1. My twin leaves in a rocket, travels at constant speed $v = 0.95$ for a year, turns around, and travels for a year at $v = -0.95$, and returns home. Define these events. Event A: my twin leaves. Event B: my twin just before turning around. Event C: my twin just after turning around. Event D: my twin returns.

   (a) (2 pts.) Draw a space-time diagram in which I am at rest. On the diagram, show events A–D.

   (b) (2 pts.) On the space-time diagram, show the coordinate frame in which my twin is at rest on the outbound trip. Set the origin at event B.

   (c) (2 pts.) On the space-time diagram, show the coordinate frame in which my twin is at rest on the inbound trip. Set the origin at event C.

   (d) (5 pts.) According to my twin, how much did I age at the instant of turnaround?

   (e) (3 pts.) How much did my twin age during the trip?

2. Two clocks move from event A to event B. The first moves along the straight wordline $x = v_0 t$, where $v_0$ is a constant. The second moves as $x = \frac{1}{2}at^2$ with constant acceleration. Space is flat.

   (a) (3 pts.) Without doing any calculation, figure out which clock shows a longer proper time.

   (b) (7 pts.) Find the elapsed proper time between events A and B for both worldlines.

3. Schwarzschild metric. Consider circles with radial coordinates $r_1$ and $r_2$. For the sun, $M = 1.5\,\mathrm{km}$ and the radius of the surface is $r_0 = 7.0 \times 10^5\,\mathrm{km}$. For a neutron star, $M = 1.5\,\mathrm{km}$ and the radius of the surface is $r_0 = 10\,\mathrm{km}$.

   (a) (5 pts.) Find an expression for the circumference $C_1$ of circle 1. Find an expression for the radial distance between the two circles $R_2 - R_1$. (You need keep only the terms that are first order in $M/r$.)

   (b) (4 pts.) Compute $R_2 - R_1 - \frac{1}{2\pi}(C_2 - C_1)$ where $R_1$ is at the surface of the sun and $R_2 = 1.5 \times 10^8\,\mathrm{km}$, the radius of the earth's orbit. Do the same calculation for a neutron star and a hypothetical earth in orbit.

4. Read the paper "Apprarent weight of photons," Pound, R., V., & Rebka, jr., G., A., 1960, PRL 4, 337. (There is a link on the syllabus on angel.)

(a) (5 pts.) Summarize the paper in a few sentences. Write for a senior physics major.

(b) (5 pts.) Pound & Rebka modulated the absorption of the gamma rays by moving the source or the detector. How fast did they move the absorber to cause absorption to disappear? How fast did they move the absorber to cause absorption to be maximum?

(c) (5 pts.) Pound & Rebka state (3rd full paragraph on page 339) that a 1-C temperature difference between the source and absorber is sufficient to mask the effect that they are measuring. Estimate the size of the second order Doppler effect to prove their statement. (We derived the transverse Doppler effect in class.)