You may use 1 sheet of notes during this test. You may not have any books or other notes.

You may not borrow a calculator from another student. You may borrow a calculator from the front bench.

Write brief answers. Your time is limited, and the graders do not like to read answers that address off-topic ideas.

You may pick up the exam next year in room 3260. You may see your grade on angel after Tues, Dec. 21.

Good luck.

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Kepler’s 3rd Law \[ P^2 = R^3/M \] (in AU, year, & \( M_{\text{sun}} \))

Hubble’s Law \[ v = H D \]

Mass \( M = 233 v^2 R \) (in parsec, km/s, & \( M_{\text{sun}} \))

Wien’s Law \[ \lambda_{\text{peak}} T = 2.9 \text{mm K} \]

Redshift \[ z = 1/a - 1; a = 1/(1+z) \]

Hubble’s Constant \[ 60 \text{ km/s/Mpc} \]

Wavelength \[ v = c z \]

Speed of Light \[ v = c (\lambda_{\text{rec}}/\lambda_{\text{emit}} - 1) \]

Hubble’s Constant \[ 300,000 \text{ km/s} \]

Present temperature of the cosmic background radiation 2.728 K

Parsec \[ 3.09 \times 10^{13} \text{ km} \]

AU \[ 1.50 \times 10^8 \text{ km} \]

Year \[ 3.16 \times 10^7 \text{ s} \]
1. The figure shows the Sun, Earth, and a star. It is midnight on 21 March.
   a. (3 pts.) Draw the Sun, Earth, and the star three months and 3 hours later (3am on 21 June) according to Ptolemy. On the same diagram, draw Earth and the Sun an hour later at 4am on 21 June. Explain how you placed each object.

   First figure out the locations according to Copernicus. Earth goes $\frac{1}{4}$ of the way around the sun to account for the date, and spins $\frac{1}{8}$ of a revolution to account for the time. Since Earth is stationary in Ptolemy’s model, rotate to get Earth and the guy (the line) back to the same position.

   ![Diagram of Earth, Sun, and Star at 3:00am on 21 June according to Copernicus](image)

   ![Diagram of Ptolemy’s model at 3:00am on 21 June. The circle is the sun’s orbit. At 4:00am, the sun has moved 15° clockwise](image)

   b. (2 pts.) What observation did Copernicus explain in a more natural way than did Ptolemy? Explain in what way Copernicus’ explanation is more natural.

   To explain the motion of the planets with respect to the stars, Ptolemy needed two separate motions, the motion of the epicycle and the motion of the planet on the epicycle. On the other hand, Copernicus explained it with a single motion, that of all planets around the sun.

2. Consider the Hertzsprung-Russell diagram of the star cluster Pleiades.
   a. (2 pts.) On the H-R diagram of Pleiades, circle a star, if there is one, that is burning hydrogen in its core. (There may be none.) Explain.

   All of the stars are on the main sequence, where stars are burning hydrogen in the core.

   b. (2 pts.) On the H-R diagram of Pleiades, circle a star, if there is one, that is burning
helium. (There may be none.) Explain.

There are no giants.

c. (2 pts.) On the H-R diagram of Pleiades, draw the main sequence of Pleiades billions of years from now. Explain your reasoning.

The hotter main-sequence stars have used up their fuel. The main sequence is truncated.

3. About the sun. Write very brief answers.

a. (2 pts.) At the present time, how does the sun produce energy? What is used up, and what is created?

Hydrogen fused to become helium. H is used up. \(^{4}\text{He}\) is created.

b. (1 pt.) What is the approximate age of the sun and earth?

4.5Byr.

c. (3 pt.) When the sun runs out of a fuel, it shrinks until it burns another fuel, which heats the gas to provide pressure. When it completely exhausts its fuel, what prevents it from shrinking to a point, in spite of it having no fuel to make energy? Explain.

In a normal gas, energy is required to have pressure. In a degenerate electron gas, the electrons move because they are confined to a small space. No energy is needed to make the electrons move.

4. Unfortunate planet U orbits the black hole in the center of the Milky Way Galaxy. The length of the semi-major axis is 1AU. The mass of the black hole is 3 million times the mass of the sun.

a. (1 pt.) Is the period of the orbit of planet U longer or shorter than a year? Explain without doing any calculation.

It takes Earth a year to orbit the sun. With a larger mass, the force of gravity is bigger. Since the acceleration is \((\text{distance}/\text{time}^2)\), the time is shorter than a year.

b. (2 pts.) Find the period of the orbit of planet U.

Kepler’s 3\(^{rd}\) Law.

\[ P^2 = \frac{R^3}{M} \]

\[ P = 1\text{year}/(3\times10^6)^{1/2} \]

\[ = 0.58\text{e-4year}=5\text{hour}. \]

5. You are transported back in time to when the universe was 13 million years old and the expansion parameter was 1/100.

a. (3 pts.) What was the temperature of the radiation from the Big Bang? (You must show your work.) What has a comparable temperature?

\[ T = \frac{T_{\text{now}}}{a} = \frac{2.728}{100} = 273\text{K}, \] which is the freezing point of water.

b. (2 pts.) Name an element that is found in this room but did not exist at that time. Explain.

At that time, H and \(^{4}\text{He}\) had formed, but there were no stars. O, C, N, Ca, and Fe did not exist at that time.
c. (2 pts.) What was the ratio of neutrons to protons? Explain.

The ratio n/p was 1/7 from when the universe was 3-min old up to the present. At 3min, the vast majority of the neutrons became locked in $^4\text{He}$ nuclei. The number of neutrons has increased slightly as $^4\text{He}$ is made in stars.

6. The galaxy NGC3672. Use the graphs in Figure 1.

a. (2 pts.) Calculate the distance between us and that galaxy?

Look the spectrum along the minor axis, where the motion is due primarily to the expansion of the universe.

$$D=v/H=1850\text{km/s}/(60\text{km/s}/\text{Mpc})=31\text{Mpc}.$$  

b. (2 pt.) If NGC3672 had no dark matter, would the lower panel change? Explain.

No change. The rotational motion, which is affected by the mass of the galaxy, does not affect motion in the line of sight.

c. (3 pts.) Find the ratio $M_{18}/M_9$, where $M_{18}$ is the mass enclosed within 18 kpc of the center, and $M_9$ is the mass enclosed within 9 kpc.

The rotational speed is constant from 9 to 18kpc. Then Kepler’s 3rd Law says $M(R)=\text{constant} \cdot R$. Since R doubles, $M_{18}/M_9=2$.

7. The angular size of the hot spots in the cosmic background radiation is 1 degree. The cosmic background radiation shows the universe as it was at recombination.

a. (2 pts.) The linear size of the hot spots is a known length. Explain why that is known.

The linear size is $(\text{speed of light}) \times (\text{age of universe})$

b. (4 pt.) If the universe has less mass density than it does, would the angular size be larger or smaller than 1 degree? Explain.

Angular size=$\frac{\text{linear size}}{\text{distance}}$. If universe has less mass density, the time to expand by a factor of 1000 is greater. Therefore the distance = time×(speed of light) is greater, and the angle is smaller.

8. The three major contributors to the mass density of the universe are ordinary matter, dark matter, and dark energy.
a. (3 pts.) What is the approximate value of the density parameter for each?
Density parameter of ordinary matter, dark matter, and dark energy is 0.05, 0.23, & 0.73.

b. (3 pts.) How is dark energy different from ordinary matter?
Dark energy has negative pressure and repulsive gravity.

9. A galaxy is moving at 1000km/s with respect to the frame in which the Big Bang is at rest. It is moving toward 0hr right ascension and 0° declination, written 0hr+0°.

a. (2 pts.) What is the temperature of the radiation from the Big Bang in the direction 6hr+0° as measured by astronomers in that galaxy? Explain.

Temperature is 2.728K, the average temperature. The reason is that our motion is in the perpendicular direction, which shows no Doppler shift.

b. (4 pts.) What is the temperature of the radiation from the Big Bang in the direction 0hr+0° as measured by astronomers in that galaxy? Explain.

In this direction, the Doppler shift
\[ \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} = 1 - \frac{v}{c} = 1 - \frac{1000 \text{ km/s}}{300,000 \text{ km/s}} = 1 - \frac{1}{300}. \]

Because of Wien’s Law, the temperature is
\[ 2.728 \times \left(1 - \frac{1}{300}\right) = (2.728 + 0.009) \text{K} \]