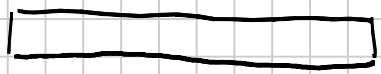
LECTURE #6

Note Title 1/25/2010

T_2  T_1

$$\vec{J}_Q = n\vec{v}\mathcal{E} = -\kappa \nabla T$$

κ THERMAL CONDUCTIVITY (ELECTRONIC CONTRIBUTION)

$$\kappa = \frac{1}{3} v^2 C_V \tau$$

v^2 (UNDER-ESTIMATED BY DRUDE BY 100)

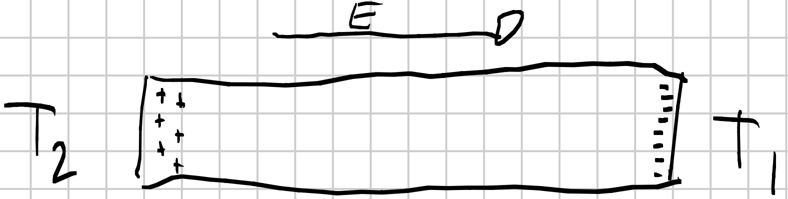
C_V (OVER-ESTIMATED " " " ")

WIEDEMANN - FRANZ LAW

$$\boxed{\frac{\kappa}{\sigma_e} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T}$$

THERMOPOWER

SEEBECK EFFECT

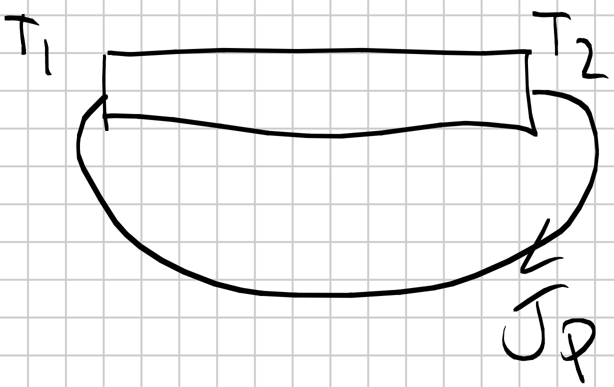


$$\vec{E} = Q \nabla T$$

$\vec{v}_Q$ L → R DUE TO ∇T

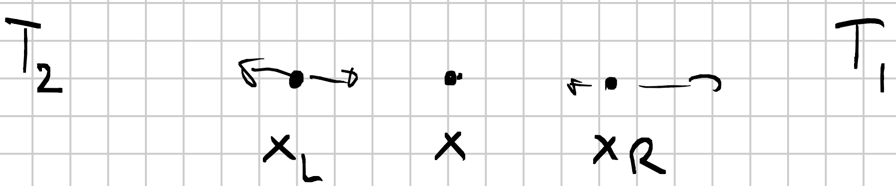
$\vec{v}_E$ R → L DUE TO $\vec{E}$

AT EQULIBRIUM $\vec{v}_Q + \vec{v}_E = 0$



$J^{Peltier} \propto \nabla T$ $\qquad J^P \propto v_Q$

PELTIER EFFECT



x

$$v_Q(x) = \frac{v(x_L)}{2} - \frac{v(x_R)}{2} = \frac{v(x - v\tau)}{2} - \frac{v(x + v\tau)}{2}$$

$v\tau \ll 1$

$$v_Q(x) = -v\tau \frac{dv}{dx} = -\frac{\tau}{2} \frac{dv^2}{dx} \qquad \text{(1D CASE)}$$

$$\langle |\vec{v}|^2 \rangle = \frac{2}{m} \langle \mathcal{E} \rangle$$

1D CASE $\to$ 3D CASE

$$\langle v^2 \rangle \longrightarrow \frac{\langle v^2 \rangle}{3}$$

$$\vec{v}_Q = -\frac{\tau}{6} \frac{d\langle |v|^2 \rangle}{dx}$$

$$\frac{d\mathcal{E}}{dx} = \frac{d\mathcal{E}}{dT}\frac{dT}{dx}$$

$$\vec{v}_E = -\frac{e\vec{E}\tau}{m}$$

$$\vec{v}_Q + \vec{v}_E = 0 \Rightarrow -\frac{\tau}{3m}\frac{d\mathcal{E}}{dT}\cancel{\nabla T} - \frac{e\tau}{m} Q \cancel{\nabla T} = 0$$

$$\frac{d\mathcal{E}}{dT} = \frac{c_V}{n} \qquad c_V = \frac{3}{2} k_B n \quad \left(\text{CLASSICAL GAS}\right)$$

$$Q = -\frac{1}{3}\frac{c_V}{en} = -\frac{1}{2}\left(\frac{k_B}{e}\right)$$

TOO BIG BY TWO ORDERS OF

$\Rightarrow$ QUANTUM GAS OF ELECTRONS

SOMMERFELD MODEL

$$f_{MB}(\varepsilon) \sim \underbrace{n\lambda_T^3}\, e^{-\beta\varepsilon}$$

$$\beta = \frac{1}{k_B T}$$

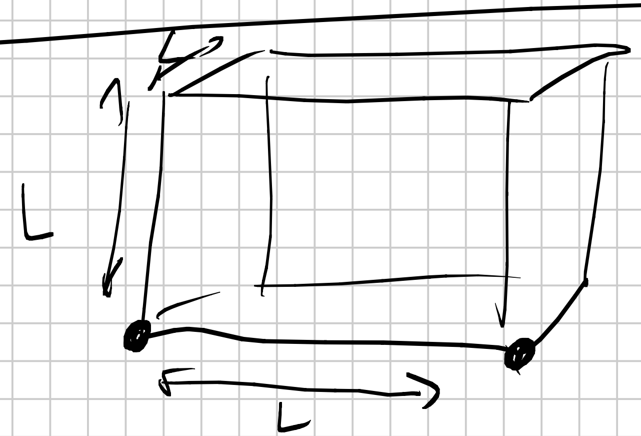
$$\frac{\hbar^2}{2m}\frac{1}{\lambda_T^2} = k_B T$$

$$\downarrow$$

$$\boxed{f_{FD}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}+1}}$$

$\lambda_T =$ THERMAL WAVELENGTH $= \sqrt{\dfrac{\hbar^2}{2m k_B T}}$

$n\lambda_T \ll 1 \Rightarrow f_{MB}$ IS OK



$$\psi_{\vec{k}}(\vec{r}) = \frac{1}{\sqrt{L^3}}\, e^{i\vec{k}\cdot\vec{r}}$$

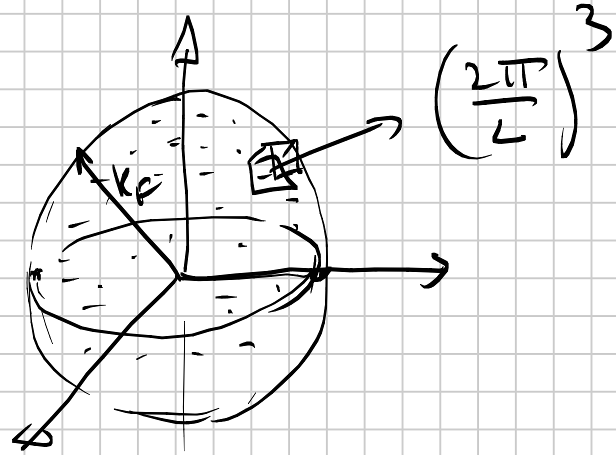
BORN VON KARMAN BOUNDARY CONDITIONS $\psi_k(L,0,0) = \psi_k(0,0,0)$

$e^{ik_x \cdot L} = 1$ $\qquad$ $k_x = m\dfrac{2\pi}{L}$

$\forall\ \vec{k} \to \left(\dfrac{2\pi}{L}\right)^3$ ELEMENT OF VOLUME

$T = 0$ PROPERTIES

FERMI SPHERE



$$N = \frac{\frac{4}{3}\pi k_F^3}{\left(\frac{2\pi}{L}\right)^3} \times \overset{\text{SPIN}}{\overset{\downarrow}{2}} \Rightarrow \frac{N}{L} = \boxed{m = \frac{k_F^3}{3\pi^2}}$$

$$E = 2 \int_{\substack{\text{FERMI} \\ \text{SPHERE}}} \frac{\hbar^2 k^2}{2m} \frac{d^3k}{\left(\frac{2\pi}{L}\right)^3} \Rightarrow \frac{1}{\pi^2} \frac{\hbar^2 k_F^5}{10\, m} L^3$$

$$\varepsilon_F = \frac{\hbar^2 k_F^2}{2m} \qquad K_B T_F = \varepsilon_F$$

$$\frac{E}{N} = \frac{3}{5} K_B T_F \qquad \langle |v|^2 \rangle = \frac{2\langle \varepsilon \rangle}{m}$$

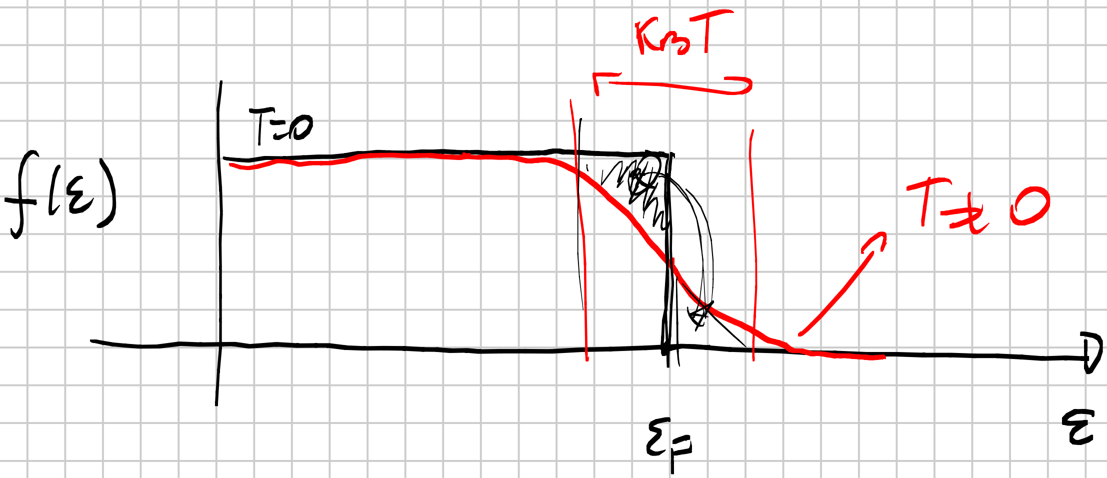
CLASSICAL CASE $\quad \langle v^2 \rangle \propto \frac{E}{N} \propto \frac{3}{2} K_B T$

QUANTUM CASE $\quad \langle v^2 \rangle \propto \frac{E}{N} \quad \frac{3}{5} K_B T_F \qquad T_F \sim 10^4\, K$

SPECIFIC HEAT

$$C_V = \frac{1}{V}\frac{dE}{dT}$$

QUALITATIVE ESTIMATION

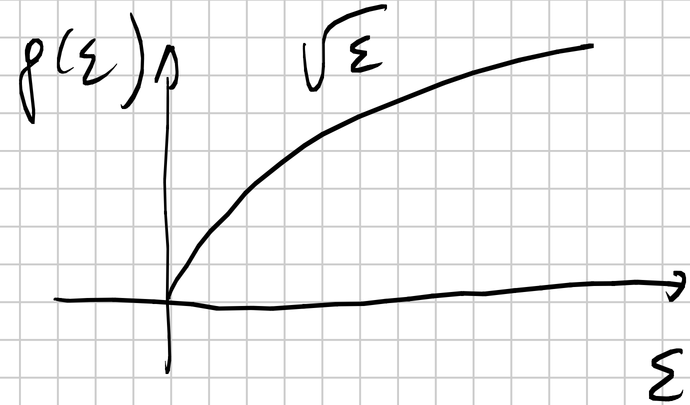


DENSITY OF STATES AT ε_F

$$E(T) \sim E(T=0) + \#\,\text{ELECTRONS EXCITED} \times K_B T = E(0) + \frac{g(\varepsilon_F) \times K_B T \times K_B T}{2}$$

$$g(\varepsilon) = 2\int \frac{d^3k}{(2\pi)^3}\,\delta\!\left(\varepsilon - \frac{\hbar^2 k^2}{2m}\right)$$ DENSITY OF STATES.

$$g(\varepsilon) = \frac{3}{2}\left(\frac{n}{\varepsilon_F}\right)\left(\frac{\varepsilon}{\varepsilon_F}\right)^{1/2}$$



IN 3D SYSTEMS

$$g(\varepsilon_F)$$

$$E(T) \sim E_0 + \frac{3}{2}\left(\frac{n}{\varepsilon_F}\right)\left(\frac{\varepsilon_F}{\varepsilon_F}\right)^{1/2} \frac{(k_B T)^2}{2}$$

$$\frac{1}{V}\frac{dE(T)}{dT} \sim \frac{3}{2} n k_B \left(\frac{k_B T}{\varepsilon_F}\right) = C_V$$

CLASSICAL CASE $\quad C_V = \frac{3}{2} n k_B$

QUANTUM CASE $\quad C_V \sim \frac{3}{2} n k_B \cdot \left(\frac{k_B T}{k_B T_F}\right)$