LECTURE #18

Note Title 2/22/2010

SEMICLASSICAL APPROX FOR BLOCH ELECTRON DYNAMICS

ψ_{mk} $\vec{v}_m(k) = \frac{1}{\hbar}\frac{\partial \varepsilon_m(\vec{k})}{\partial \vec{k}}$ $\left(\tau_{IONS} = \infty\right)$

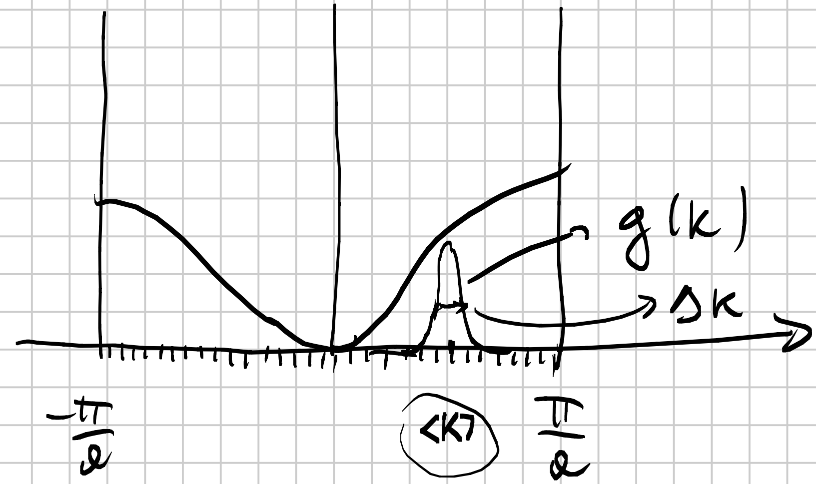
τ FOR BLOCH ELECTRONS $< \infty$

1. IMPURITIES AND DEFECTS
2. LATTICE VIBRATION

DESCRIBE DYNAMICS OF BLOCH ELECTRONS BETWEEN COLLISIONS OF TYPE 1 OR 2

$$\psi_m(\vec{r}) = \sum_{\vec{k}} g(\vec{k})\, \overline{\psi_{mk}(\vec{r})}$$

$$\psi_n(\vec{r},t) = \sum_k g(k)\, e^{\frac{i\varepsilon_n(k)t}{\hbar}}\, \psi_{nk}(\vec{r})$$



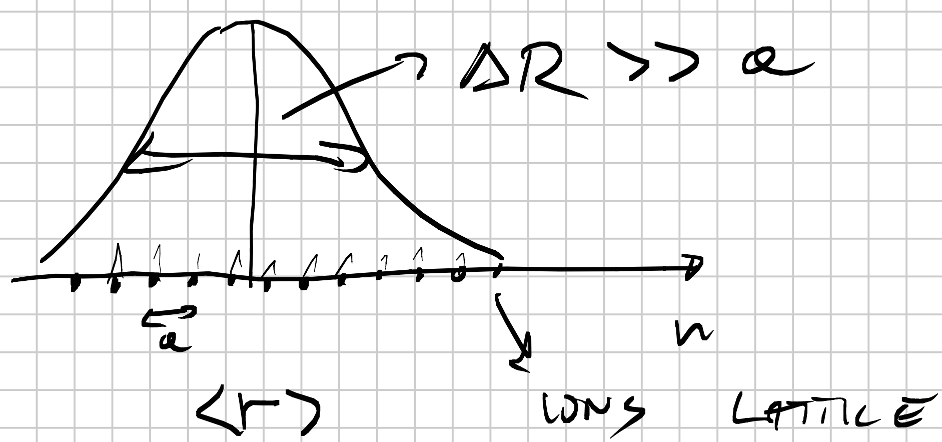
$$v_g = \frac{1}{\hbar} \frac{\partial \varepsilon_n(k)}{\partial \vec{k}}\Bigg|_{k=\langle k \rangle}$$

ΔK FOR WAVE PACKET $\ll \frac{2\pi}{a}$ SIZE OF BZ

$$\Delta R\, \Delta K \sim 1 \quad \Rightarrow \quad \Delta R \gg \frac{a}{2\pi}$$

$\downarrow$

SIZE OF ELECTRON WF $\gg$ LATTICE SPACING



SEMICLASSICAL EQUATIONS OF MOTION

① FIX THE BAND

② $$\frac{d\langle\vec{r}\rangle}{dt} = \frac{1}{\hbar}\frac{\partial \varepsilon(\vec{k})}{\partial \vec{k}} \qquad \left(\langle\dot{r}\rangle = v_g \right)$$

③ $$\hbar\langle\dot{\vec{K}}\rangle = \overbrace{-e\left(\vec{E} + \frac{v_g(k)}{c}\times\vec{H}\right)}^{\text{FORCE}}$$

DIFFERENT THAN $m\dot{v} = \vec{F}$ IN THE FREE CASE.

$$\langle\vec{K}+\vec{G}\rangle = \langle\vec{k}\rangle$$

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① + ② + ③ CAN BE DERIVED USING ENERGY CONSERVATION

CASE WITH $\vec{E}$ ONLY $\rightarrow$ $\vec{E} = -\nabla\phi(\vec{r})$

$\mathcal{E}_m(K) - e\phi(\vec{r}) = \text{TOTAL ENERGY OF ELECTRON} = E_{TOT}$

$$\frac{dE_{TOT}}{dt} = 0 \Rightarrow \frac{d\,\mathcal{E}_m(k)}{\hbar\, d\vec{k}}\,\hbar\frac{d\langle\vec{k}\rangle}{dt} - e\frac{d\phi(\vec{r})}{d\vec{r}}\,\frac{d\langle\vec{r}\rangle}{dt} = 0$$

①

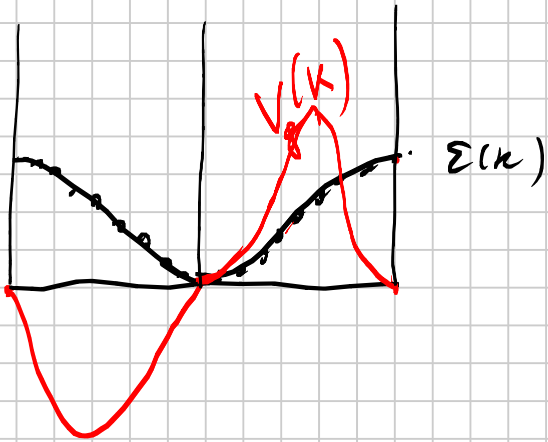
②

$$\hbar\frac{d\langle k\rangle}{dt} = e\frac{d\phi(\vec{r})}{d\vec{r}} = -eE$$

$$\hbar\langle\dot{k}\rangle = \text{FORCE}$$

FILLED BANDS GIVE NO TOTAL CURRENT

$$\vec{J} = -en\vec{v} \Rightarrow \vec{J}_m = -e\int_{\text{OCCUPIED STATES}} v_g(k)\,\frac{d^3k}{(2\pi)^2}$$

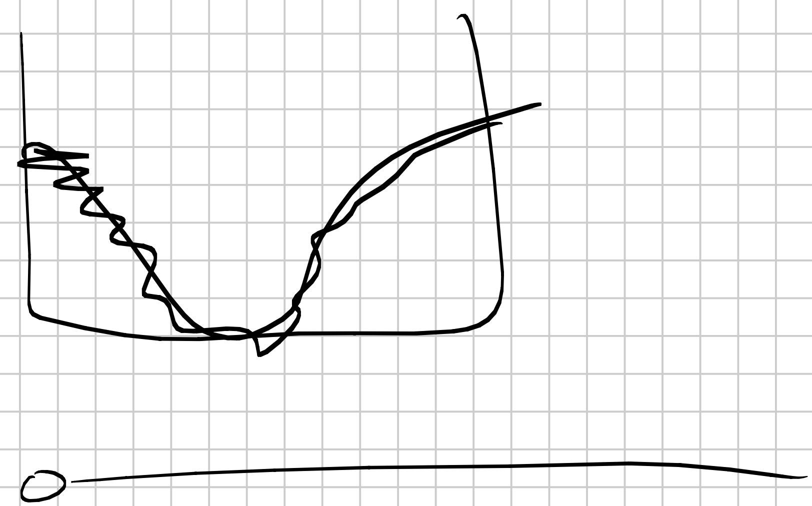


$$J = \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \cancel{dk}\, \frac{d\varepsilon(k)}{\cancel{dk}} = \varepsilon\left(+\frac{\pi}{a}\right) - \varepsilon\left(-\frac{\pi}{a}\right)$$

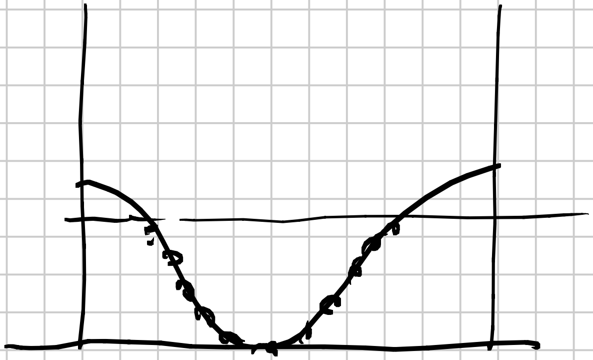
$$= 0$$

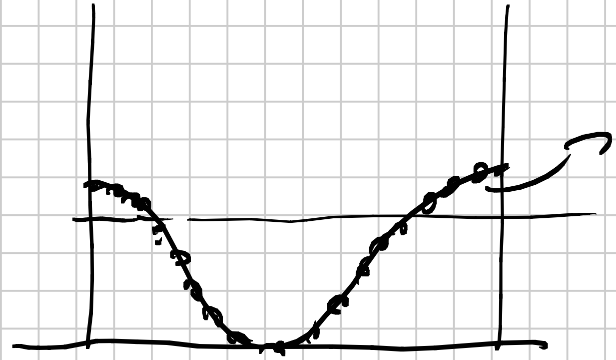
J_{TOT} IS 0 BECAUSE OF PERIODICITY OF $\varepsilon(\vec{k})$!

$$\varepsilon(\vec{k}) = \varepsilon(\vec{k} + \vec{G})$$



HOLES





ADD ELECTRONS TO FILL BAND

$$J = -e \int_{\substack{\text{OCCUPIED} \\ \text{STATES}}} \frac{d^3k}{4\pi^3}\, v(k)$$

$$J_{\text{FULL}} = -e \int_{\substack{\text{OCCUPIED} \\ \text{STATES}}} \frac{d^3k}{4\pi^3}\, v_g(k) - e \int_{\substack{\text{UNOCCUPIED} \\ \text{STATES}}} \frac{d^3k}{4\pi^3}\, v_g k = 0$$

$$\Rightarrow -e \int_{\substack{\text{OCCUPIED} \\ \text{STATES}}} = \overset{\boxed{\text{HOLES}}}{+e} \int_{\substack{\text{UNOCCUPIED} \\ \text{STATES}}}$$

FREE ELECTRON CASE:

$$\dot{p} = \hbar \dot{k} = -eE$$

$$\dot{p} = m \frac{d\vec{v}}{dt}$$

$\dot{k}$ IS PARALLEL TO THE ACCELERATION

BLOCH ELECTRON $\langle \dot{k} \rangle$ CAN BE ANTIPARALLEL TO ACCELERATION

$$\dot{k} = F$$

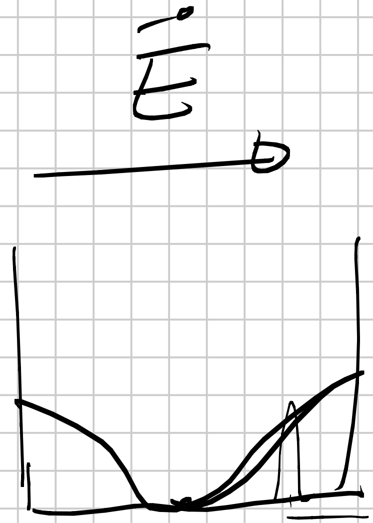
$$\frac{d v_g(k)}{dt} = a(k) = \frac{1}{\hbar} \frac{\partial v_g(k)}{dk} \frac{dk}{dt}$$

$$\frac{1}{\hbar} \frac{d v_g(k)}{dk} = \frac{1}{\hbar^2} \frac{d^2 \varepsilon(k)}{dk^2} = \frac{1}{m} < 0$$

$$\frac{d^2 \varepsilon}{dk^2} < 0$$

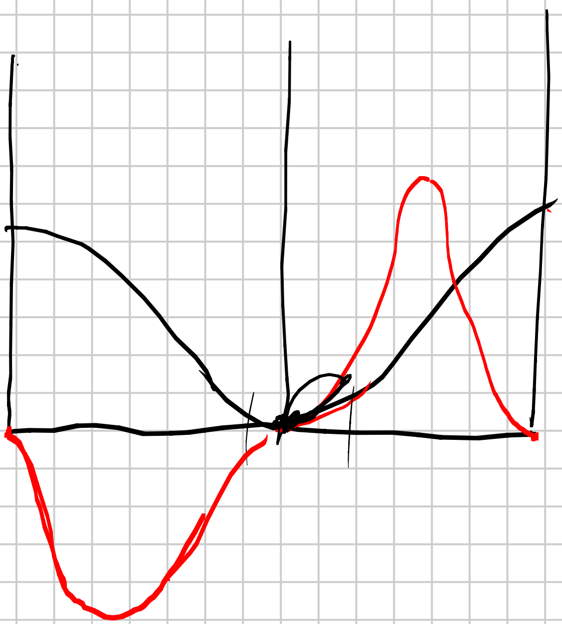
ELECTRON ACCELERATION OPPOSITE TO FORCE

BLOCH OSCILLATIONS



$$\hbar \dot{k} = -eE \Rightarrow k(t) = k(0) - \frac{eEt}{\hbar}$$

$$v(k) = v(k(t)) = v\left(k(0) - \frac{eEt}{\hbar}\right)$$



ELECTRON OSCILLATES WITH PERIOD

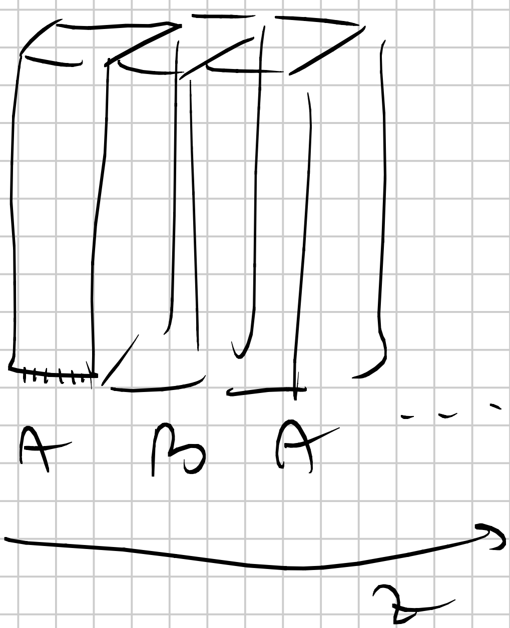
$$-\frac{eEt}{\hbar} = \frac{2\pi}{a}$$

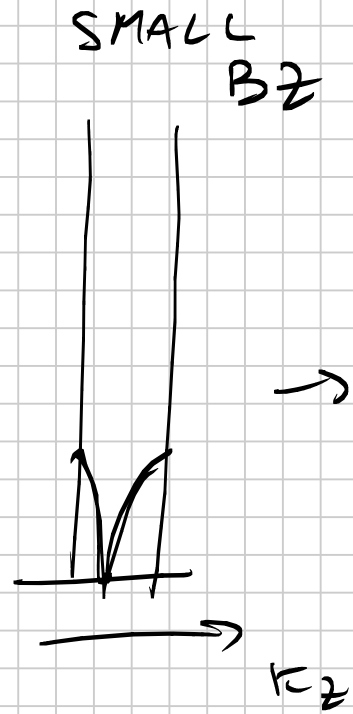
DC FIELD $\Rightarrow$ AC CURRENT

$$T = \frac{2\pi\hbar}{eEa} \Rightarrow \underline{\text{BLOCH OSCILLATIONS}}$$

HARD TO SEE IN REAL CRYSTALS BECAUSE OF RELAXATION

SUPERLATTICES





$\rightarrow$ BLOCH OSCILLATIONS CAN BE OBSERVED