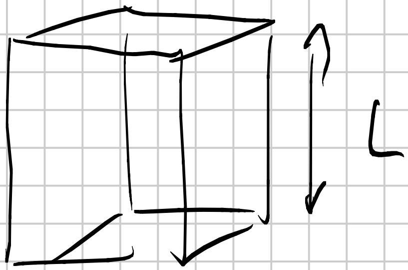
LECTURE #7

$$E = Q \nabla T \qquad Q = -\frac{1}{3}\frac{c_V}{em}$$

TOO BIG WITH DRUDE MODEL

SOMMERFELD THEORY OF METAL



$$\vec{K} = \frac{2\pi}{L}\left(m_x, m_y, m_z\right)$$

m_i: INTEGERS

FERMI SPHERE $\quad |\vec{K}|_{max} = K_F$

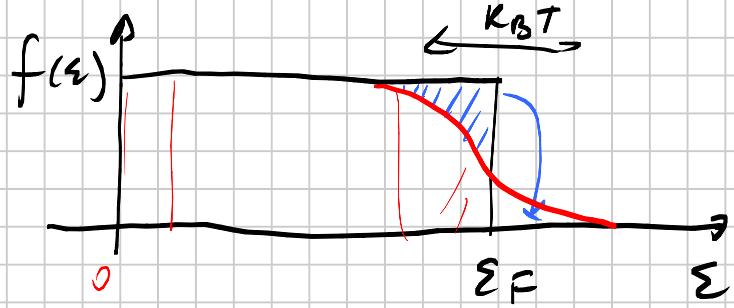
$$m = \frac{N}{V} = \frac{K_F^2}{3\pi^2}$$

(SPIN DEGENERACY)

$$\langle E \rangle_{SOMMERFELD} = N \frac{3}{5} K_B T_F \qquad (T = 0)$$

$$\neq \langle E \rangle_{DRUDE} = N \frac{3}{2} K_B T$$

$$C_V = \frac{1}{V}\frac{dE(T)}{dT}$$



$$\Delta E \sim g(\varepsilon \sim \varepsilon_F)\cdot(k_BT)^2$$

$g(\varepsilon)$ DENSITY OF STATES

$$\frac{C_V^{\text{SOMMERFELD}}}{C_V^{\text{DRUDE}}} \sim \left(\frac{T}{T_F}\right) \sim 10^{-2}$$

SOMMERFELD EXPANSION

$E(T)$

$\downarrow$

$$E(T) = \int_0^\infty d\varepsilon \; g(\varepsilon)\frac{\varepsilon}{e^{\frac{(\varepsilon-\mu)}{k_BT}}+1}$$

$$\frac{N}{V} = \overset{\text{FIXED}}{n} = \int_0^\infty g(\varepsilon)\frac{1}{e^{\frac{(\varepsilon-\mu)}{k_BT}}+1}\,d\varepsilon$$

$\mu(T)$

SOMMERFELD EXPANSION IS AN ASYMPTOTIC SERIES

$$\boxed{e^{x} = 1 + x + \frac{x^2}{2} + \ldots = \sum_m \frac{x^m}{m!} \qquad \text{TRUE FOR ANY } x \Downarrow \text{ CONVERGING SERIES}}$$

$$f(x) \longrightarrow \sum_m a_m g_m(x) \qquad \text{ONLY FOR } x \to \infty$$

$$\boxed{k_B = 1}$$

$$f(\varepsilon) \nearrow \propto \sqrt{\varepsilon} \text{ FOR } \mu(T)$$
$$f(\varepsilon) \searrow \varepsilon^{3/2} \quad E(T)$$

$$I(T) = \int_0^\infty f(\varepsilon) \frac{d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}} + 1}$$

$$z = \frac{\varepsilon - \mu}{T} \qquad d\varepsilon = T\, dz$$

$$I(T) = T \int_{-\frac{\mu}{T}}^{\infty} dz \frac{f(\mu + zT)}{e^{z} + 1} = T \Big[\overset{I}{\int_{-\frac{\mu}{T}}^{0} dz \ldots} + \overset{II}{\int_0^\infty dz \ldots} \Big] =$$

CHANGE $z \to -z$ IN I

$$= T\left[\int_0^{\mu/T} \frac{f(\mu - zT)}{e^{-z}+1}\,dz + \int_0^{\infty} \frac{f(\mu+zT)}{e^{z}+1}\,dz\right] =$$

$$\left[\frac{1}{e^{-z}+1} \to \underbrace{1}_{\sim} - \frac{1}{e^{z}+1} \quad \text{USE THIS IDENTITY}\right]$$

$\mu \sim \varepsilon_F \gg T \qquad \mu/T \gg 1 \sim \infty$

$\mu/T \to \sim\infty$

$$= I(T) = \underbrace{T\int_0^{\mu/T} f(\mu - zT)\,dz}_{} - T\int_0^{\mu/T} \frac{f(\mu - Tz)}{e^{z}+1}\,dz + T\int_0^{\infty}\frac{f(\mu+Tz)}{e^{z}+1}\,dz$$

$$= \underbrace{\int_0^{\mu} f(\varepsilon)\,d\varepsilon}_{\sim T=0 \text{ VALUE}} + \underline{T}\int_0^{\infty} \frac{f(\mu+zT) - f(\mu - zT)}{e^{z}+1}\,dz$$

EVEN TERMS IN zT CANCEL

$$f(\mu + zT) = \boxed{f(\mu) + zT\, f'(\mu) + \frac{1}{2}(zT)^2 f''(\mu) \ldots}$$

$$I(T) = \int_0^{\mu} f(\varepsilon)\, d\varepsilon + 2T^2 f'(\mu) \underbrace{\boxed{\int_0^{\infty} \frac{z}{e^z+1}\, dz}}_{\pi^2/12} + \frac{1}{3} T^4 f'''(\mu) \underbrace{\boxed{\int_0^{\infty} \frac{z^3}{e^z+1}\, dz}}_{\pi^4/120}$$

$$C_V = \left(\frac{\pi^2}{2}\right) \frac{k_B T}{\varepsilon_F}\, n k_B$$

FROM SOMMERFELD EXPANSION

Q

$$R_H = -\frac{1}{nec} \qquad Q = -\frac{1}{3} \frac{C_V}{ne}$$

IN SOME MATERIALS

$R_H > 0$

$Q > 0$

HOLES

$\boxed{DC}$

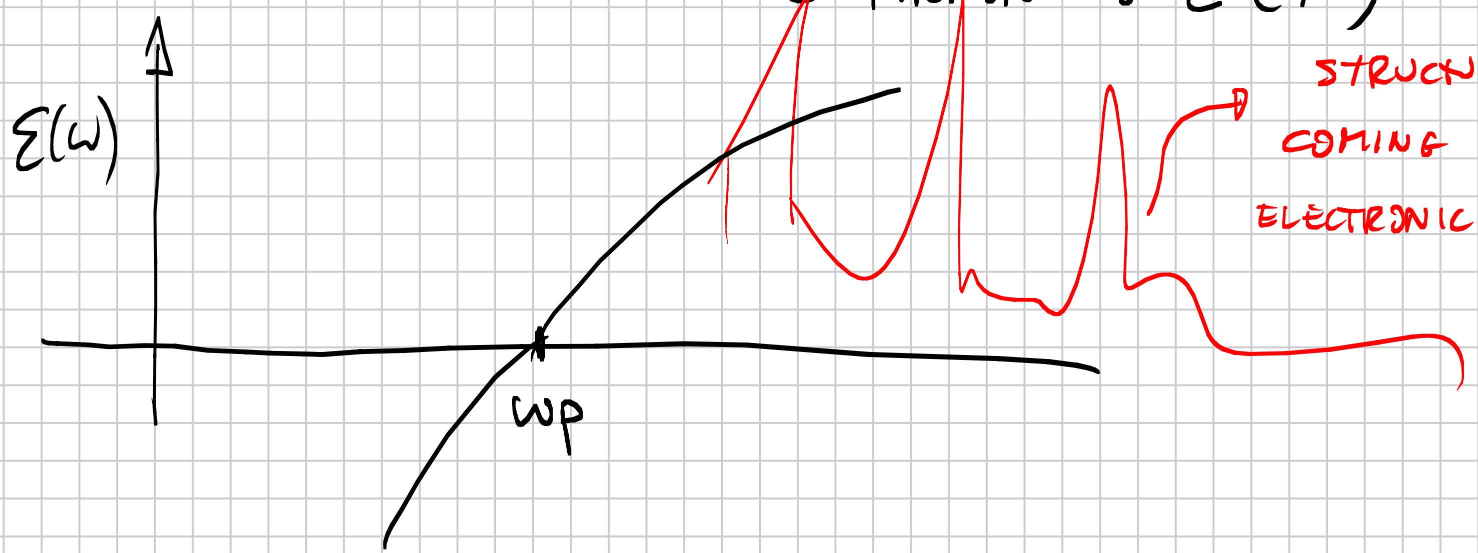
$$\sigma_0 = \frac{n e^2 \tau}{m} \qquad \sigma_0(T)$$

DUE TO TEMPERATURE DEPENDENCE OF τ

e-PHONON $\to \tau(T)$

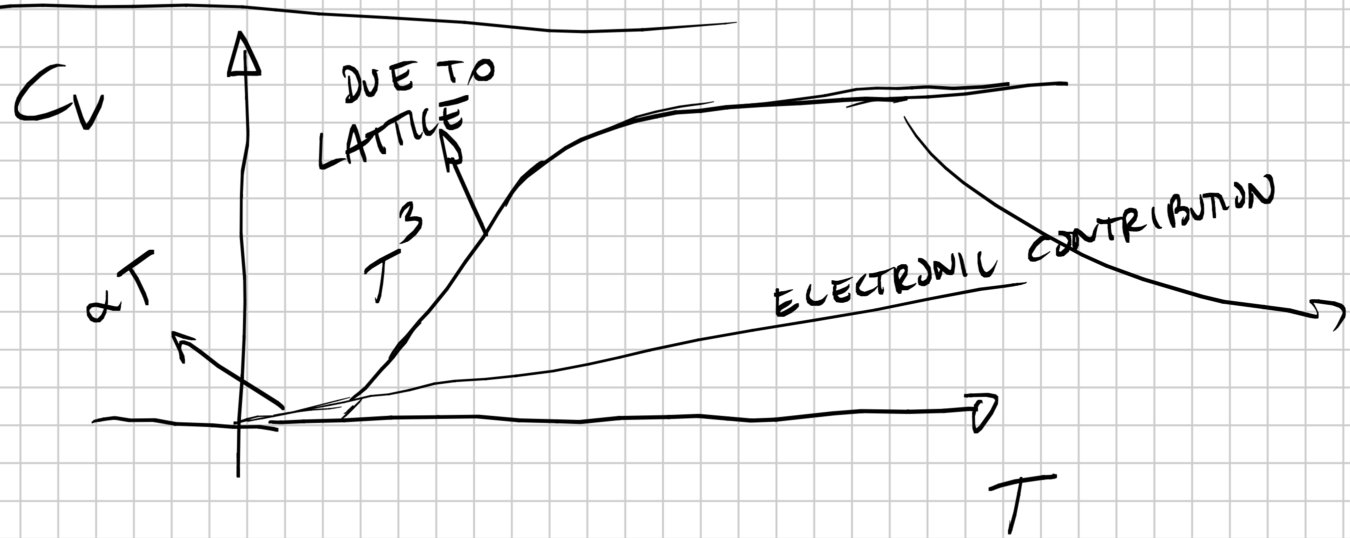
$\boxed{AC}$

$\sigma(\omega)$



STRUCTURES COMING FROM ELECTRONIC BANDS

C_V IN METALS



PHONONS

- WHY INSULATORS $\longrightarrow$ BAND THEORY

THEORY OF BRAVAIS LATTICES