Newton on K's 3rd Law, To Frame the World—30 Sept

- Announcements
  - Test 1 will be returned on Wed.
  - Homework 4 is due on 7th.
  - Observatory open house
    - Fri & Sat 9:00-11:00, weather permitting
- Outline
  - Newton amends Kepler's 3rd Law (left over from last week)
  - Kepler found orbit of Mars relative to earth's orbit.
  - Goal was to measure the absolute distance (in miles or km) of the solar system
  - Cassini & Richer 1672

Giovanni Domenico Cassini, (1625 - 1712) engraving by N. Dupuis
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Newton explains Kepler's 3rd Law

- Kepler's 3rd Law for objects orbiting the sun

  $$P^2 = R^3$$

  - P is the period in years. R is the semi major axis in AU.
- Newton derived K's 3rd Law from his mechanics and his law of gravity. He found
  - Kepler's 3rd Law applies for any object in orbit around a much more massive one. Even Ida and Dactyl.

    $$P^2 = R^3/M$$

  - M is the mass in units of the mass of the sun.
- Key idea for measuring mass
  - Used for all astronomical objects.

Ida & its moon Dactyl. Galileo Mission, NASA

Newton's Laws imply Kepler's 3rd Law

- Newton's Law of Gravity: Force between sun and planet

  Force = G $mass_{Sun}$ $mass_{Planet}$/$Distance^2$

$$F = G\,M\,m/R^2$$

- Newton's 2^{nd} Law

  Force = $mass_{Planet}$ acceleration

$$F = m\,a$$

- Simplified derivation. Assume the orbit is a circle.

1. S1: The motion is "natural." S2: The gravity of the sun is causing the direction of the motion to change.
   A. TT. B. TF. C. FT. D. FF

Newton's Laws imply Kepler's 3rd Law

- Newton's Law of Gravity: Force between sun and planet

  Force = G $mass_{Sun}$ $mass_{Planet}$/$Distance^2$

$$F = G\,M\,m/R^2$$

- Newton's 2^{nd} Law

  Force = $mass_{Planet}$ acceleration

$$F = m\,a$$

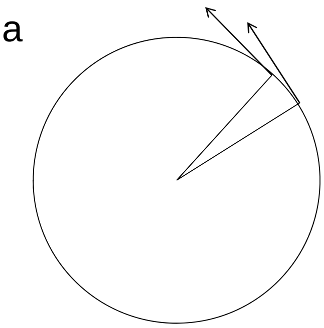
- Simplified derivation. Assume the orbit is a circle.
- The force of the gravity of the sun is causing the direction of the motion to change.

$$a = G\,M/R^2$$

- Is it surprising that the acceleration does not depend on the mass of the planet? Where have we seen this before?

Acceleration of motion in a circle

1. A planet is moving in a circle at the same speed. Is it accelerating?
   A. Y
   B. N



Acceleration of motion in a circle

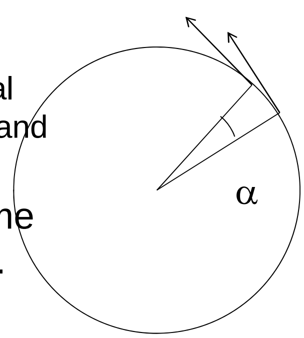
1. How do you find the acceleration?
   A. Draw an arrow from tip of initial velocity to tip of final velocity and divide by the time.

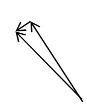
- Angle between velocity is same as angle between radial lines.

$$\alpha = 2\pi\, t/P$$

$a = v\, \alpha/t$ (small angle formula)

$a = \frac{2\pi v}{P} = 4\pi^2 R/P^2$ since $v = 2\pi R/P$





Newton's Laws imply Kepler's 3rd Law

$$a = G\,M/R^2$$
$$a = 4\pi^2 R/P^2$$

- Finally

$$P^2 = (4\pi^2/G)\,R^3/M$$

$4\pi^2/G = 2\text{x}10^{30}$ kg yr^2/AU^3 = 1 M_{sun} yr^2/AU^3

- If P is in years, R in AU, and M is in M_{sun}, then

$$P^2 = R^3/M$$

1. In what way is Newton's form of Kepler's 3rd Law different from Kepler's?
   A. The two are the same.
   B. Newton says it depends on the mass of the star.
   C. N found what it is in human-based units.
   D. More than one of the other answers is correct.

Kepler's 3rd Law, according to Newton

- Phy 321 derivation

$$P^2 = (4\pi^2/G)\,R^3/(M+m)$$

- The mass is the mass of the star and planet.
- If period is measured in years, the semi-major axis is measured in AU, and mass of star is measured in M_{sun}, and mass of planet is negligible,

$$P^2 = \frac{R^3}{M_{star}}$$

- Kepler's 3rd Law is the special case of a planet with negligible mass in orbit around a star with the mass of the sun.

1. A planet orbits a star with the same orbit as the earth. Its period is 2 years. The mass of the star is__ M_{sun}.
   A. 2
   B. 4
   C. ½
   D. 1/4

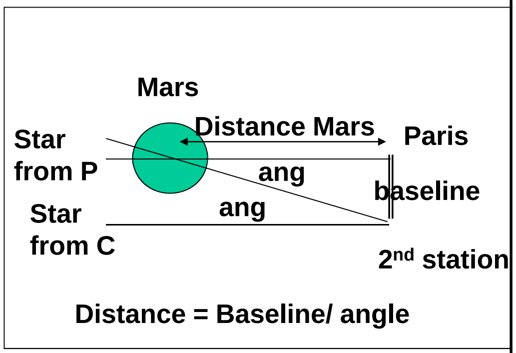
To Frame the World

- We pretend to be Jean (Giovanni) Cassini, Director of the Paris Observatory.
- We propose a grand plan to "Frame the World" to the Louis XIV. We will determine the distance to the outermost planet in the solar system!

Giovanni Domenico Cassini, (1625 - 1712)
engraving by N. Dupuis
www.sil.si.edu/digitalcollections/hst/scientific-identity/fullsize/SIL14-C1-1

Cassini & Richer 1672

- Angle=baseline/distance
- What baseline should C&R use to measure distance to Mars?
  - Cassini was the director of the Paris Observatory.
  - Richer was his colleague.



Cassini & Richer 1672

- Angle=baseline/distance
- What baseline should C&R use to measure distance to Mars?
- Cayenne-Paris baseline is 7000km.
  - Angle=baseline/distance=7000km/60,000,000km
    = 120μrad
  - Shift is 25 times width of a star seen with modern telescope.

Cassini & Richer

- We are pretending to be Jean Richer and Giovanni Cassini in 1672. We are measuring the distance to Mars by making observations from Paris and Cayenne.
- Facing the screen, hold a pencil at arms length. Without moving the pencil, look at it with your left and then your right eye. The pencil tip shifts with respect to something on the screen.

1. What corresponds to Mars?
   A. Left eye or right eye
   B. Tip of pencil
   C. Something in the screen
   D. The shift of the pencil tip with respect to the screen.
2. What corresponds to Paris?
3. What corresponds to the star?
4. What is proportional to the parallax angle?