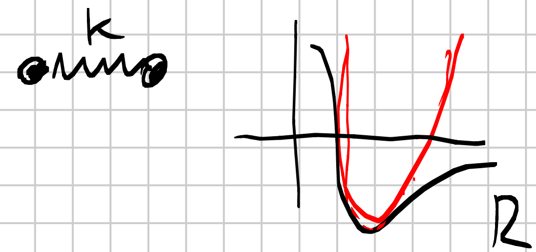
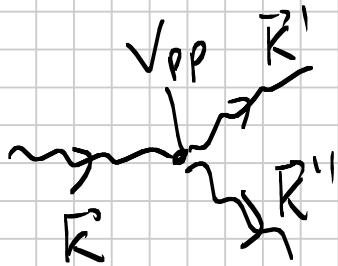
LECTURE #32

Note Title 4/7/2010

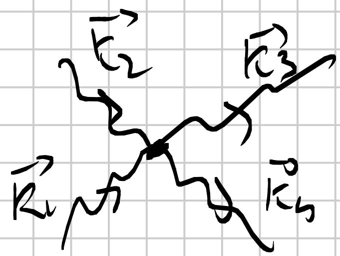
ANHARMONIC EFFECTS (CH 25)



$V_{PHONON-PHONON}$



CUBIC

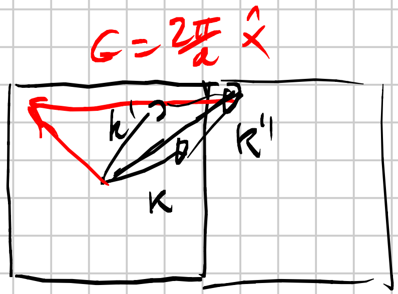


QUARTIC

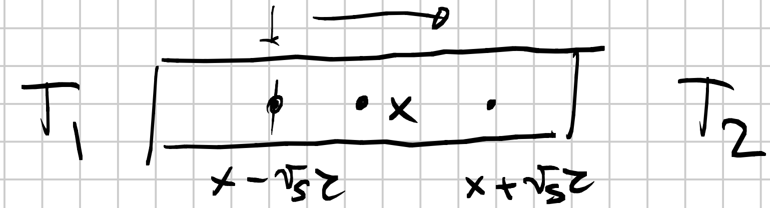
↓

$\vec{K} = \vec{K}' + \vec{K}''$ (NORMAL PROCESS)

$\vec{K} = \vec{K}' + \vec{K}'' + \vec{G}$ ($\vec{G} \neq 0$ UMKLAPP PROCESS)



THERMAL CONDUCTIVITY



$T_1 > T_2$

FLOW OF ENERGY

τ AVERAGE TIME BETWEEN PHONON-PHONON SCATTERING EVENTS

$$J_Q = \frac{v_s}{2}\left[E(x - v_s\tau) - E(x + v_s\tau)\right]$$

$$\left[E(x) - v_s\tau\frac{dE}{dx} - \left[E(x) + v_s\tau\frac{dE}{dx}\right]\right] \to$$

EXPAND IN $v_s\tau \ll L$

1D

$$\to J_Q = -v_s^2\tau\frac{dE}{dx} \to v_s^2\tau\left(\frac{dE}{dT}\right)\left(-\frac{dT}{dx}\right)$$

$$J_Q = -\underbrace{\frac{v_s^2}{3}\tau^L C_v^L}_{\kappa^L}\nabla T$$

$$J_a^e = \frac{1}{3} v_e^2 C_v^e \tau^e$$

$$\varkappa^L(T) \sim C_v(T)\, \tau^L(T)$$

$C_v(T) \to T^3$

HOW DOES τ^L DEPEND ON T

① $T \ll T_D$

$\to$ UMKLAPP PROCESSES ARE FROZEN

$$\langle P_{TOT} \rangle = \sum_k \hbar k \, n(k)$$ CONSERVED

⇒ NO CONTRIBUTION TO THERMAL CONDUCTIVITY

⇒ τ IS AFFECTED ONLY BY SURFACE EFFECTS OR IMPURITIES ↗ τ_0 T-INDEPENDENT

$T \sim T_D$ UMKLAPP PROCESSES ALLOWED

SCATTERING RATE $\propto$ # HIGH ENERGY PHONONS

$$\sim n(q = q_D) \sim \frac{1}{e^{\frac{\hbar \omega_D}{k_B T}} - 1}$$

FOR $T \lesssim T_D$ → $e^{-\frac{T_D}{T}}$

$T \gg T_D$ → $\left(\frac{T}{T_D}\right)$

$$k_B T_D = \hbar \omega_D = \hbar v_s q_D$$

$$\frac{1}{e^{\frac{\hbar\omega_D}{k_B T}} - 1} \sim \frac{1}{\cancel{1} + \underbrace{\left(\frac{\hbar\omega_D}{k_B T}\right)}_{\ll 1} - \cancel{1}} = \left(\frac{T}{T_D}\right)$$

$\frac{1}{\tau}$ = SCATTERING RATE $\propto n(q = q_D)$

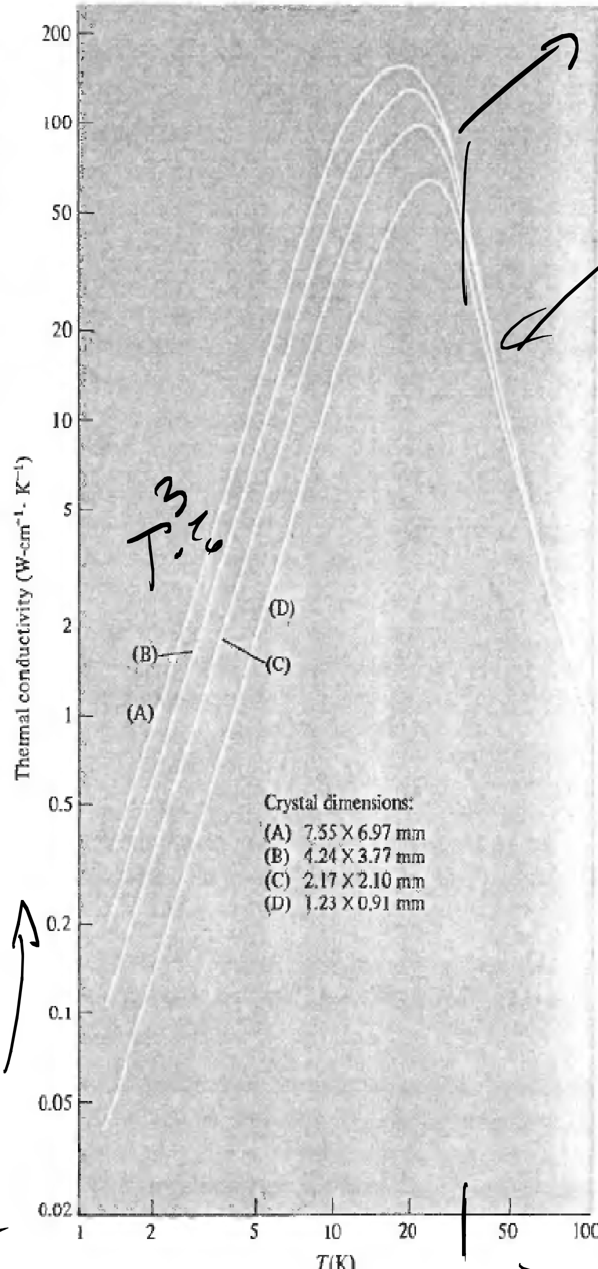
$$\tau \begin{cases} \tau \sim e^{\frac{T_D}{T}} & T \lesssim T_D \\ \tau \sim \frac{T_D}{T} & T \gg T_D \end{cases}$$

$e^{\frac{1}{T}}$

UMKLAPP ACTIVATED

$T^3 \times e^{\frac{T_D}{T}}$

Figure 25.5
Thermal conductivity of isotopically pure crystals of LiF. Below about 10 K the conductivity is limited by surface scattering. Therefore the temperature dependence comes entirely from the T^3 dependence of the specific heat, and the larger the cross-sectional area of the sample, the larger the conductivity. As the temperature rises, umklapp processes become less rare, and the conductivity reaches a maximum when the mean free path due to phonon-phonon scattering is comparable to that due to surface scattering. At still higher temperatures the conductivity falls because the phonon-phonon scattering rate is rapidly increasing, while the phonon specific heat is starting to level off. (After P. D. Thatcher, *Phys. Rev.* 156, 975 (1967).)



$T^{3.16}$

Log κ

$\frac{1}{\tau(x)} \sim C_V(T) \cdot \frac{1}{T}$

SURFACE + IMPURITY

T_D

ANHARMONIC EFFECT

ELECTRIC CONDUCTIVITY

Z ELECTRONS → ELECTRON – PHONON SCATTERING