Radiation from a source —15 Apr 2010

- Outline
  - Introduction (§16)
    How to detect gravity waves
    Order-of-magnitude strains
    Polarization
  - Wave equation (§21.5)
  - Source of gravitational waves (§23) (Today)
  - The Hulse-Taylor pulsar

The wave equation

$$\boxed{\Box h_{\mu\kappa} \equiv \left(-\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) h_{\mu\kappa} = -16\,\pi\, G\, S_{\mu\kappa}}.$$

In E&M, you learned the solution to the wave equation. This is derived in Hartle (§23.2)

$$h_{\alpha\beta}\left(t,\ \vec{x}\right) = 4\,G \int \frac{S_{\alpha\beta}\left(t',\vec{x}'\right)}{\left|\vec{x}-\vec{x}'\right|}\, d^3\, x'$$

Recall the source $S_{\alpha\beta} = T_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta}\, T_\lambda^\lambda$.

Is this plausible?

Q: For static masses, what should $h_{\alpha\beta}$be? Recall the Schwarzschild metric.

□□

For static masses, $T_{\alpha\beta} = \mathrm{diag}(\rho,\ 0,\ 0,\ 0)$, and $T_\lambda^\lambda = -\rho$. So $S_{\alpha\beta} = \frac{1}{2}\ \mathrm{diag}(\rho,\ -\rho,\ -\rho,\ -\rho)$.

$$h_{\alpha\beta} = 4 \int S_{\alpha\beta} \big/ r\ \mathrm{dVol} = -\frac{2\,M}{r}\ \mathrm{diag}(1,\ -1,\ -1,\ -1) = 2\ \mathrm{diag}(\phi,\ -\phi,\ -\phi,\ -\phi).$$

Q: There is a time t' in the integral. How do you define t'?

The quadrupole formula. Binary star

Consider a slowly moving source that is confined to a region much smaller than the distance to the observer. Then

$$h_{\alpha\beta}\left(t, \vec{x}\right) = 4\,G \int \frac{S_{\alpha\beta}\left(t', \vec{x}'\right)}{\left|\vec{x} - \vec{x}'\right|}\, d^3\, x'$$

yields

$$h_{i\,j}(t,\, r) = G\,\frac{1}{r}\,\frac{d^2}{dt^2}\,I_{i\,j}(t - r),$$

where the

$$I^{\text{ij}} \;=\; \int x^i\, x^j\, \rho\left(t,\, \vec{x}\right)\, d^3\, x.$$

For a binary star system, the energy radiated in gravitational waves causes the period to change (Taylor & Weisberg)

$$\dot{P} \equiv \frac{dP}{dt} = -\frac{192}{5}\left(\frac{2\,\pi}{P}\right)^{5/3} m_1\, m_2\, M^{-1/3}\; f(\epsilon)$$

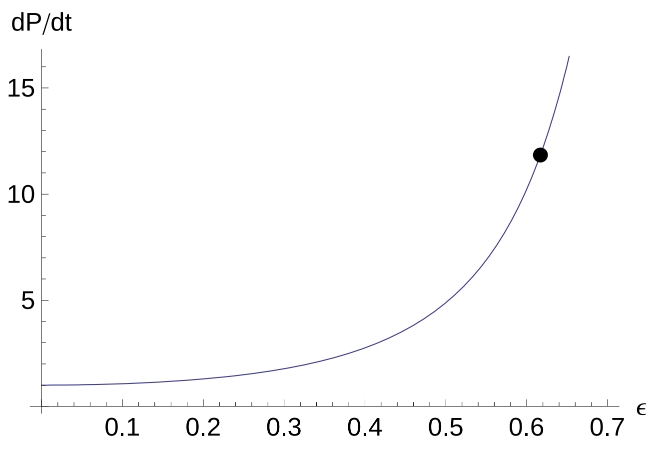
The masses of the stars are $m_1$and m_2. $M = m_1 + m_2$. The eccentricity is ϵ.

$$f\;(\epsilon) = \left(1 + \frac{73}{24}\,\epsilon^2 + \frac{37}{96}\,\epsilon^4\right)\left(1 - \epsilon^2\right)^{-7/2}$$

Q: Simplicio: If a system loses energy, it should slow down, not speed up. What is wrong with Simplicio's thinking when applied to the binary pulsar?

```
f[ϵ_] := (1 + 73/24 ϵ^2 + 37/96 ϵ^4) (1 - ϵ^2)^(-7/2)

Plot[f[ϵ], {ϵ, 0, .7}, AxesLabel → {"ϵ", "dP/dt"},
 Evaluate@baseStyle, Epilog → {PointSize[Large], Point[{.617, f[.617]}]}]
```



Q: Why is the period derivative (and gravitational radiation) large for high eccentricity?

Binary pulsar 1913+16

Observations of the binary pulsar 1913+16 was the first detection of the effect of gravitational waves.
The pulsar emits a burst of radio waves every 59ms.
Because of gravitational radiation, the system is losing energy, and the orbital period becomes shorter.

Ref: Taylor & Weisberg 1989, ApJ, 345, 434.
Taylor 1994, RMP 66.
Hulse 1994, RMP 66.

Q: Why were the effects of gravity waves not seen in the thousands of observations of other stars? What were keys about binary pulsars?

Q: How does that fact that the pulsar moves show up in the observations?

Q: The period of 1913+16 is 27906.980894(2)s (11 digits), about 8hr. The uncertainty in the time of arrival (5-min average) in 1988 was 16μs. How well can the position of the pulsar be determined?

```
In[2]:= 16*^-6 × 3.*^5 km
Out[2]= 4.8 km
```

Q: The size of the orbit is 2.3s

```
In[3]:= a = 2.3 × 3*^5 km
Out[3]= 690 000. km
```

What fraction of the orbit is the positional accuracy? A part in

```
In[4]:= % / %%
Out[4]= 143 750.
```

The eccentricity is 0.6171472(10). The gravitational redshift is

$$-\frac{M}{r}$$

It changes because $(1-\epsilon)\,a < r < (1+\epsilon)\,a$

```
ϵ = 0.6171472;
1.5 km * 1.4 / a / {1 - ϵ, 1 + ϵ}
```

$\{7.81359\times10^{-6},\ 1.84983\times10^{-6}\}$

Q: How would gravitational redshift appear in the observations?

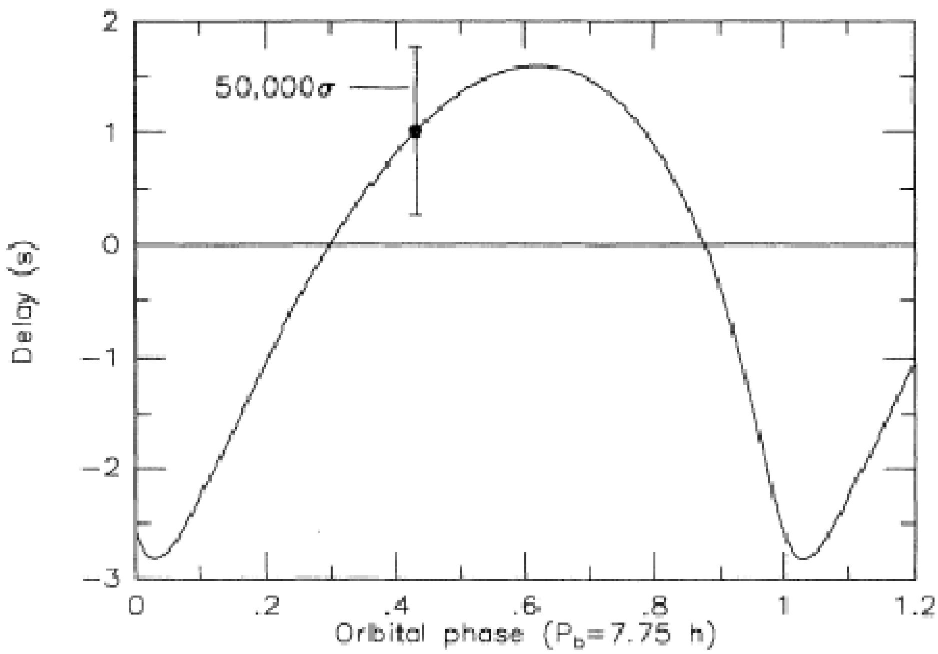


FIG. 5. Orbital delays observed for PSR 1913+16 during July, 1988. The uncertainty of an individual five-minute measurement is typically 50 000 times smaller than the error bar shown.

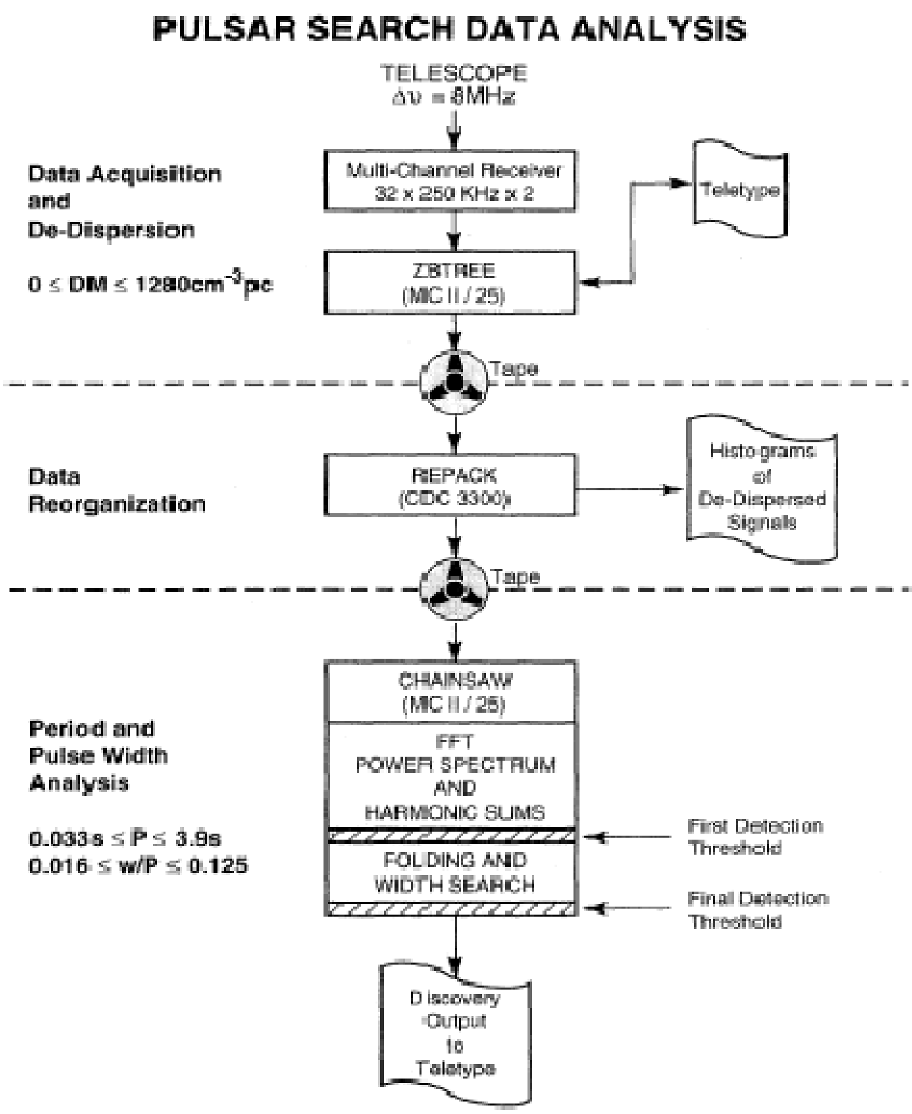
Taylor 1994, RMP

Discovery

Russell Hulse and Joe Taylor in 1974. Figures from Hulse's Nobel paper in RMP

In principle, finding pulsars is straightforward. Look for periodic signals from the sky.
The problem is dispersion by interstellar electrons. The velocity of the radio waves is proportional to $1/f^2$. A "dedisperser" removes the dispesion of the interstellar medium.

The dedisperser is done by Chainsaw.
Q: How much memory did the Chainsaw computer have? 32kB, 32MB, 32GB?



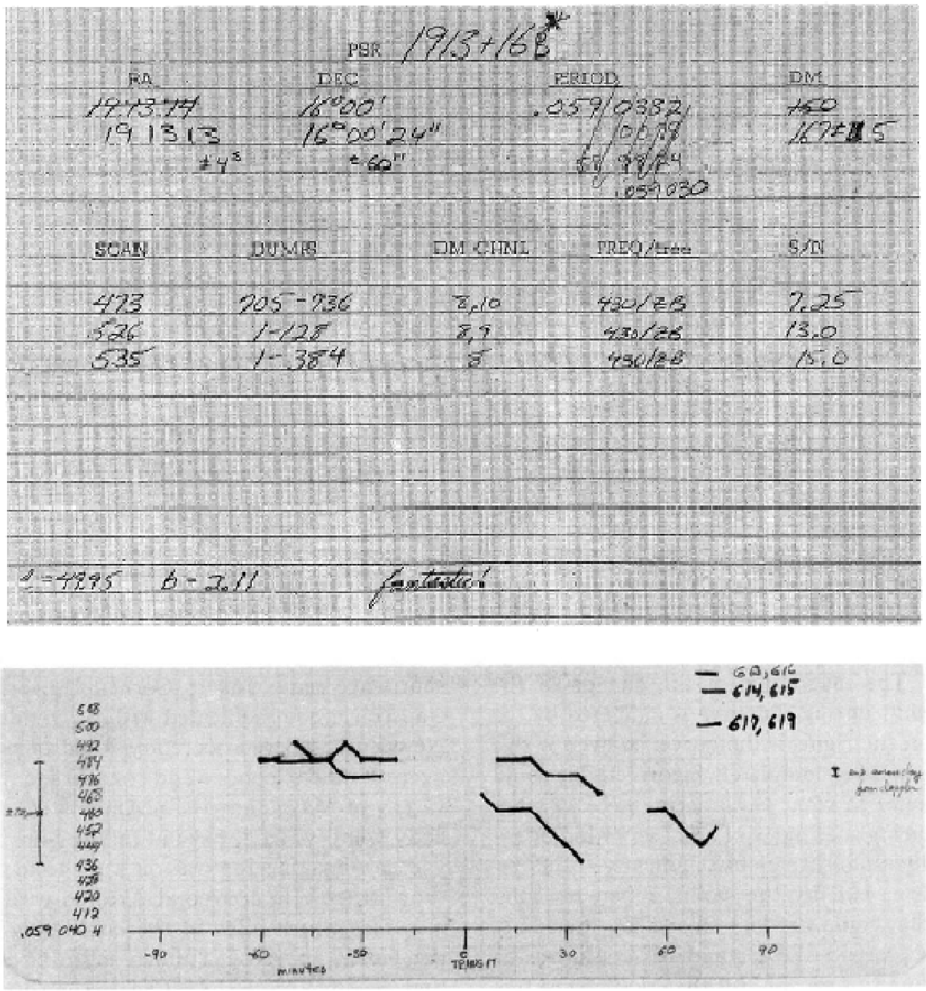


FIG. 9. The first two observations of PSR 1913+16 using the improved system specially designed to resolve the difficulties with determining the pulsation period for this pulsar. The observed pulsation period in successive 5-minute integrations is plotted versus time before and after transit. A calculation showing the magnitude of the change in the earth's Doppler shift is also seen on the right. Looking at this plot of data from September 1 and September 2, I realized that by shifting the second of these curves by 45 minutes the two curves would overlap. This was a key moment in deciphering the binary nature of PSR 1913+16.

Technical improvements for timing

The dedisperser (Taylor & Weisberg)

TABLE 1

OBSERVING SYSTEMS USED AND SUMMARY OF AVAILABLE DATA

Dates	Frequency (MHz)	Total Bandwidth (MHz)	Frequency Channels	System Noise Temperature (K)	Time Resolution (μs)	TOA Uncertainty (μs)	Number of Observations
A. 1974 Sep–Dec[a]	430	8.0	32	175	5000	275	524
B. 1975 Apr–1976 Nov[b]	430	0.64, 3.2	32	175	2000	310	112
C. 1975 Jun–1976 Feb	430	0.25	1	175	2000	890	75
D. 1976 Nov–Dec[a,b]	430	0.64	32	175	750	155	73
E. 1977 Jul–Aug[b]	430	0.64	32	175	340	150	52
F. 1978 Jun–1981 Feb	430	3.34	504	175	43	75	573
G. 1977 Jul–Aug	1410	8.0	32	80	125	75	57
H. 1977 Dec	1410	8.0	32	80	125	55	72
I. 1978 Mar–Apr[a]	1410	8.0	32	80	125	50	116
J. 1980 Jul–1981 Feb	1410	8.0	32	80, 40	200	85	312
Mark I. 1981 Feb–1984 Dec	1410	16.0	64	40	125	20	1719
Mark II. 1984 Oct–1988 Jul	1408	8.0	32	40	125	31	638
Mark III. 1988 Jul–	1404	40.0	32	40	640	16	159

Q: The uncertainty in the time of arrival improved from 275μs to 16μs. How does this help?

TABLE 3

ASTROMETRIC AND SPIN PARAMETERS OF PSR 1913+16

PARAMETER	COORDINATE SYSTEM B1950.0(CfA)	J2000.0(JPL)
α	$19^h13^m12^s.46549(15)$	$19^h15^m28^s.00018(15)$
δ	$16°01'08''.189(3)$	$16°06'27''.4043(3)$
μ_α (mas yr^{-1})	-3.2 ± 1.8	-3.2 ± 1.8
μ_δ (mas yr^{-1})	1.2 ± 2.0	1.2 ± 2.0
ν(s^{-1})	16.940539303217(2)	16.940539303295(2)
$\dot{\nu}$(10^{-15} s^{-2})	−2.47559(2)	−2.47583(2)
$\lvert\ddot{\nu}\rvert$(10^{-27} s^{-3})	<6	<6
t_0(JED 2,445,888+)	0.745517962	0.745517886
P(ms)	59.029997929883(7)	59.029997929613(7)
$\dot{P}$(10^{-18} s s^{-1})	8.62629(8)	8.62713(8)
$\lvert\ddot{P}\rvert$(10^{-29} s s^{-1})	<2	<2

NOTE.—Figures in parentheses are uncertainties in the last digits quoted.

Q: On the SOAR Telescope, I am able to measure the position of a star to 10mas. How can the location of the pulsar be measured to 0.15mas?

Orbital parameters

TABLE 4

KEPLERIAN ORBITAL PARAMETERS

Solution	$x = (a_1 \sin i)/c$ (s)	e	T_0 (JED 2,445,888 +)	P_b (s)	ω_0 (degrees)
BT(1)	2.341774(9)	0.6171472(10)	0.61724862(8)	27906.980894(2)	220.1426(2)
BT(2)	2.34178(10)	0.6171467(11)	0.61724862(8)	27906.980894(2)	220.143(2)
EH	2.341749(12)	0.617127(5)	0.61724861(8)	27906.980895(2)	220.1428(2)
H88	2.341752(19)	0.617128(6)	0.61724859(10)	27906.980895(2)	220.1428(3)
DDGR	2.341754(9)	0.6171314(10)	0.61724857(7)	27906.980891(2)	220.1428(2)
DD(1)[a]	2.341754(9)	0.6171313(10)	0.61724860(8)	27906.980895(2)	220.1428(2)
DD(2)[a]	2.34176(10)	0.617132(3)	0.6172486(2)	27906.980895(2)	220.143(2)
DD(3)	2.341761(9)	0.6171304(10)	0.61724861(8)	27906.980894(2)	220.1426(2)

NOTE.—Figures in parentheses are uncertainties in the last digits quoted.
[a] Preferred solutions.

TABLE 5

POST-KEPLERIAN PARAMETERS

Solution	$\dot{\omega}$ (degrees yr^{-1})	γ (ms)	$\dot{P}_b$ (10^{-12})	$s = \sin i$	r (μs)	M ($M_\odot$)	m_2 ($M_\odot$)	$\lvert\dot{x}\rvert$ (10^{-13})	$\lvert\dot{e}\rvert$ (10^{-14} s^{-1})	Instrumental Parameters	χ^2	dof
BT(1)	4.22660(4)	4.288(10)	−2.428(26)	...	...	...	...	0[a]	0[a]	1	2617.8	2500
BT(2)	4.22660(18)	4.29(11)	−2.427(30)	...	...	...	...	<2.7	<4.3	1	2613.3	2498
EH	4.22661(4)	4.281(10)	−2.429(26)	0.73(4)	...	...	...	0[a]	0[a]	1	2551.8	2499
H88	4.22660(4)	4.281(10)	−2.429(27)	$0.71^{+0.06}_{-0.10}$	...	...	...	0[a]	0[a]	1	2551.6	2499
DDGR	...	...	...	...	...	2.82837(4)	1.386(3)	0[a]	0[a]	1	2556.6	2501
DD(1)[b]	4.22659(4)	4.289(10)	−2.427(26)	0.734[a]	6.83[a]	...	...	0[a]	0[a]	1	2552.0	2500
DD(2)[b]	4.22659(18)	4.29(11)	−2.428(34)	[c]	[c]	...	...	<2.4	<1.9	1	2551.9	2496
DD(3)	4.22656(4)	4.296(11)	−2.435(34)	0.734[a]	6.83[a]	...	...	0[a]	0[a]	5	4878.7	4001

NOTE.—Figures in parentheses are uncertainties in the last digits quoted.
[a] Parameter held fixed at this value.
[b] Preferred solutions.
[c] See text and Fig. 7.

Q: What is $\dot{\omega}$ for Mercury?

Q: The inclination i is the angle between the plane of the orbit and the plane of the sky. For what sin i is the Shapiro effect the greatest?