

# Physics 472 – Spring 2010

## Homework #3, due Friday, January 29

(Point values are in square brackets.)

1. [4] Griffiths problem 4.34. We did this in class, but it's a good warm-up for the next two problems.
2. [5] Consider an electron in a hydrogen atom state with  $l=1$ . The total angular momentum is represented by the operator  $\vec{J} = \vec{L} + \vec{S}$ . The corresponding quantum number  $j$  can take the values  $j=3/2$  or  $j=1/2$ . In class we showed how to use the lowering operator to express the eigenstates of  $J^2$  and  $J_z$ , labeled  $|j, m_j\rangle$ , in terms of linear combinations of the states  $|l, m_l\rangle \otimes |s, m_s\rangle$ . Follow the same procedure here to construct all four states of the  $j=3/2$  ladder and both states of the  $j=1/2$  ladder. Check your results using the Clebsch-Gordan coefficients in Table 4.8.
3. [4] Griffiths problem 4.36. For part (a) you may use Table 4.8. For part (b), you may use either Table 4.8 or the results you derived in the previous problem. If you choose the latter, you need to invert the basis transformation, i.e. you need to express the appropriate  $|l, m_l\rangle \otimes |s, m_s\rangle$  state as a linear combination of the  $|j, m_j\rangle$  states.
4. [7] Griffiths problem 4.55.

For part (e), use Table 4.8. The state given in the problem contains tensor product terms of the form  $|l, m_l\rangle \otimes |s, m_s\rangle$ . You need to express those in terms of the eigenstates of  $J^2$  and  $J_z$ , i.e. the states  $|j, m_j\rangle$ . The radial quantum number  $n$  plays no role in this basis transformation. To aid in keeping track of all the quantum numbers, write each of the two terms of the quantum state first in this form:  $|n, l, m_l\rangle \otimes |s, m_s\rangle$ , then express those as linear combinations of the states  $|n, j, m_j\rangle$ .

Part (g) is interesting. Should you add the two wavefunctions, then square the sum? Or should you square them first, then add the squares? Since the two wavefunctions are associated with different spin states, they are (in principle) distinguishable. That means you should square first, then add. (It is like 2-slit diffraction in the case where you know which slit each electron goes through.) Use Tables 4.7 and 4.3 to express your final answer as a simple function of  $(r, \theta, \phi)$ .