You may not use books or notes.
Do the easy questions first. Then go back to the harder ones.

Eccentricity (dist between foci)/(major axis)
Small-angle Ange=baseline/distance
approx.
Area of triangle

$$
A=\frac{1}{2} \text { base height }
$$

Degree-radian conversion
$180^{\circ}=\pi$ radian
$57^{\circ}=1$ radian

| Name |  |
| :--- | ---: |
| PID |  |
| 1 | $/ 5$ |
| 2 | $/ 4$ |
| 3 | $/ 3$ |
| 4 | $/ 8$ |
| 5 | $/ 25$ |
| Total |  |

Center of mass $\quad m_{1} d_{1}=m_{2} d_{2} \quad m$ are masses. $d$ are distance to the center of mass

Sidereal day $\quad 24 \mathrm{hr} 4 \mathrm{~min}$

| Kepler's $3^{\text {rd }}$ law | $P^{2}=a^{3}$ | P is the period in years and a <br> is the semi major axis in AU |
| :--- | :--- | :--- |
| Aphelion distance | $R_{a}=a(1+\epsilon)$ | a is the semi major axis and $\varepsilon$ <br> is the eccentricity |
| Perihelion distance | $R_{p}=a(1-\epsilon)$ | a is the semi major axis and $\varepsilon$ <br> is the eccentricity |
| Kepler's $2^{\text {nd }}$ law at <br> perihelion or | $A=\frac{1}{2} \mathcal{v} D$ | vis speed, t is time, and D is <br> distance | aphelion

Cap Sag Sco


1. Astronomers have been measuring the speed of star $X$. They find that its speed changes periodically just as that of 51 Peg does. The period is 4 day 5 hr , which is the same as that of 51 Peg . The maximum speed is $60 \mathrm{~km} / \mathrm{s}$, whereas the maximum for 51 Peg is $60 \mathrm{~m} / \mathrm{s}$. In both cases, the visible star and something much fainter orbit each other. Assume the orbits are circles.
a. (2 pt.) Find the ratio of the radii of the orbits of these two stars, $R_{X} / R_{51 \text { Peg }}$.

Since the periods are the same, the ratio of the distances traveled in one period is the same as the ratio of the speeds, and $\frac{R_{X}}{R_{51 \text { Peg }}}=\frac{v_{\chi}}{v_{51 \text { Peg }}}=\frac{60 \mathrm{~km} / \mathrm{s}}{60 \mathrm{~m} / \mathrm{s}}=1000$
Many computed the numerical value of the distance traveled and then found the ratio. That is a lot of work and prone to error. Save yourself some time by canceling the common factors before you use your calculator.
b. (3 pts.) Devise a model for star X that explains its faster speed. Explain your reasoning.
Star X is on a bigger orbit. The radius of the orbit is the distance $d_{s}$ from the star to the center of mass of the star-companion system. Since $d_{s}=m_{c} / m_{s} d_{c}$, the mass of the companion is 1000 bigger for star X than for 51 Peg. Star X is orbiting another star, not a planet.
2. Use the drawing on the front page, which shows the constellations of the Zodiac. A bright star in the constellation Leo is a "morning star." A morning star is a bright star that is seen close to the sun in the predawn sky.
a. ( 3 pts.) What is the approximate date? Explain.

I drew the horizon on $9 / 7$ just before dawn. (The date must be after $8 / 21$, the date on which the sun is in front of Leo. It must not be too much after $8 / 21$, since the sun and Leo will not be close.) The sun is below the horizon, and Leo is just above the horizon. I know this is near sunrise because a little later, earth spins a bit and the sun will be above the horizon. Both the sun and Leo are in nearly the same direction.

The most common difficulty is not being able to figure out the horizon on a time and date and not being able to getting information from the drawing.
b. ( 2 pts.) Is a morning star ever an evening star at some other time of year? An evening star is a bright star that is seen close to the sun in the evening twilight sky. Explain.

Yes. A month earlier, the star in Leo was an evening star. Draw the earth on $8 / 7$, me and the horizon just after sunset.
3. You are teaching triangles in high-school geometry. Your class learned that knowing the angles and the length $B$ of one side, one is able to determine the lengths of the other sides. You tell your class that Hipparchus used this knowledge about triangles to find the distance to the moon.
a. (2 pts.) Draw the triangle that Hipparchus used, and label the corners.
b. (2 pts.) He knew one angle because he knew the direction of the moon. How did he measure the other, small angle?


Hipparchus measured the small angle, called "ang" in the drawing, by comparing the solar eclipses at Alexandria and Hellespont. The eclipse was total at Hellespont, and in Alexandra, the sun shifted $1 / 4$ of the full moon, which is $1 / 2^{\circ}$. Therefore ang $=1 / 4$ $\times 1 / 2^{\circ}$.

Some assumed that one angle was $90^{\circ}$, which isn't necessarily so. Some found ang from the distance to the moon. That is backwards: he measured the angle to find the distance.
4. (3 pts.) Suppose a magician turned off the light of the sun. Would you see different stars at different times of the year? Explain.

You would not see different stars at different times of the year. For example, on 9/21, you would not normally see Virgo because of the light of the sun. With the magician turning off the sun, you could see Virgo, Except for the stars blocked by the sun itself.
5. In 2003, astronomers discovered Sedna, a new object in the solar system. Its semi major axis is 520 AU , and its distance from the sun at perihelion is 76 AU .
a. (2 pts.) Find its eccentricity.
(distance between a focus and the center)=eccentricity (semi major axis)
distance between a focus and the center= (semi major axis) - (perihelion distance)
Eccentricity $=(520-76) / 520=0.85$.
b. ( 3 pts .) On the day of discovery, its motion indicated its distance was 100 AU from the sun. Its actual semi major axis and eccentricity were unknown, because those required measurements over time. Pretend that you were a member of the discovery team on the day of discovery. CNN demands to know its period. Estimate the period of the object. What assumption did you have to make? Explain.
I assume that the orbit is a circle, because I have no other information. Then I use Kepler's $3^{\text {rd }}$ Law to find $P=a^{3 / 2}=100^{3 / 2}=1000$ year
c. (3 pts.) Astronomers say they were lucky to see it during the brief time when it is close to the sun (and also Earth). Explain why it is close to Earth for only a brief time.
Sedna was seen when its distance was 100AU, which is close to its perihelion distance of 76AU. Kepler's $2^{\text {nd }}$ Law says its speed is fast at perihelion. The length of its path far from the sun is greater. Therefore for these two reasons, Sedna spends most of its time much farther from the sun.

