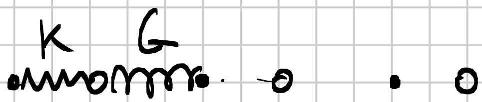
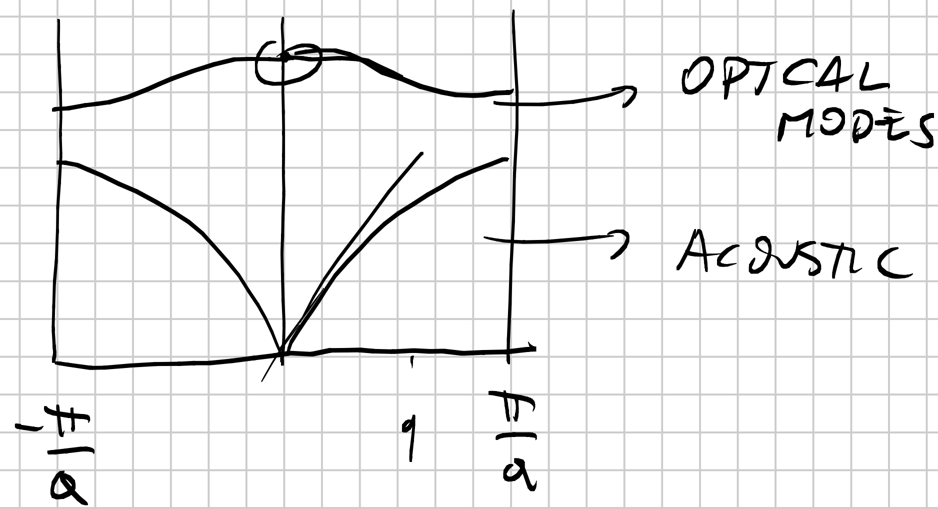
LECTURE #29

Note Title 3/31/2010

THU 29 2:30 – 3:20





CLASSICAL HARMONIC CRYSTAL

$$\langle E \rangle = \langle T \rangle + \langle V \rangle$$

$$\langle T \rangle = \frac{k_B T}{2} \times 3 \quad (3 \text{ DIMENSIONS}) \qquad \frac{3}{2} k_B T$$

$\langle V \rangle$

VIRIAL THEOREM

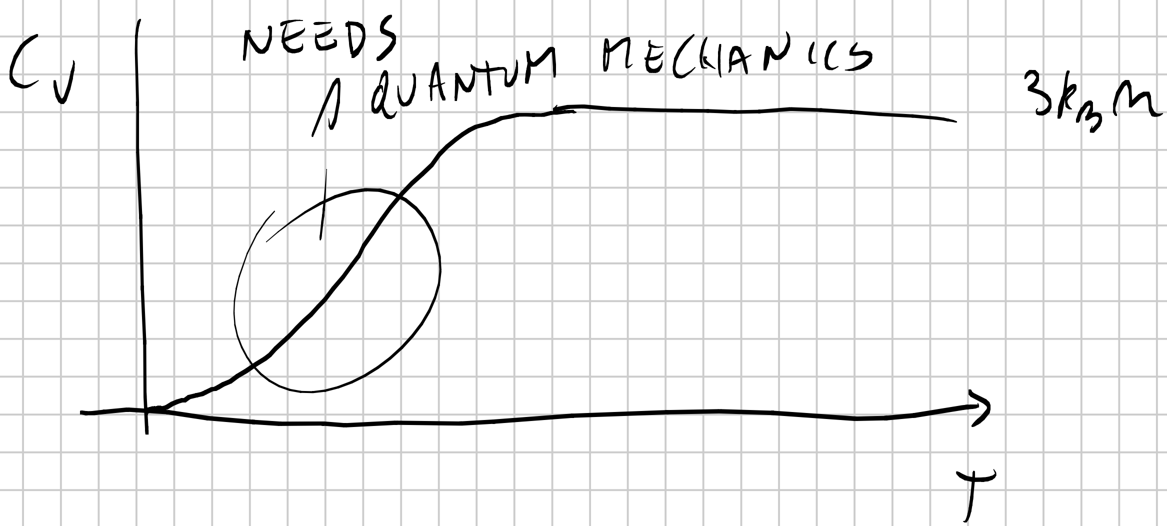
IF $V \propto (\vec{r}_i - \vec{r}_j)^\lambda \Rightarrow \langle V \rangle = \frac{2}{\lambda} \langle T \rangle$

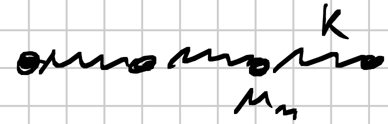
$\lambda = 2 \Rightarrow \langle T \rangle = \langle V \rangle$

$\langle E \rangle = \langle V \rangle + \langle T \rangle = 3 k_B T N$

\# OF PARTICLES

$$C_V = \frac{1}{V} \frac{\partial \langle E \rangle}{\partial T} = 3 k_B \frac{N}{V} = 3 k_B n$$





$$H = \sum_n \frac{p_n^2}{2M} + \frac{1}{2}K\left(u_n - u_{n+1}\right)^2$$

$$H = \frac{p^2}{2M} + \frac{1}{2}MKx^2$$

$$\hat{x} \qquad \hat{p}$$

$$\boxed{\hat{a} = \frac{\hat{x}}{x_0} + \frac{\hat{p}}{p_0}}$$

$$x_0 = \sqrt{\frac{2\hbar}{M\omega}}$$

$$\boxed{x_0 p_0 = \hbar}$$

$$[\hat{x}\ \hat{p}] = i\hbar$$

$$[\hat{a}\ \hat{a}^\dagger] = 1$$

$$\Rightarrow \quad H = \hbar\omega\left(a^\dagger a + \frac{1}{2}\right)$$

$$\hat{a}_q = \frac{1}{\sqrt{N}} \sum_n e^{i\vec{q}\cdot\vec{R}_n}\left(\frac{\hat{u}_n}{u_0} + i\,\frac{\hat{p}_n}{p_0}\right)$$

$$u_0 = \sqrt{\frac{2\hbar}{M\omega(q)}} \quad \rightarrow \quad H^{TOT} = \sum_q \hbar\omega(q)\left(a_q^\dagger a_q + \frac{1}{2}\right)$$

$$[a_q \; a^+_{q'}] = \delta_{qq'}$$

BOSE STATISTICS

$$\langle a^+_q a_q \rangle = \langle n_q \rangle = \frac{1}{e^{\frac{\hbar\omega(q)}{k_B T}} - 1}$$

$$\frac{1}{e^{\frac{\varepsilon-\mu}{k_B T}} - 1}$$

$\mu = 0$

TOTAL # OF PHONONS IS NOT CONSERVED

$$a^{+n}_q |0\rangle = |n_q\rangle$$

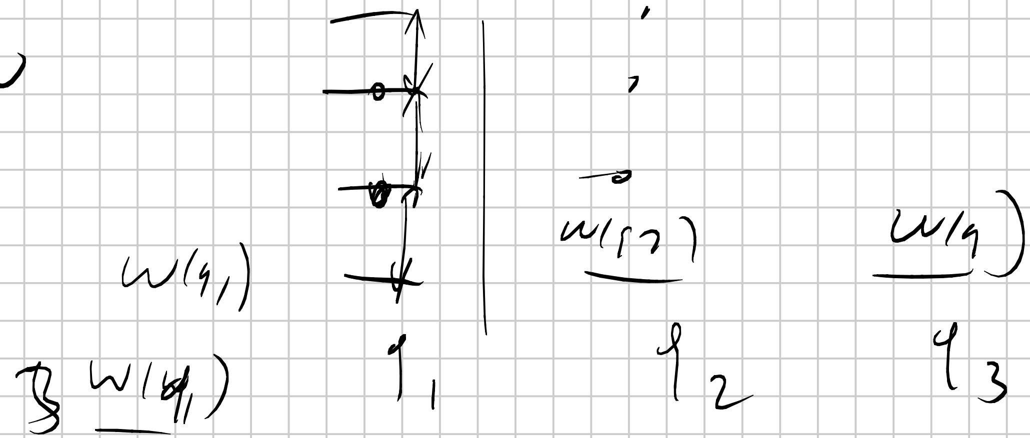
$$a_q |n_q\rangle \to |n_q - 1\rangle$$

$$\langle E \rangle = \sum_q \hbar\omega(q) \langle n_q \rangle = \sum_q \hbar\omega(q) \frac{1}{e^{\frac{\hbar\omega(q)}{k_B T}} - 1}$$

COHERENT

$$|\alpha_q\rangle = \sum_n c_n a^{+n}_q |0\rangle \qquad a_q |\alpha_q\rangle = \alpha_q |\alpha_q\rangle$$

① EINSTEIN



$$|2q_1\rangle \otimes |q_2\rangle \otimes |q_3\rangle$$

$$\frac{\hbar\omega(q_1)\left(\langle n_q\rangle + \frac{1}{2}\right)}{\hbar\omega(q_i)}$$

$$\hbar\omega(q) = \hbar\omega_E$$

\# ATOMS

$$\langle E\rangle = \sum_q \frac{\hbar\omega_E}{e^{\frac{\hbar\omega_E}{k_BT}} - 1} \longrightarrow 3N \frac{\hbar\omega_E}{e^{\frac{\hbar\omega_E}{k_BT}} - 1}$$

$$\frac{\hbar\omega_E}{k_B T} = x$$

$$\langle E\rangle = 3N\, k_B T \frac{x}{e^{x} - 1}$$

$k_B T \gg \hbar\omega_E \qquad x \ll 1$

$$\frac{x}{e^{x}-1} \sim \frac{x}{\cancel{1}+x-\cancel{1}} = 1$$

$$\langle E\rangle = 3N k_B T$$

$x \gg 1 \qquad k_B T \ll \hbar\omega_E$

$$\langle E\rangle \sim 3N k_B T\, x\, e^{-x}$$

$$e^{-\frac{\alpha}{T}}$$

$$C_V = \frac{1}{V}\frac{\partial\langle E\rangle}{dT}$$

T^3

② DEBYE MODEL

$$\langle E \rangle = \sum_q \hbar\omega(q) \langle n_q \rangle$$