Anisotropies in the cosmic background radiation —23 Feb 2010

- Homework 4
  - Problem 2c
    Recall for $\Omega_0 > 0$, the comoving coordinate is double valued. Use the variable $\xi = \arcsin H_0 r$.
  - Problem 6 (Hartle 18.3)
    Assume pressureless matter. Assume the speed is nonrelativistic $V_* \ll 1$.
    Hint: What is conserved?
  - Problem 7 (Harthe 18.20)
    You may ask *Mathematica* to do the integral.
- Wilkinson Microwave Anisotropy Probe (WMAP) satellite
- "Dipole" anisotropy
- Compute angular scale of "acoustic peak," the largest anisotropy beside the dipole anisotropy.

Values of the cosmological parameters

- Hubble's constant
  - $H_0 = (72 \pm 7)$km/s/Mpc
  - $H_0^{-1} = 14$ Gyr
    $= 4300$ Mpc
- Density parameter
  - $\Omega_{v0} = 0.74$ vacuum, also called cosmological constant Λ, also called dark energy
  - $\Omega_{m0} = 0.26$ pressureless matter, mostly dark matter, matter that does not interact with light
  - $\Omega_{b0} = 0.043$ baryons, ordinary matter
  - $\Omega_{r0} = 1.2 \times 10^{-5}$ radiation

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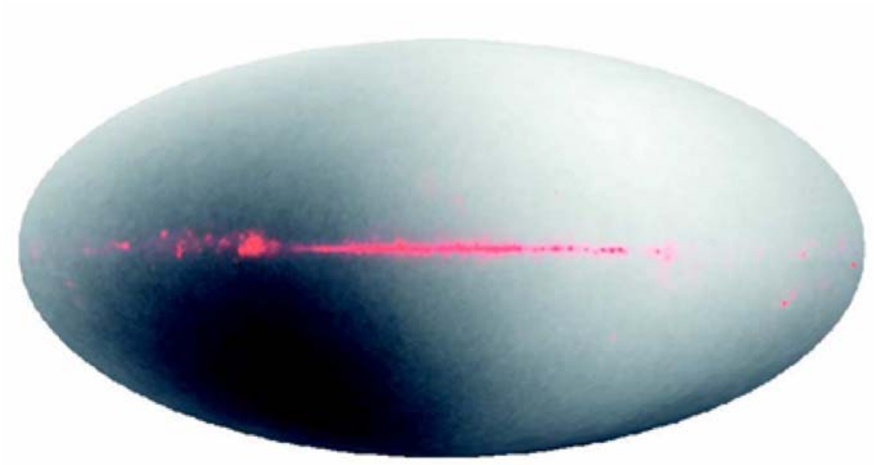
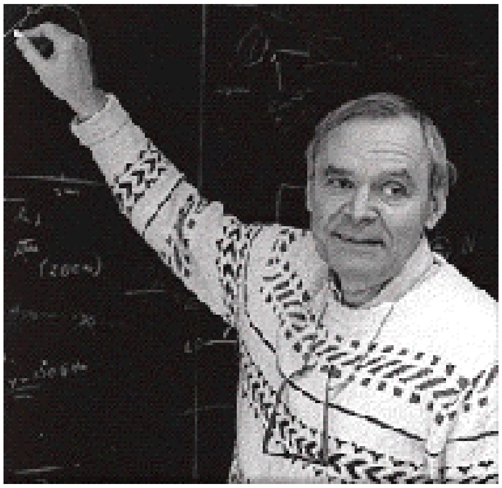
WMAP satellite



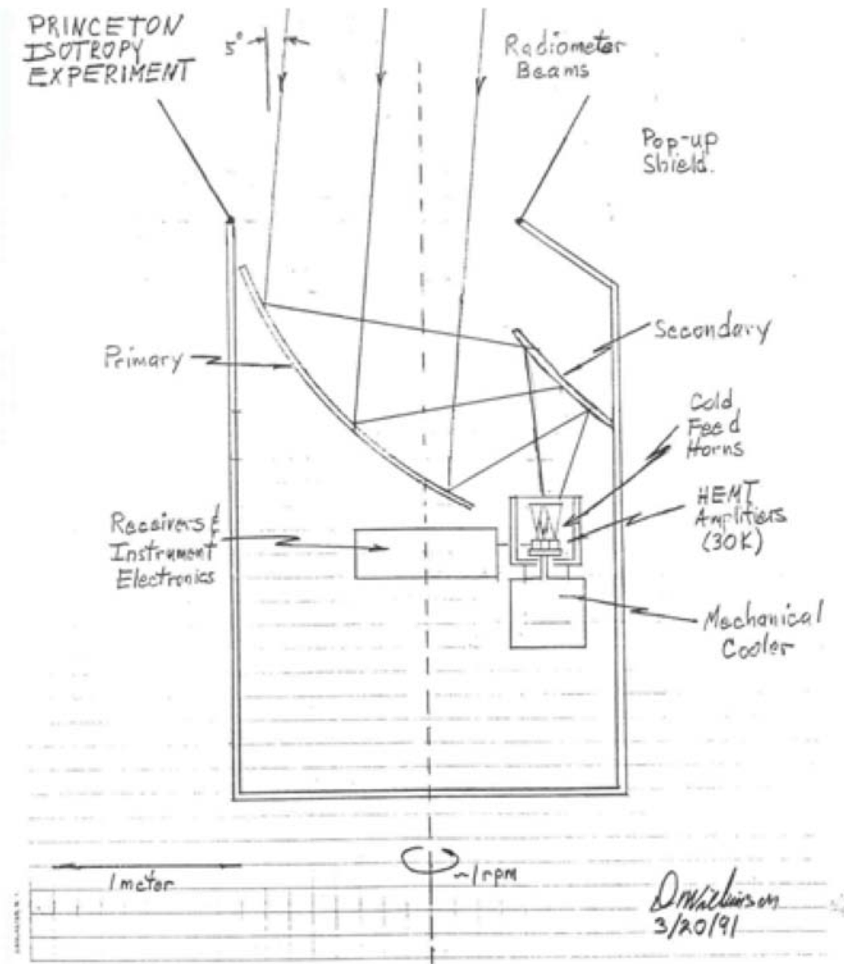
Temperature of the entire sky. Hottest spot is 3mK hotter than the average.
Pink is radiation of the Milky Way Galaxy, which has a different spectrum.

Measure spatial variations in temperature of the CBR

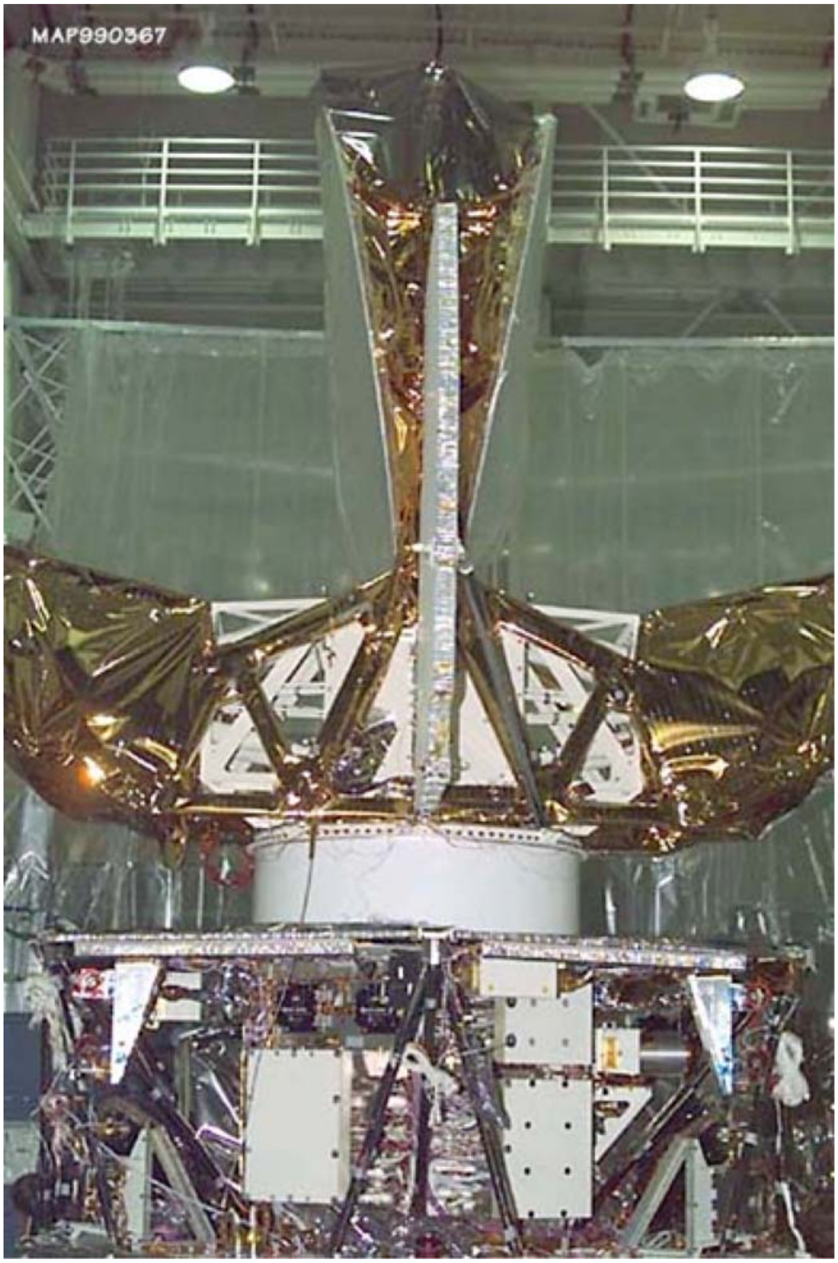
- Sensitivity is 0.000035K (a part in 100,000).
- Anything in the instrument even 0.0001K warmer is fatal.
- Symmetric design
- Record temperature difference between left & right channels. Temperature difference is small.
- Rotate entire instrument.
- Instrumental problems do not change; radiation from the sky does change.



Dave Wilkinson
1935-2002, b. Hillsdale MI



Dave's notebook, (Greg Tucker)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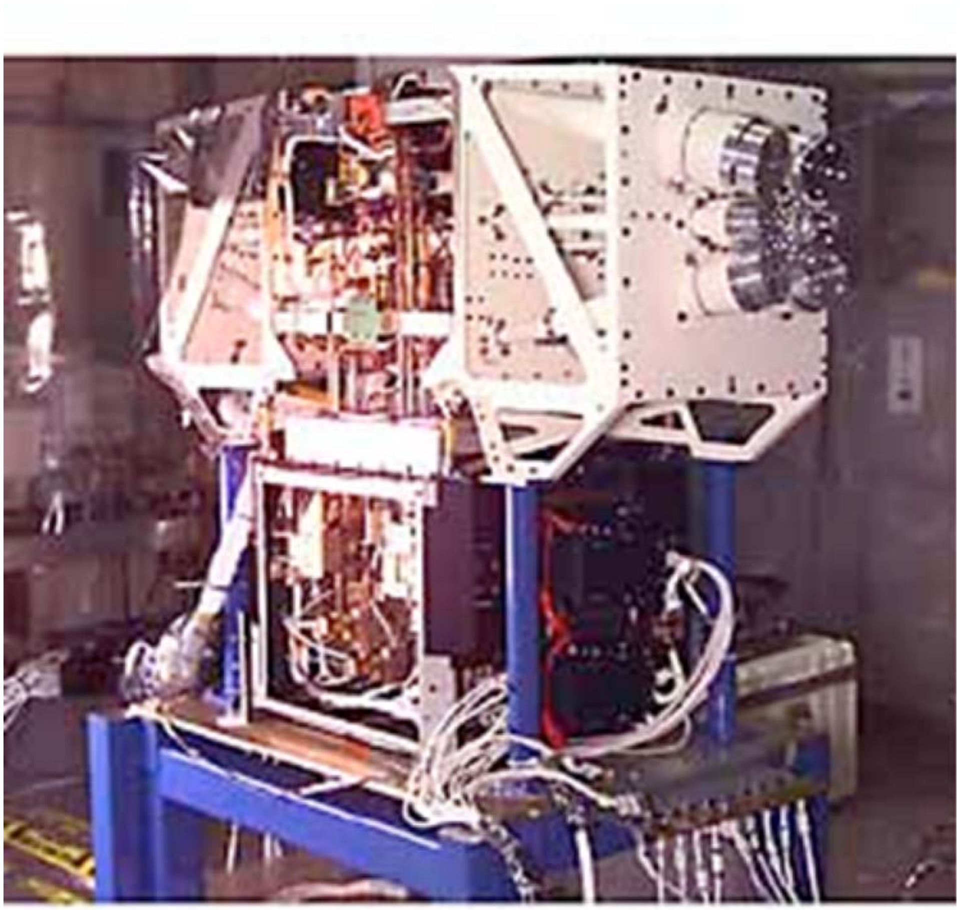

Five wavelength bands.
Each assembly compares two regions of the sky separated by 140°.
Q: Why does each assembly compare two regions of the sky?
One assembly operates at 22 GHz, one at 30 GHz, two at 40 GHz, two at 60 GHz, and four at 90 GHz.
Q: Why is there only one assembly at 22GHz yet there are four at 90GHz?

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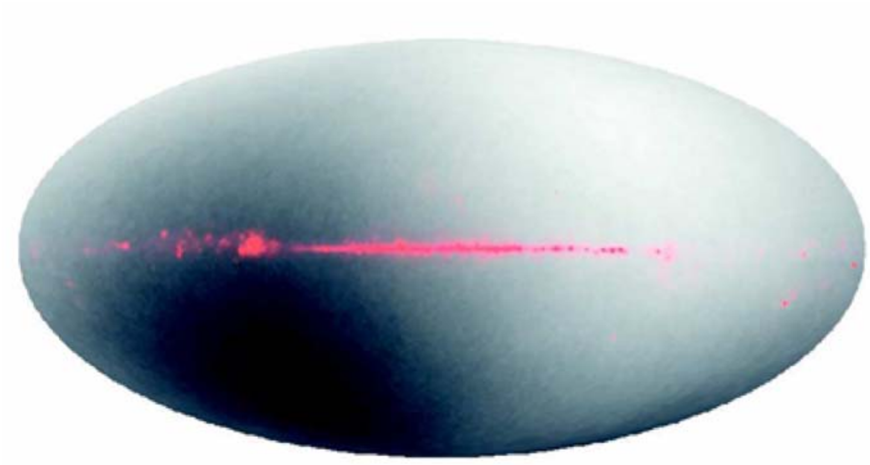
Dipole anisotropy



Temperature of the entire sky. Hottest spot is 3mK hotter than the average.
Pink is radiation of the Milky Way Galaxy, which has a different spectrum.

There is a special frame in which the universe is at rest.

We are moving with respect to this frame.
On 21 June, we are moving toward Pisces at 30 km/s = 0.0001.

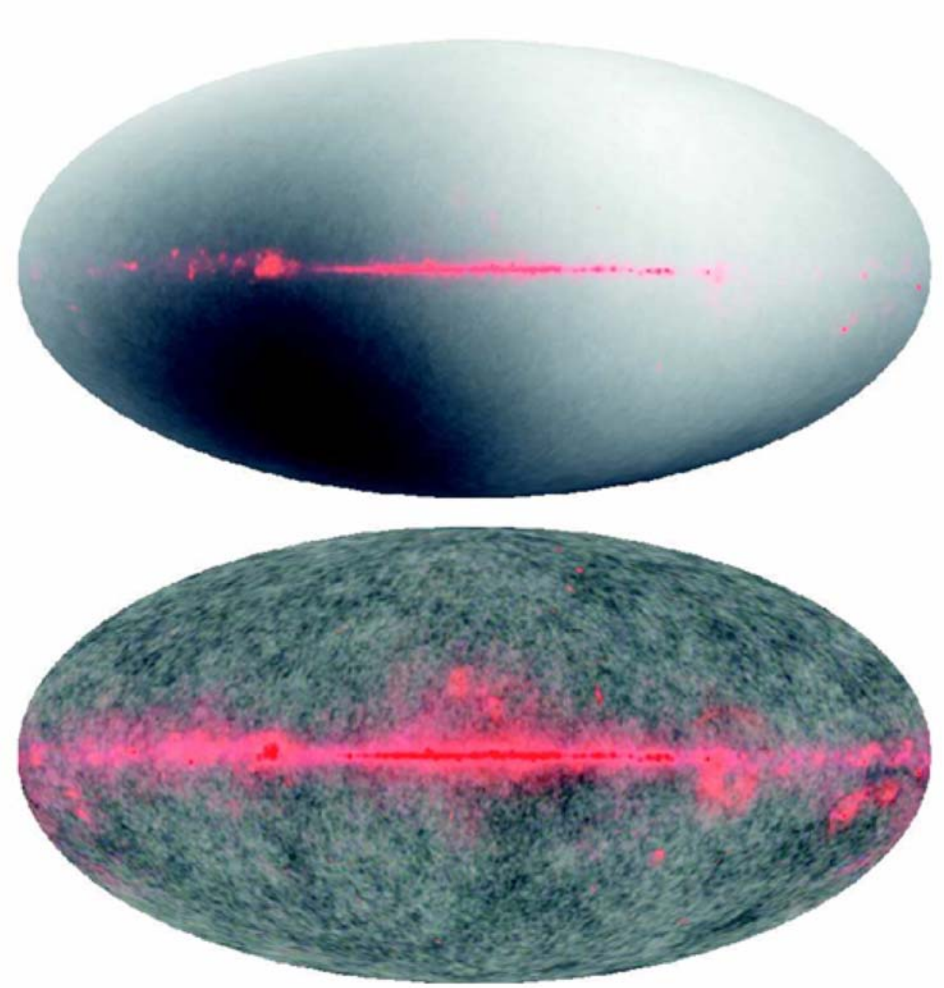
Q: On 21 June, what is the Doppler shift of the 2.7-K photons looking toward Pisces?
90° from Pisces? 180° from Pisces?

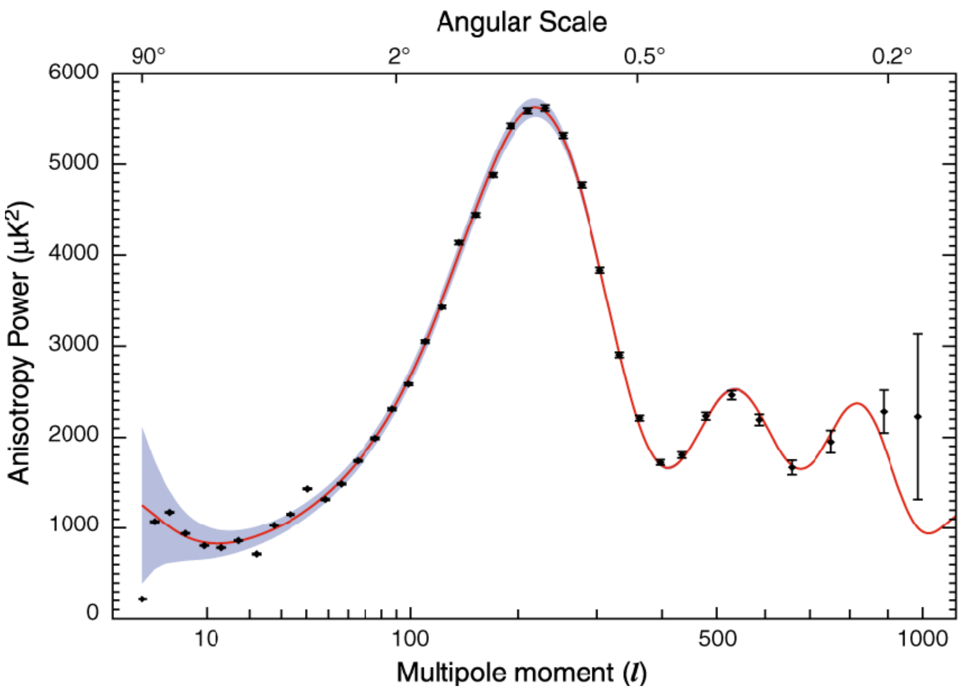
Q: On 21 June, what is the temperature shift of the CBR toward Pisces?
90° from Pisces? 180° from Pisces?

The peculiar velocities of galaxies are about 300 km/s = 0.001. A not unreasonable value of the dipole anisotropy is 2.7 mK.

The dipole anisotropy is 3.346 ± 0.017 mK toward (l, b) = (263.85 ± 0.1 °, 48.25 ± 0.04 °) (Bennett et al., 2003, ApJS, 148, 1). Motion of the sun in the Milky Way is 215 km/s. Milky Way moves at 200 km/s towards Andromeda. Net velocity of the local group of galaxies is 627 ± 22 km/s toward (l, b) = (276 ± 3 °, 30 ± 3 °) (Smoot et al, 1991, ApJ 371, L1,Kogut et al, 1993, ApJ 419, 1)

WMAP angular size of fluctuations





WMAP: http://map.gsfc.nasa.gov/media/060911/PowerSpectrum150.png

Plan: Why is the largest anisotropy at an angular scale of 1°? Fluctuations can grow for the age of the universe. The size is about $c\,t$. We calculated the age of the universe (for a given $\Omega_{0\,r}$). Then calculate the size L precisely. We already know that the angle subtended by a ruler at expansion paremeter a and comoving distance r is $\theta = L/(a\,r)$.

Exercises

Friedman's equation

$$\left(\frac{d\,a}{d\,H_0\,t}\right)^2 - \frac{8\pi G}{3 H_0^2}\,\rho\,a^2 = -\frac{1}{H_0^2\,r_0^2}$$

$$\left(\frac{d\,a}{H_0\,d\,t}\right)^2 - \Omega_0\,\frac{\rho}{\rho_0}\,a^2 = (1-\Omega_0)$$

Explicitly put in 3 types of stuff, matter (pressureless), radiation, and vacuum.

$\rho_m \propto a^{-3}$

$\rho_r \propto a^{-4}$

Q: $\rho_{\text{vac}} \propto a^{?}$

$$\left(\frac{d\,a}{H_0\,d\,t}\right)^2 - \left(\Omega_{\text{m0}}\,a^{-1} + \Omega_{\text{r0}}\,a^{-2} + \Omega_{\text{v0}}\,a^2\right) = (1 - \Omega_{\text{m0}} - \Omega_{\text{r0}} - \Omega_{\text{v0}})$$

Other terminology:

$(1 - \Omega_{\text{m0}} - \Omega_{\text{r0}} - \Omega_{\text{v0}}) = \Omega_k$

Letter "k" stands for curvature. Recall r_0 is the radius of curvature. $H_0^2\,r_0^2 = (1 - \Omega_{\text{m0}} - \Omega_{\text{r0}} - \Omega_{\text{v0}})$

$$\left(\frac{d\,a}{H_0\,d\,t}\right)^2 = \left(\Omega_{\text{k0}} + \Omega_{\text{m0}}\,a^{-1} + \Omega_{\text{r0}}\,a^{-2} + \Omega_{\text{v0}}\,a^2\right)$$

Density parameter and Hubble's constant change

Recall $H = \frac{1}{a}\,\frac{d\,a}{d\,t}$

$$\left(\frac{H}{H_0}\right)^2 = \left(\Omega_{\text{k0}}\,a^{-2} + \Omega_{\text{m0}}\,a^{-3} + \Omega_{\text{r0}}\,a^{-4} + \Omega_{\text{v0}}\right)$$

Q: Does Hubble's constant change? How does it change with a for pressureless matter? for radiation?

Expansion vs time

Friedman's equation relates the expansion parameter $a(t)$, matter density $\rho(t)$, and the radius of curvature r_0. The position of a galaxy at comoving coordinate r is $r\,a(t)$.

We want to integrate Friedman's equation to find $t(a)$.

$$\left(\frac{d\,a}{H_0\,d\,t}\right)^2 = \left(\Omega_{k0} + \Omega_{m0}\,a^{-1} + \Omega_{r0}\,a^{-2} + \Omega_{v0}\,a^2\right)$$

$$H_0\,d\,t = \left(\Omega_{k0} + \Omega_{m0}\,a^{-1} + \Omega_{r0}\,a^{-2} + \Omega_{v0}\,a^2\right)^{-1/2} d\,a$$

$$H_0\,t(a) = \int_0^a \left(\Omega_{k0} + \Omega_{m0}\,x^{-1} + \Omega_{r0}\,x^{-2} + \Omega_{v0}\,x^2\right)^{-1/2} d\,x$$

Important & instructive cases:
Matter:

$$H_0\,t(a) = \int_0^a \left(1 - \Omega_{m0} + \Omega_{m0}\,x^{-1}\right)^{-1/2} d\,x$$

```
Integrate[1 / Sqrt[(1 - o) + o x^-1], {x, 0, a}, Assumptions → {1 > o > 0, 1 > a > 0}]
```

$$\left(-\sqrt{a\,o\,(a + o - a\,o)} + o\,\sqrt{-\frac{o}{-1+o}}\,\mathrm{ArcSinh}\left[\sqrt{a\left(-1+\frac{1}{o}\right)}\,\right]\right) \Big/ \left((-1+o)\,\sqrt{o}\right)$$

More specifically, for $\mathbf{\Omega_{m0} = 0}$,

$$H_0\,t = a$$

For $\mathbf{\Omega_{m0} = 1}$,

$$H_0\,t = \frac{2}{3}\,a^{3/2}$$

For $\mathbf{\Omega_{m0} = 2}$,

$$H_0\,t = \int_0^a \left[-1 + 2\,x^{-1}\right]^{-1/2} d\,x$$

$$= \int_0^a \frac{x}{\left[2\,x - x^2\right]^{1/2}}\,d\,x = \int_0^a \frac{1-(1-x)}{\left[1-(1-x)^2\right]^{1/2}}\,d\,x$$

$$= \arccos(1-x)\,\big|_0^a + \left[1-(1-x)^2\right]^{1/2}\,\big|_0^a$$

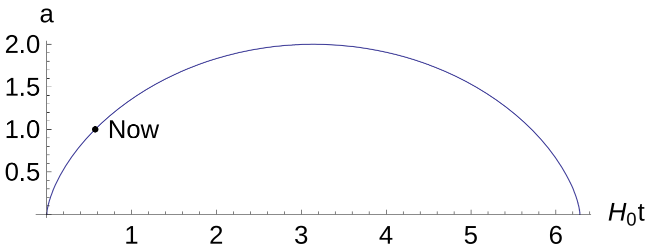
Or.

$$a = 1 - \cos\eta$$

$$t = H_0^{-1}(\eta - \sin\eta)$$

What is the maximum value for the expansion parameter?

At the present ($a = 1$), the age of the universe is $H_0^{-1}\left(\frac{\pi}{2} - 1\right)$.



Radiation with $\Omega_{r0} = 1$:

$$H_0\, t(a) = \int_0^a \left(x^{-2}\right)^{-1/2} d\,x = \int_0^a x\, d\,x$$

$$H_0\, t = \frac{1}{2} a^2$$

Q: Summarize the results.

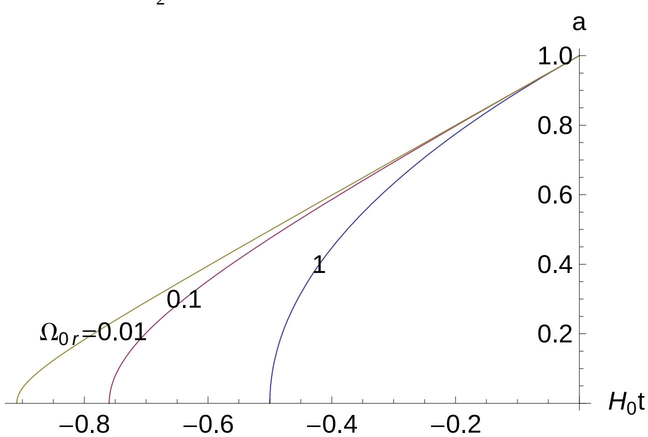
```
Integrate[x / Sqrt[x^2 (1 - o) + o], {x, 0, a}, Assumptions → {o > 1, 1 > a > 0}]
```

$$\frac{\sqrt{o} - \sqrt{-a^2 (-1 + o) + o}}{-1 + o}$$

$$H_0\, t = \left[\Omega_0{}^{1/2} - \left(\Omega_0 - a^2(\Omega_0 - 1)\right)^{1/2}\right] \Big/ (\Omega_0 - 1)$$

For $\Omega_0 = 1$,

$$H_0\, t = \frac{1}{2} a^2$$



Caption: time vs expansion parameter for a universe with radiation only.

Q: All of the models have the same slope at the present time. Why?

Q: Why does it take less time for the universe to expand with a larger density of radiation?

```
age[a_, o_] := (Sqrt[o] - Sqrt[-a^2 (-1 + o) + o]) / (-1 + o)

dAge[a_, o_] := age[a, o] - age[1, o]

fig[] = ParametricPlot[{{dAge[a, .999], a}, {dAge[a, .1], a}, {dAge[a, .01], a}},
  {a, 0, 1}, AxesLabel → {"H0t", "a"}, AspectRatio → 1 / GoldenRatio,
  BaseStyle → {FontSize → Medium, FontFamily → "Helvetica"},
  Epilog → {Text["1", {dAge[.4, .999], .4}],
    Text["0.1", {dAge[.3, .1], .3}], Text["Ω0 r=0.01", {dAge[.2, .01], .2}]}]
```

$$\text{age}[a\_, o\_] := \frac{\sqrt{o} - \sqrt{-a^2(-1+o)+o}}{-1+o}$$