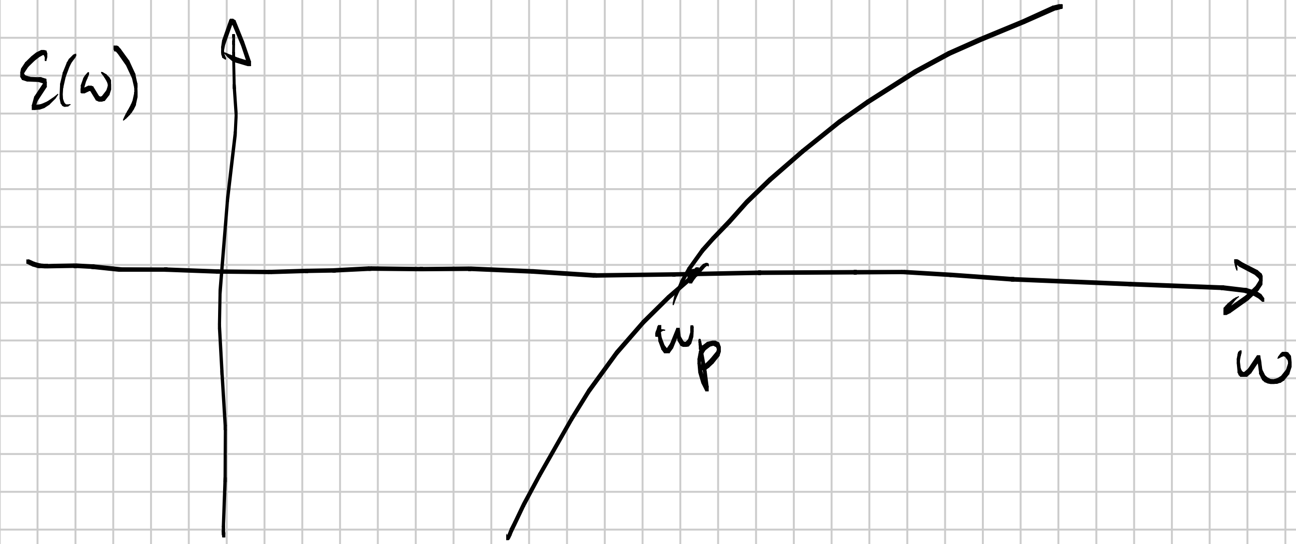
DRUDE MODEL

- $\vec{J} = \sigma_0 E \qquad \sigma_0 = \dfrac{ne^2\tau}{m}$
- HALL EFFECT $\qquad R_H = -\dfrac{1}{nec}$
- AC $\quad \sigma(\omega) = \dfrac{\sigma_0}{1 - i\omega\tau} \quad \longleftrightarrow \quad \varepsilon(\omega)$

$$\varepsilon(\omega) = 1 + \frac{4\pi i \sigma(\omega)}{\omega}$$

LIMIT $\omega\tau \gg 1 \qquad \varepsilon(\omega) \sim 1 - \left(\dfrac{\omega_p}{\omega}\right)^2$

ω_p PLASMA FREQUENCY $\qquad \omega_p^2 = \dfrac{4\pi e^2 n}{m}$



PROPAGATING WAVES

$$\vec{\nabla}\cdot\vec{E} = 0$$

ω_p ALSO DESCRIBES A CHARGE DENSITY OSCILLATION / PLASMON

$$\rho(t) \sim \rho(\omega)\, e^{-i\omega t}$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho(t) \qquad \vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0$$

$$\vec{J}(\omega) = \sigma(\omega)\,\vec{E}(\omega)$$

$$\vec{\nabla}\cdot\sigma(\omega)\,\vec{E}(\omega) = i\omega\rho(\omega)$$

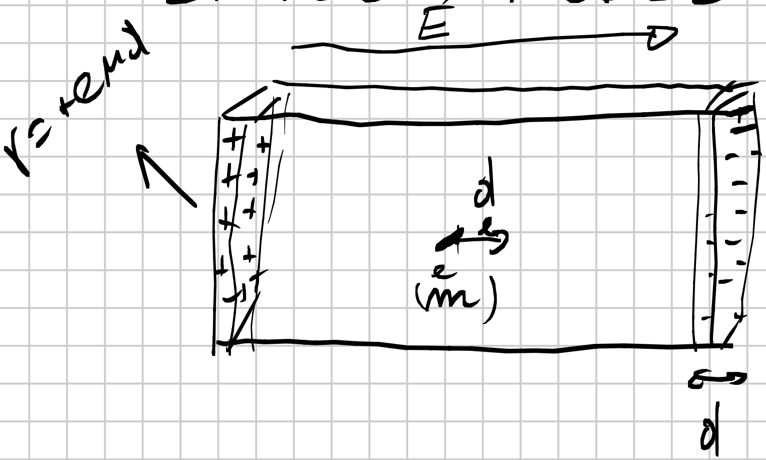
$$4\pi\,\sigma(\omega)\,\rho(\omega) = i\omega\,\rho(\omega)$$

$$i\omega\left(1+\frac{4\pi i\,\sigma(\omega)}{\omega}\right)\rho(\omega)=0$$

I CAN HAVE $\rho(\omega)\neq 0$ ONLY IF $\left(1+\frac{4\pi i\sigma(\omega)}{\omega}\right)=\varepsilon(\omega)=0$

$\varepsilon(\omega)=0$ ONLY FOR $\omega=\omega_p$

SIMPLE MODEL FOR PLASMON



ACCUMULATION OF ELECTRONS

SURFACE CHARGE

$\gamma = -e\,n\cdot d$

$\vec{E}$ IS THE ELECTRIC FIELD $\Rightarrow \vec{E} = 4\pi|\gamma|$

$$m\ddot{d} = -eE = -e\,4\pi\,(e n d) \Rightarrow \ddot{d} = \frac{4\pi e^2 n}{m}\, d$$

$$\frac{4\pi e^2 n}{m} \rightarrow \omega_p^2$$

$\vec{J}_e = (-e) n \vec{v}$ FLUX OF CHARGE

T_2

T_1 $T_2 > T_1$

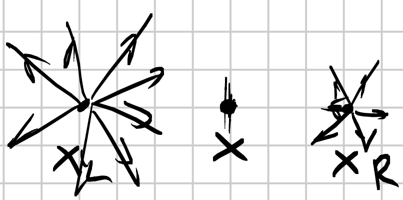
STUDY THE FLOW OF ENERGY FROM LEFT → RIGHT

$\mathcal{E}$ → KINETIC ENERGY OF ELECTRONS

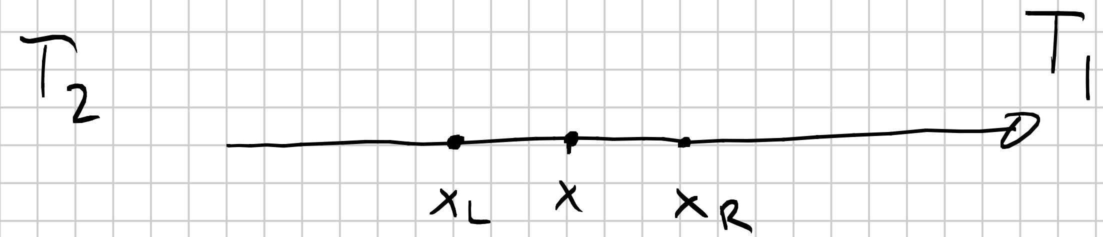
$$\vec{J}_Q = \mathcal{E} n \vec{v}$$

$$\vec{J}_Q = -\kappa \nabla T$$

κ → THERMAL CONDUCTIVITY



→ MAKE 1D MODEL



$$J_Q(x) = \frac{n}{2}\varepsilon(x_L)v + \frac{n}{2}\varepsilon(x_R)(-v) =$$

$$J_Q(x) = \frac{nv}{2}\left[\varepsilon(x-\tau v) - \varepsilon(x+\tau v)\right] \qquad (\tau v \ll 1)$$

$$\to J_Q(x) = \frac{nv}{2}\left[\cancel{\varepsilon(x)} - \frac{d\varepsilon}{dx}\tau v - \cancel{\varepsilon(x)} - \frac{d\varepsilon}{dx}\tau v\right]$$

$$J_Q(x) = -\overbrace{n}^{\frac{N}{V}} v \frac{d\varepsilon}{dx} v\tau = -\overbrace{\boxed{\frac{1}{V}\frac{dN\varepsilon}{dT}}}^{=\frac{1}{V}\frac{dE_{tot}}{dT} = c_V}\frac{dT}{dx} v^2\tau$$

$$J_Q = - C_V v^2 \tau \frac{dT}{dx}$$

$$v^2 = v_x^2$$

$$3D \rightarrow \quad v_x^2 = v_y^2 = v_z^2$$

$$v_x^2 = \frac{|\vec{v}|^2}{3}$$

$$\vec{J}_Q = - C_V \frac{v^2}{3} \tau \nabla T \Rightarrow \kappa = C_V \frac{v^2}{3} \tau$$

ELECTRONIC THERMAL CONDUCTIVITY

$$\langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} k_B T$$

CLASSICAL MODEL FOR ELECTRONS

$$\langle v^2 \rangle = 3 k_B T / m$$

$$C_V = \frac{3}{2} k_B n \Rightarrow \kappa = \boxed{\frac{3}{2} k_B^2 \frac{n \tau}{m} T}$$

$$\frac{\kappa}{\sigma_0} = \frac{3}{2}\left(\frac{K_B}{e}\right)^2 T$$ WIEDEMANN - FRANZ LAW

κ COMPARES WELL WITH EXPERIMENTS

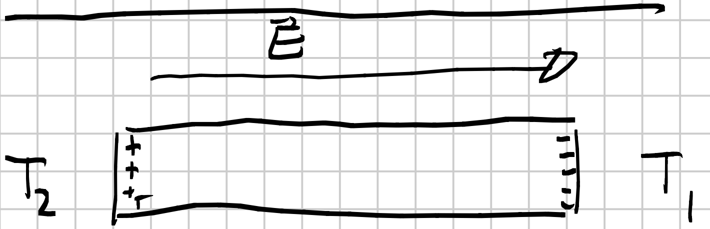
EXPLAINS WIEDEMANN - FRANZ

$\kappa \propto C_V \cdot v^2$ $\rightarrow$ TWO ERRORS COMPENSATE

	CLASSICAL	FERMIONS	
v^2	$\propto K_B T$	$K_B T_F$	$\frac{v^2_{CLASS}}{v^2_{FERMI}} \sim \frac{1}{100}$
C_V	$\frac{3}{2} n K_B$	$\sim \boxed{\frac{3}{2} n K_B \left(\frac{K_B T}{K_B T_F}\right)}$	$\frac{C_V^{CLASS}}{C_V^{FERMIONS}} \sim 100$

THERMOPOWER

(SEEBECK EFFECT)



$$\vec{E} = Q\,\nabla T$$

Q THERMOPOWER