Physics 472 Final Exam – May 3, 2010

Total points = 30. Show all your work!

Useful relations about Angular Momentum:

$[J_x, J_y] = i\hbar J_z$ and cyclic permutations.

$J^2 |j,m\rangle = \hbar^2 j(j+1)|j,m\rangle \qquad J_z |j,m\rangle = \hbar m |j,m\rangle$

$J_+ = J_x + iJ_y \qquad J_- = J_x - iJ_y$

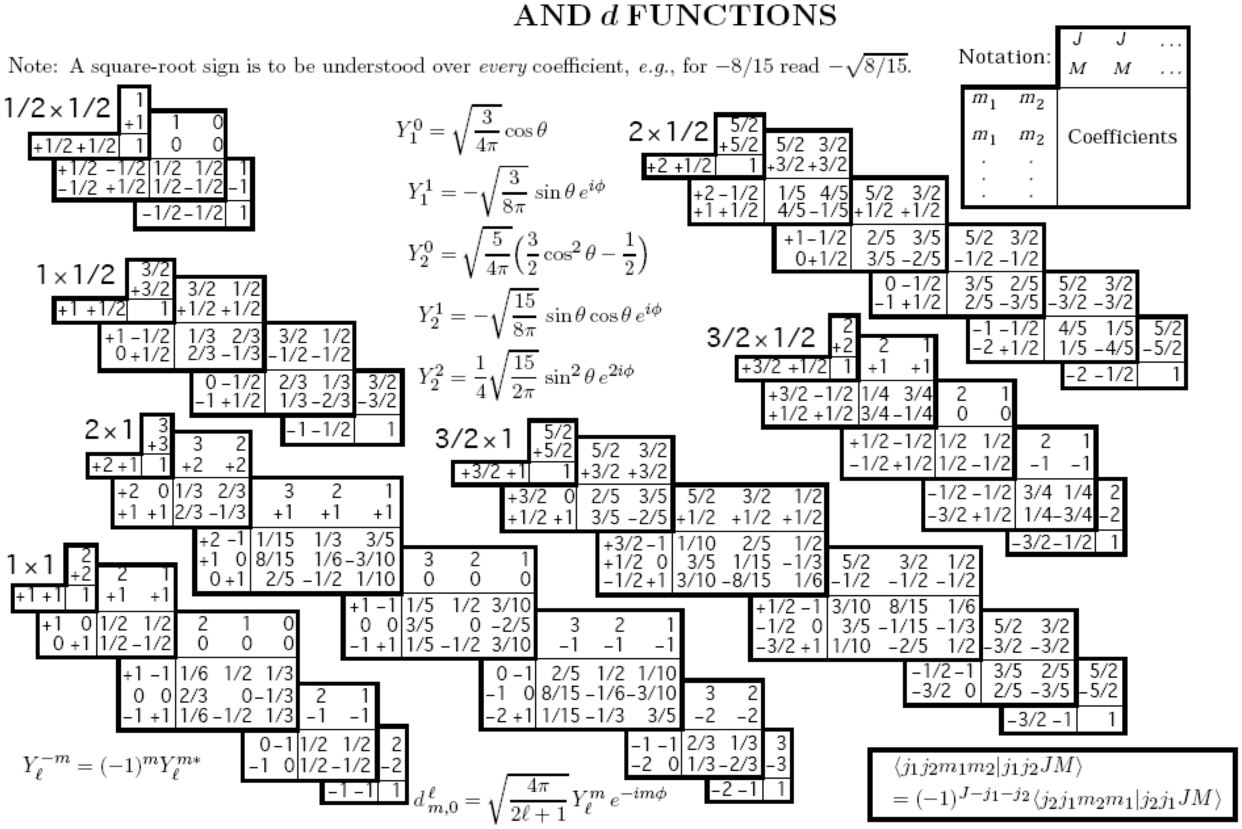
$J_+|j,m\rangle = \hbar\sqrt{j(j+1)-m(m+1)}\,|j,m+1\rangle \qquad J_-|j,m\rangle = \hbar\sqrt{j(j+1)-m(m-1)}\,|j,m-1\rangle$

Spin-1/2:

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma}, \qquad \text{where} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.



Non-Degenerate Perturbation Theory: $H = H^0 + \lambda H'$ $H^0 \left|\psi_n^{(0)}\right\rangle = E_n^{(0)} \left|\psi_n^{(0)}\right\rangle$

$$E_n = E_n^{(0)} + \lambda \left\langle \psi_n^{(0)} \middle| H' \middle| \psi_n^{(0)} \right\rangle + \lambda^2 \sum_{m \neq n} \frac{\left| \left\langle \psi_m^{(0)} \middle| H' \middle| \psi_n^{(0)} \right\rangle \right|^2}{E_n^{(0)} - E_m^{(0)}} + \ldots$$

$$\left|\psi_n\right\rangle = \left|\psi_n^{(0)}\right\rangle + \lambda \sum_{m \neq n} \left|\psi_m^{(0)}\right\rangle \frac{\left\langle \psi_m^{(0)} \middle| H' \middle| \psi_n^{(0)} \right\rangle}{E_n^{(0)} - E_m^{(0)}} + \ldots$$

Degenerate Perturbation Theory: If an energy level is n-fold degenerate with respect to H^0, then the first order energy shifts and corresponding eigenstates of H are given by the eigenvalues and eigenvectors of the $n \times n$ matrix with elements $H'_{jk} = \left\langle \psi_j^{(0)} \middle| H' \middle| \psi_k^{(0)} \right\rangle$, where $j, k = 1, 2, \ldots, n$.

Symmetries are a powerful tool to help determine which matrix elements are zero. For example, for hydrogen eigenstates $\left| nlm \right\rangle$, $[L_z, z] = 0$ implies $(m - m')\left\langle n'l'm' \middle| z \middle| nlm \right\rangle = 0$, and the parity transformation $\Pi z \Pi = -z$ implies $(-1)^{l+l'} \left\langle n'l'm' \middle| z \middle| nlm \right\rangle = -\left\langle n'l'm' \middle| z \middle| nlm \right\rangle$.

Variational Method: If the ground state energy of H is E_0, then for any normalized state $\left|\psi\right\rangle$,

$$E_0 \leq \left\langle \psi \middle| H \middle| \psi \right\rangle$$

Hydrogen Atom: $H = \dfrac{p^2}{2m} - \dfrac{e_2}{4\pi\varepsilon_0 r}$ $E_n = \dfrac{-Ry}{n^2}$ where $Ry = \dfrac{1}{2}\dfrac{e^2}{4\pi\varepsilon\, a_0} = \dfrac{\hbar^2}{2ma_0^2} = 13.6\,\text{eV}$

$$R_{10} = 2a^{-3/2} e^{-r/a} \qquad R_{20} = \tfrac{1}{\sqrt{2}} a^{-3/2} \left(1 - \tfrac{1}{2}\tfrac{r}{a}\right) e^{-r/2a} \qquad R_{21} = \tfrac{1}{\sqrt{24}} a^{-3/2} \tfrac{r}{a} e^{-r/2a}$$

$$Y_0^0 = \left(\tfrac{1}{4\pi}\right)^{1/2} \qquad Y_1^0 = \left(\tfrac{3}{4\pi}\right)^{1/2} \cos\theta \qquad Y_1^{\pm 1} = \mp \left(\tfrac{3}{8\pi}\right)^{1/2} \sin\theta\, e^{\pm i\phi}$$

Harmonic Oscillator: $H = \dfrac{p^2}{2m} + \tfrac{1}{2} m\omega^2 x^2$ $E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega$

Infinite Square Well for $0 < x < a$: $\psi_n(x) = \sqrt{\dfrac{2}{a}} \sin\left(\dfrac{n\pi x}{a}\right)$ $E_n = \dfrac{n^2 \pi^2 \hbar^2}{2ma^2}$

Integrals: $\displaystyle\int_0^\infty x^n e^{-x/a} dx = n! a^{n+1}$ $\displaystyle\int_0^\infty x^{2n} e^{-x^2/a^2} dx = \sqrt{\pi} \frac{(2n)!}{n!} \left(\frac{a}{2}\right)^{2n+1}$ $\displaystyle\int_0^\infty x^{2n+1} e^{-x^2/a^2} dx = \frac{n!}{2} a^{2n+2}$

Time-dependent perturbation theory: If a system in an eigenstate of energy E_i is subjected to a time-dependent perturbation $H'(t)$ starting at time t = 0, the first-order probability amplitude for a transition to an eigenstate with energy E_f is:

$$c_f(t) = \frac{-i}{\hbar} \int_0^t \left\langle f \middle| H'(t') \middle| i \right\rangle e^{i(E_f - E_i)t'/\hbar}\, dt'$$