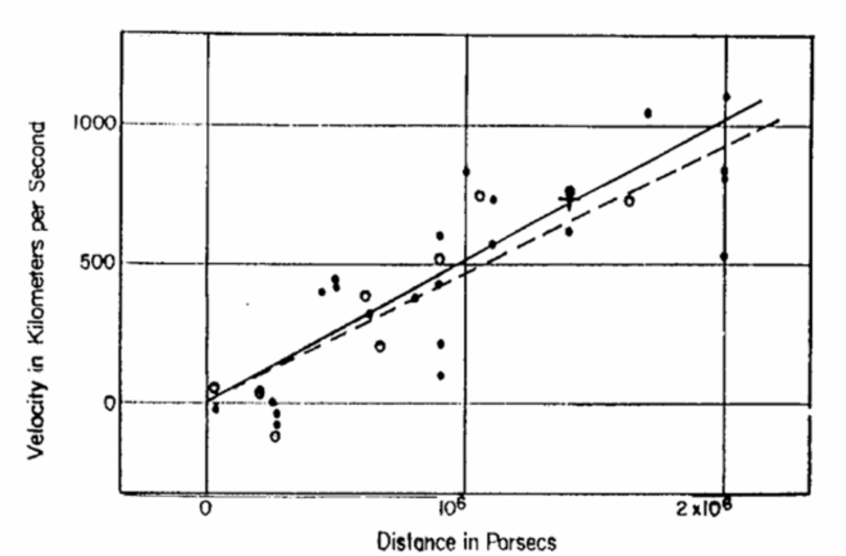
The universe

In the 1920's, V. M. Slipher measured velocities of nearby galaxies. Hubble estimated their distances. Hubble (Hubble, E., 1929, Proc. Nat. Acad. Sci. 15, 168) found velocities v are proportional to distances D.



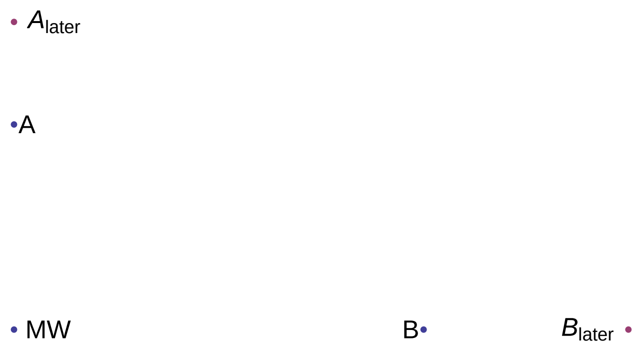
This is called Hubble's Law.

$$v = H D$$

Hubble expansion is special

In[24]:=
```
ListPlot[{{{0, 0}, {0, 1}, {2, 0}}, 1.5 {{0, 1}, {2, 0}}}, Epilog →
  {Text["MW", {0, 0}, {-1.5, 0}], Text["A", {0, 1}, {-1.5, 0}], Text["B", {2, 0}, {1.5, 0}],
   Text["A_later", 1.5 {0, 1}, {-1.5, 0}], Text["B_later", 1.5 {2, 0}, {1.5, 0}]},
 AspectRatio → Automatic, Axes → False, ImageSize → 300,
 BaseStyle → {FontFamily → "Helvetica", FontSize → Medium}]
```

Out[24]=



Expansion by Hubble's Law is very special. Consider Milky Way, galaxy A at distance 1, and galaxy B at distance 2.

1. Expansion is by a scale factor. $v_A = 1$, and $v_B = 2$. Suppose some time passed, and A has moved by 0.5 to 1.5. Then B has moved by 1 to 3. B remains twice as far as A.

2. Centerless expansion.

A is $5^{1/2}$ from B. MW is 2 from B.

After time passed, A is $\left(3^2 + 1.5^2\right)^{1/2} = \frac{3}{2}\, 5^{1/2}$ from B, and MW is $\frac{3}{2}\, 2$ from B.

Galaxy B is the center of the expansion too.

3. There exists a beginning, when the scale factor is 0. In this example, let time be -1.

4. Hubble did not find special directions. The universe is isotropic.

Puzzle

Can galaxies go faster than light? H=70km/s/Mpc. (A parsec is $180\times3600/\pi$AU.) A galaxy is at 6,000Mpc. Its speed $v = \mathrm{HD} = 420,000\ \mathrm{km}/s$ is faster than the speed of light. How is that possible? What could happen if I could go faster than the speed of light?

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Isotropic & homogeneous spaces

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The universe is the same everywhere (homogeneous) and the same in all directions (isotropic).

Simplicio: We see light from distant galaxies that were forming stars for the first time. There is not the same as here.

Can you think of a 2-d homogeneous & isotropic space?

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Friedman-Robertson-Walker spaces

A 3-d space that is homogeneous and isotropic has a special choice of time. A choice of coordinates is

(t, r, θ, ϕ)

and the metric is

$$\mathrm{ds}^2 = -\mathrm{dt}^2 + a(t)^2 \left[d\, r^2 \big/ \left(1 - (r/r_0)^2\right) + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\, \mathrm{d}\phi^2\right)\right].$$

(r, θ, ϕ) is called the comoving coordinate. A galaxy stays at the same position; time changes.

r_0^2 can have any value, positive or negative.

$a(t)$ is called the expansion parameter. Descibe the effect of the expansion parameter.

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In[12]:=
```
fig[α_: π / 6] := Module[{c = {0, 0}},
  ListPlot[c, AspectRatio → Automatic, PlotRange → {1.05 {-1, 1}, 1.05 {-1, 1}},
   Epilog → {Circle[c, 1], Circle[c, .3, {π / 2 - α, π / 2}],
     Text["ξ", .3 {Sin@α / 2, Cos@α}, {0, -2}], Dashed, Line[{c, {0, 1}}],
     Line[{c, {Sin@α, Cos@α}, {0, Cos@α}}]}, ImageSize → 200, Axes → False]]
```

What is r_0^2?

$ds^2 = -dt^2 + a(t)^2 \left[d\,r^2 \big/ \left(1 - (r/r_0)^2\right) + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right]$

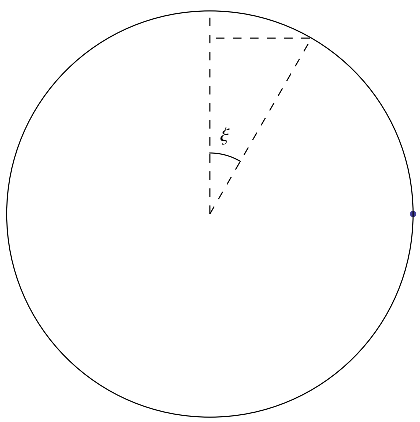
Let time be fixed. Consider the spatial part of the metric.

$$d\,r^2 \big/ \left(1 - (r/r_0)^2\right) + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

Suppose $r_0^2 \to \infty$. What is the space?

Suppose $r_0^2 > 0$. What is the space?
Map 3-d space into 2-d by supressing ϕ. The space is the 2-d surface of a sphere. I draw a slice.

In[13]:= `fig[]`

Out[13]=


Define $\sin\xi = r/r_0$. $r_0\, d\,\xi = d\,r/\cos\xi = d\,r \big/ \left[1 - (r/r_0)^2\right]$. Therefore

$$ds^2 = r_0^2\, d\text{ latitude}^2 + r^2\, d\text{ longitude}^2.$$

Lesson: If $r_0^2 > 0$, then r_0 is the radius of curvature of the 3-d space. If ${r_0}^2 < 0$, the space is like a saddle, and the space is infinite.

Hubble's Law

Consider the case $r^2 \ll |r_0{}^2|$. The radial distance of a galaxy at r is

$$D(t) = a(t)\, r.$$

The galaxy moves away at speed

$$v = \frac{d}{dt} D(t) = \frac{1}{a} \frac{d\,a(t)}{dt}\, a\, r = \frac{1}{a} \frac{d\,a(t)}{dt}\, D.$$

Define Hubble's constant $H = \frac{1}{a} \frac{d\,a(t)}{dt}$. Then

$$v = H\, D$$

This is called Hubble's Law.