

# Physics 472 Midterm Exam #1 -- Monday, February 8, 2010

Total points = 20. Show all your work!

Useful relations about Angular Momentum:

$$[J_x, J_y] = i\hbar J_z \text{ and cyclic permutations.}$$

$$J^2 |j, m\rangle = \hbar^2 j(j+1) |j, m\rangle$$

$$J_z |j, m\rangle = \hbar m |j, m\rangle$$

$$J_+ = J_x + iJ_y$$

$$J_- = J_x - iJ_y$$

$$J_+ |j, m\rangle = \hbar \sqrt{j(j+1) - m(m+1)} |j, m+1\rangle$$

$$J_- |j, m\rangle = \hbar \sqrt{j(j+1) - m(m-1)} |j, m-1\rangle$$

Spin-1/2:

$$\vec{S} = \frac{\hbar}{2} \vec{\sigma}, \quad \text{where } \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

## 35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND $d$ FUNCTIONS

Note: A square-root sign is to be understood over every coefficient, e.g., for  $-8/15$  read  $-\sqrt{8/15}$ .

Notation:  $\begin{matrix} J & J & \dots \\ M & M & \dots \end{matrix}$

$1/2 \times 1/2$

1		
+1/2	1/2	0
-1/2	1/2	0

$1 \times 1/2$

3/2	3/2	1/2
+1	+1/2	1/2
+1	-1/2	1/3
0	+1/2	2/3

$2 \times 1$

3	3	2
+2	+1	1
+2	0	1/3
+1	+1	2/3

$1 \times 1$

2	2	1
+1	+1	1
+1	0	1/2
0	+1	1/2

$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$

$Y_1^1 = -\sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$

$Y_2^0 = \sqrt{\frac{5}{4\pi}} \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$

$Y_2^1 = -\sqrt{\frac{15}{8\pi}} \sin \theta \cos \theta e^{i\phi}$

$Y_2^2 = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{2i\phi}$

$2 \times 1/2$

5/2	5/2	3/2
+2	+1/2	1
+2	-1/2	1/5
+1	+1/2	4/5

$3/2 \times 1/2$

2	2	1
+3/2	+1/2	1
+3/2	-1/2	1/4
+1/2	+1/2	3/4

$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$

$d_{m,0}^\ell = \sqrt{\frac{4\pi}{2\ell+1}} Y_\ell^m e^{-im\phi}$

$\langle j_1 j_2 m_1 m_2 | j_1 j_2 JM \rangle$   
 $= (-1)^{J-j_1-j_2} \langle j_2 j_1 m_2 m_1 | j_2 j_1 JM \rangle$