Physics 472 – Spring 2010

Homework #5, due Friday, February 19

(Point values are in parentheses.)

1. [7] On a previous problem set you found the matrices that represent the operators for the three component of spin, for a spin-1 particle. They are:

$$\hat{S}_x = \begin{pmatrix} 0 & \hbar/\sqrt{2} & 0 \\ \hbar/\sqrt{2} & 0 & \hbar/\sqrt{2} \\ 0 & \hbar/\sqrt{2} & 0 \end{pmatrix}, \ \hat{S}_y = \begin{pmatrix} 0 & -i\hbar/\sqrt{2} & 0 \\ i\hbar/\sqrt{2} & 0 & -i\hbar/\sqrt{2} \\ 0 & i\hbar/\sqrt{2} & 0 \end{pmatrix}, \ \hat{S}_z = \begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix}$$

A beam of spin-1 atoms is prepared in the initial spin state: $|\chi\rangle = \begin{pmatrix} -3i/4 \\ \sqrt{6}/4 \\ i/4 \end{pmatrix}$.

a) [1] If you send the beam into a SG apparatus with its spin axis oriented along the z-direction, calculate the probabilities associated with each of the three output ports of the apparatus, i.e. calculate the probabilities that measurement of S_z will produce $\hbar$, 0, and $-\hbar$. (You should be able to do this just by looking at $|\chi\rangle$, with little calculation.)

b) [2] Now change the spin orientation of your SG apparatus to the x-axis, and calculate the probabilities associated with each of the three output ports. This time you will need to do a real calculation.

c) [2] Do the same thing with the SG apparatus oriented along the y-axis.

d) [2] Calculate the expectation values, $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$, using the probabilities you calculated in parts (a) – (c). Check your answers using direct matrix multiplication.

2. [4] Griffiths problem 6.1

3. [4] Griffiths problem 6.2. The easiest way to do part (b) is to express the $\hat{x}$ operator in terms of $\hat{a}$ and $\hat{a}^+$, as we have done in class.

4. [5] Griffiths problem 6.4. You do not have to sum the series in part (a), but try to if you want a challenge.