

You may not use books or notes.

Do the easy questions first. Then go back to the harder ones.

If you are not sure about the wording of a question, ask.

If a part of a question depends on the numerical value of a previous part that you cannot do, assume a value and state that you have done so.

If a calculation is not possible on your calculator, assume close values for which you are able to calculate, and state that you have done so.

«Student Name»	
Signature	
1	/ 10
2	/ 5
3	/ 5
4	/ 5
5	/ 6
6	/ 3
Total	/ 34

Eccentricity (dist between foci)/(major axis)

Small-angle approx. Angle=baseline/distance  
 $a(\text{arcsec})=1\text{AU}/D(\text{parsec})$

Area of triangle  $A = \frac{1}{2}\text{base height}$

Degree-radian conversion  $180^\circ = \pi \text{ radian}$

Center of mass  $m_1 d_1 = m_2 d_2$

Sidereal day 24hr 4min

Kepler's 3<sup>rd</sup> law  $P^2 = a^3 / M$

$57^\circ = 1 \text{ radian}$

m are masses. d are distance to the center of mass

$P$  is the period in years,  $a$  is the semi major axis in AU, and  $M$  is the mass in terms of the mass of the sun.

Aphelion/perihelion distance  $R_a = a(1 + \epsilon)$   
 $R_p = a(1 - \epsilon)$

Kepler's 2<sup>nd</sup> law at perihelion or aphelion  $A = \frac{1}{2} v t D$

Force of gravity  $F = GMm/R^2$

$a$  is the semi major axis and  $\epsilon$  is the eccentricity

$v$  is speed,  $t$  is time, and  $D$  is distance

$M$  and  $m$  are the masses.  $R$  is distance.  $G$  is a constant.

Stefan-Boltzmann Law  $L = R^2 T^4$

Wien's Law  $\lambda_{\text{peak}} T = 2.9 \text{ mm K}$

Flux  $F = L/D^2$

$L$  is luminosity,  $D$  is distance

Flux and magnitude  $m_B - m_A = -2.5 \log_{10} \frac{f_B}{f_A}$

$\frac{f_B}{f_A} = 10^{-(m_B - m_A)/2.5}$

p-p chain  $2p \rightarrow {}^2\text{H} + e^+ + \text{neutrino}$   
 ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2p$

$p + {}^2\text{H} \rightarrow {}^3\text{He} + \text{photon}$

Gravity balances normal gas pressure  $M/R = T$

$M$  is mass.  $R$  is radius.  $T$  is temperature.

Energy-mass  $E = mc^2$

Kinetic energy  $KE = \frac{1}{2}mv^2$

## 1. The life of the sun

- a. (1 pt.) The sun and solar system are 4.5 Byr old. Approximately how much longer will the sun be small enough so as not to engulf Earth?

The sun will be a main-sequence star for 10 Byr, and then it becomes a giant, which engulfs Earth. Therefore the sun will stay small for another 5 Byr.

- b. (2 pts.) Why does the helium in the core of the sun not fuse at the present time? (1 pt.) When that helium does fuse eventually, what will the helium become?

The core of the sun is too cool to fuse helium. When it does fuse helium, the result will be carbon.

- c. (3 pts.) When the solar system was forming, before the sun became a star, it radiated energy into space. At that time what was the energy source? Explain why the present energy source did not apply.

Before the sun became a star, it was contracting. This converts gravitational potential energy into thermal energy. Recall  $T=M/R$ . Eventually, the center of the sun became hot enough for hydrogen to fuse.

- d. (3 pts.) The sun eventually becomes a white dwarf. Explain why it will be so much smaller than it is now.

In a white dwarf, the pressure is not caused by the motion of particles due to their energy. Instead, the particles move because of a quantum mechanical principle, which does not need energy. Therefore it is stable without an energy source.

## 2. Adams showed that Sirius B is a new type of star, which we call a white dwarf.

- a. (2 pts.) In terms of the hot-plate model, what is a white dwarf?

In the hot-plate model, the two physical parameters describing a star are its radius and temperature. A white dwarf is a star with a very small radius.

- b. (3 pts.) What hypothesis of Sirius B did Adams disprove? What was Adams' evidence?

There were two hypothesis, size and temperature, to explain the faintness of Sirius B. Adams showed that the temperatures of Sirius A and B were about the same. Therefore he disproved the hypothesis that Sirius B was faint because it was cool.

## 3. Imagine that you are reading a NASA proposal to put a telescope on Mars to measure parallax.

- a. (3 pts.) The proposal claims that the angles of parallactic shifts are larger on Mars. Explain why this assertion is true.

The angle of the parallactic shift is  $a = B/D$ , where B is the baseline and D is the distance to the star. Since Mars' orbit is bigger, the angle is greater.

- b. (2 pts.) The proposal claims that because the parallactic shifts are larger on Mars, it is worthwhile to spend a large amount of money to fund this proposal. (A telescope on Mars would cost 10,000 times as much as a telescope on Earth.) Do you agree with this assertion? Explain your reasoning.

Mars' orbit is indeed larger than Earth's, but it is not larger by a great factor. The angles are somewhat larger, but not so large that astronomers can improve the

measurements of distances by a large factor. It is not worth spending 10,000 as much money.

4. The sun will become a giant. Compared with its present values, the temperature of the surface will be cooler by a factor of 1.2, and the radius of the surface will be larger by a factor of 10.

- a. (3 pts.) By what factor will the luminosity change?

Use the hot-plate model or the Stefan-Boltzmann Law, which are the same.

Compared with the present sun, the luminosity is brighter by a factor  $L = R^2 T^4 = 10^2 \left(\frac{1}{1.2}\right)^4 = 48$ .

- b. (2 pts.) On a Hertzsprung-Russell diagram where the vertical axis is the absolute magnitude, describe how the locations differ in the vertical direction, and give a numerical value.

Since this Hertzsprung-Russell diagram uses magnitude on the vertical axis, I have to calculate the magnitude of the sun as a giant.  $m = -2.5 \log 48 = 4.2$ . The future sun is shifted 4.2 magnitudes up in the H-R diagram.

5. The moon Io orbits the planet Jupiter in 1day, 18hr, 29min, which is 0.0048year. The semi-major axis of its orbit is 422Mm, and its radius is 1.81Mm. The radius of Jupiter is 71.3Mm. A megameter (Mm) is  $10^6$  meters, and an astronomical unit is 150,000 Mm.

- a. (3 pts.) Find the mass of Jupiter in terms of the mass of the sun. Assume the orbit is circular.

I use Kepler's 3<sup>rd</sup> Law as revised by Newton. In its simplest form, I need to use AU and years. The semi-major axis is  $422\text{Mm}/(150,000\text{Mm}/\text{AU})=0.0028\text{AU}$ .  $M = \frac{a^3}{P^2} = 0.00095$  times the mass of the sun.

- b. (3 pts.) Now suppose the eccentricity is 0.5. Find the mass of Jupiter.

Since Kepler's 3<sup>rd</sup> Law is independent of eccentricity, the mass does not depend on the eccentricity.

6. Astronomers have been measuring the speed of star X. They find that its speed changes periodically just as that of 51 Peg does. The period is 4day 5hr, which is the same as that of 51 Peg. The maximum speed is 60km/s, whereas the maximum for 51 Peg is 60m/s. In both cases, the visible star and something much fainter orbit each other. Assume the orbits are circles.

- a. (3 pts.) Devise a model for star X that explains its faster speed. Explain your reasoning.

Star X is on a bigger orbit. The radius of the orbit is the distance  $d_s$  from the star to the center of mass of the star-companion system. Since the speed of star X is 1000 greater and the period is the same,  $d_s$  for star X and the mass of its companion is 1000 bigger than for 51 Peg. Star X is orbiting another star, not a planet.