Physics 472 – Spring 2010

Homework #4, due Friday, February 5

(Point values are in parentheses.)

1. [2] Griffiths problem 5.4. This problem is easier if you re-write Equation (5.10) in Dirac notation, but you may use wavefunctions if you prefer.

2. [7] The trouble with Example 5.1 and Problem 5.5 in Griffiths is that he ignores spin. So let's do this right!

   a) [2] Write down the Hamiltonian for two noninteracting identical particles in the infinite square well. Verify that what Griffiths calls the "fermion ground state" in Example 5.1 is an eigenfunction of *H*, with the appropriate eigenvalue. (Note that this is not the true ground state of this system, as you will show in the next part!)

   b) [1] As we all know, fermions have half-integer spin. Let's consider spin-1/2 particles for simplicity. Write down the true ground state of the system as a product of a spatial state times a spin state. (You may use either Dirac or wavefunction notation for the spatial state, and either Dirac or spinor notation for the spin state.)

   c) [2] Write down the energies and allowed states for the 3 lowest excited energy levels of the system of two spin-1/2 fermions. Don't forget to include both spin-singlet and spin-triplet states. Note that there are several degenerate states for some energy levels. Write down the total number of states for each energy, including all possible spin states.

   d) [2] Now consider two identical spin-1 bosons. (I chose spin-1 rather than spin-0 to make the problem interesting.) Write down the energies and allowed states for the ground state and lowest 3 excited energy levels of the system. Be sure to include all possibilities of the total spin: s = 0, s = 1, and s = 2. Use Table 4.8 to determine the exchange symmetry of the spin states for each of those three cases. Write down the total degeneracy of each energy level, as you did in part (c).

3. [4] Griffiths 5.12. Part (a) is trivial. We did some of part (b) in class, but I want you to do it to make sure you understand the logic. For each element, first write down the possible values for the total spin *s* and total orbital angular momentum *l*, then consider which values can go together consistently with the Spin-Statistics Theorem. (This is very easy until you get to carbon.) Finally, write down the possible values of *j* for each combination of *s* and *l*, then put the whole thing in spectroscopic notation. Don't do nitrogen unless you want a real challenge! Treat it as extra credit.

(over)

4. [4] Griffiths 5.13. When you get to part (d), note that I already told you to consider the Spin-Statistics Theorem in the previous problem. But now you should be able to figure out the correct electronic configuration for nitrogen, even if you had no idea how to do it in problem 5.12.

5. [3] Consider two neutrons in free space. From classical mechanics, you know that we can specify their motion using the center-of-mass coordinate $\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$ and the relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$. (To remind yourself how this works, look at Griffiths problem 5.1). For this problem, we'll assume that the center-of-mass momentum is zero, so we'll ignore $\vec{R}$ and $\vec{P}$. We could thus write the wavefunction for the <u>two-neutron</u> system in the following form:

$$\Psi_{nlms_1s_2}(r,\theta,\phi) = \Phi_{nl}(r)Y_l^m(\theta,\phi)\chi(s_1,s_2).$$

where the radial wavefunction depends both on the quantum number *n* and the orbital angular momentum *l*. Instead, let's use eigenstates of total spin S^2 and S_z, where $\vec{S} = \vec{S}_1 + \vec{S}_2$. In Dirac notation, a complete basis set of two-neutron quantum states is thus:

$$|n,l,m_l,s,m_s\rangle \equiv |n,l,m_l\rangle \otimes |s,m_s\rangle$$

When the spatial state is written in terms of the relative coordinate $\vec{r}$, spatial exchange of the two particles corresponds to the parity transformation, $\vec{r} \to -\vec{r}$. In terms of spherical polar coordinates, that corresponds to $r \to r,\ \theta \to \pi - \theta,\ \phi \to \phi + \pi$. If you look at the spherical harmonics in Table 4.3, you'll see that they have even parity if *l* is even, and odd parity if *l* is odd. In operator notation, that means $\hat{P}|l,m_l\rangle = (-1)^l |l,m_l\rangle$, where $\hat{P}$ is the parity operator.

a) [1] For each allowed value of *s*, what are the allowed values of *l* (up to *l*=4) consistent with the Spin-Statistics Theorem?

b) [2] The basis states above are not eigenstates of the total angular momentum, $\vec{J} = \vec{L} + \vec{S}$. We know how to construct states that are, using Clebsch-Gordan coefficients:

$$|n,l,s,j,m_j\rangle = \sum_{m_l+m_s=m_j} C^{l,s,j}_{m_l m_s m_j} |n,l,m_l\rangle \otimes |s,m_s\rangle$$

You don't need to look up any C-G coefficients for this problem. Instead, make a table with four columns showing all allowed combinations of *s*, *l*, and *j*, and the spectroscopic notation from atomic physics, ${}^{2S+1}L_J$.