NOMENCLATURE

Sorry… this nomenclature is not entirely consistent with Carrol & Ostlie. Among other things, it is in cgs units rather than SI as used by CO. The inside of the back cover of CO gives a table of SI to cgs conversions. Of particular interest in these notes is the unit of energy: 1 erg = 10^{-7} joule; 1 erg $s^{-1} = 10^{-7}$ watts.

Temperature:

T_e = electron temperature (K).

Densities:

N_e = electron density (electrons cm^{-3}).

$N_{H+} = H^+$ density (ionized H atoms cm^{-3}).

…etc.

Light emitted by the star:

J_υ = *mean intensity* of radiation field from star. $4\pi J_\upsilon$ = total energy in erg cm^{-2} s^{-1} Hz^{-1} of light with frequency υ flowing into a volume element (of the gas cloud) from all directions. Note that the quantity *flux* is the net flow, subtracting photons moving in one direction from those moving in the opposite direction.

L_υ = luminosity from star at frequency υ (erg s^{-1} Hz^{-1}).

Light emitted by the gas:

j_υ, $j_{recombo}$, etc = luminosity due to the various processes described in the notes, emitted per unit volume of gas. Units are erg s^{-1} cm^{-3} s^{-1} Hz^{-1} or erg s^{-1} cm^{-3} s^{-1}, depending on the context.

L_{coll}, $L_{recombo}$, etc = luminosity due to various processes, integrated over the whole ionized gas cloud. Units ergs s^{-1}.

S = surface brightness = erg cm^{-2} s^{-1} $arcsec^{-2}$ or some similar units, where the cm^{-2} part refers to the area of the receiving telescope, *not* the surface area on the nebula. The surface area element on the nebula is the $arcsec^{-2}$ part.

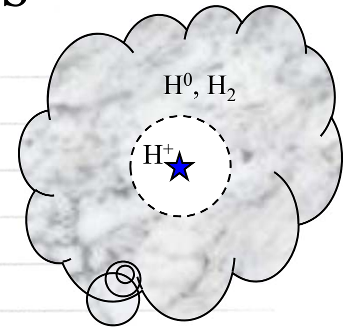
Einstein A values:

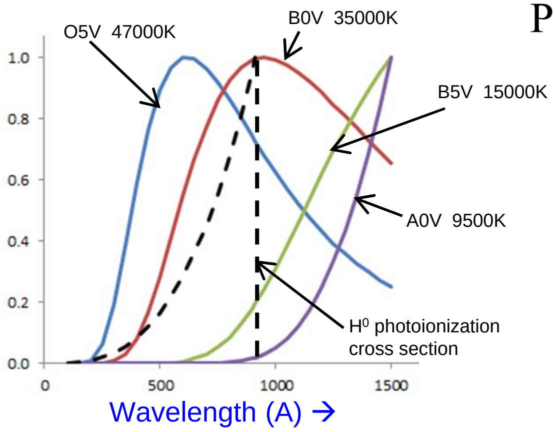
A_{ij} = transition probability of an electron jumping from level i to level j. Units s^{-1}. The quantity $1/A_{ij}$ is the characteristic lifetime of an electron in the upper level.

Cross sections:

Many of the other symbols (α, a, q) are cross sections or rates for different processes, sometimes in cm^2, sometimes convolved together with other quantities so as to be in other odd sets of units. These symbols are only intended to indicate to you that some sort of cross section or coefficient must be used and therefore must be known (either experimentally or through a calculation).

Strömgren Spheres





IONIZATION EQUILIBRUM

$$N_{H^0}\int_{\nu_0}^{\infty}\frac{4\pi J_\nu}{h\nu}\,a_\nu(H)\,d\nu = N_e N_{H^+}\,\alpha(H^0, T_e)$$

IONIZATION PARAMETER

$$U=\frac{N_{H^+}}{N_{H^0}} \propto \frac{(\text{IONIZING FLUX})}{N_e} \sim 10^3\text{-}10^4 \text{ inside a typical HII region}$$

SIZE OF IONIZED ZONE:

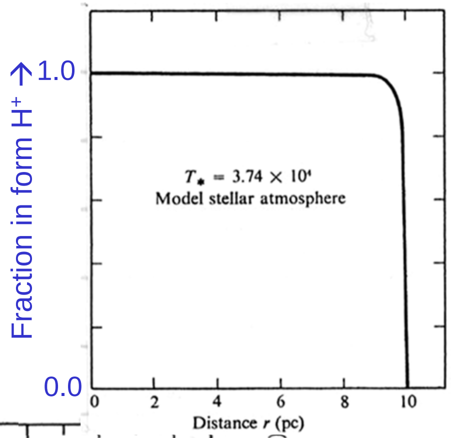
over whole HII region

$$\text{ionizations/sec} = \int\frac{L_\nu\,d\nu}{h\nu} \sim 5\times10^{48}\,s^{-1} \text{ for O6 star.}$$

$$\text{recombos/sec} = \frac{4}{3}\pi R^3\cdot N_e\cdot N_H\,\alpha$$

for $N_H = 10\,cm^{-3}$

$$R \sim \left[\frac{5\times10^{48}}{\frac{4\pi}{3}10^2\,4\times10^{-13}}\right]^{\frac{1}{3}} = 3\times10^{19}\,cm \doteq 10\,pc$$



SIZE OF TRANSITION ZONE

$$\Delta R \sim \frac{1}{N_H a_\nu} \sim \frac{1}{10\times\underbrace{6\times10^{-18}}_{cm^2}} = 2\times10^{16}\,cm$$

$$= .01\,pc$$

$$\frac{\Delta R}{R} \propto N_H^{-1/3} \sim 10^{-3}$$

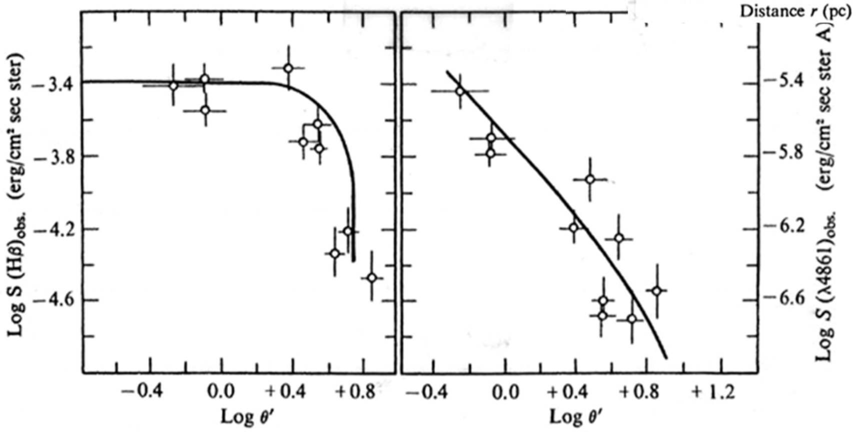
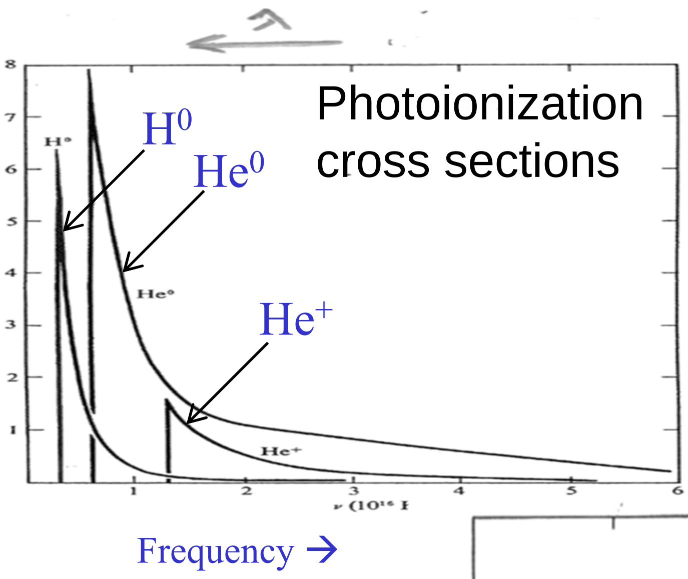
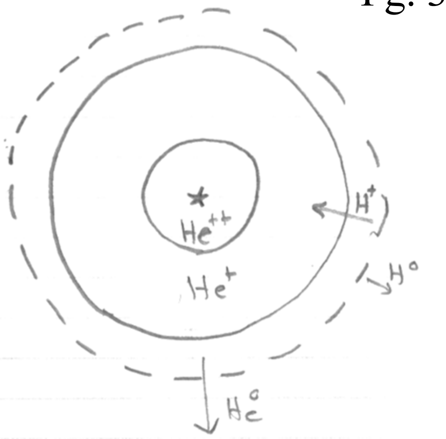
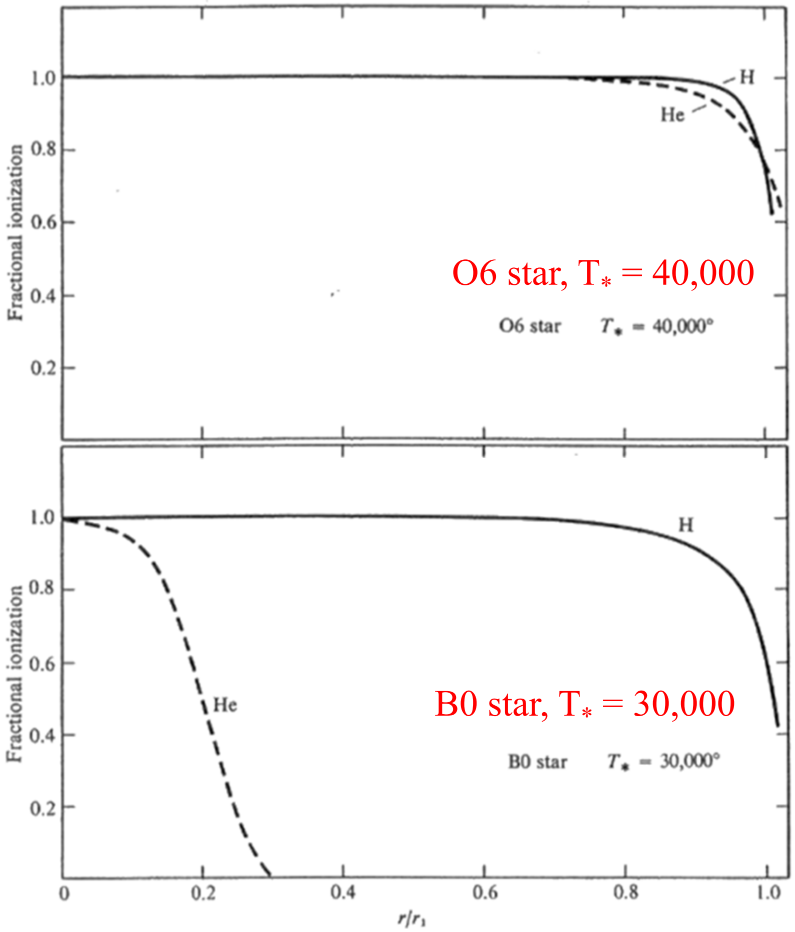


FIGURE 7.6
Diagram on left shows Hβ surface brightness as a function of angular distance from the central star in H II region NGC 6514. Diagram on right shows continuum surface brightness near Hβ (corrected for atomic continuum) as a function of angular distance from the same star.

Ionization Zones for Heavier Elements

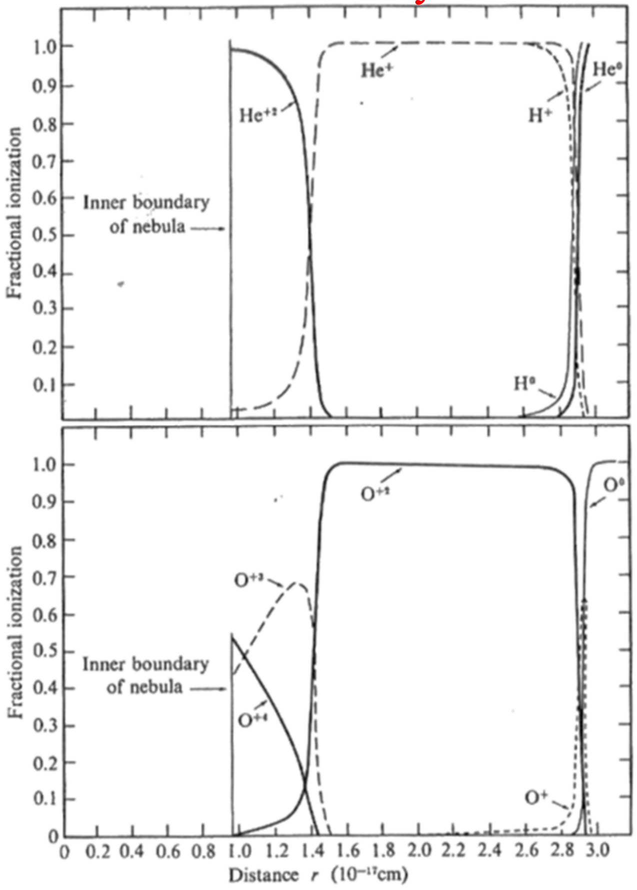

Photoionization cross sections




O6 star, T$_*$ = 40,000

B0 star, T$_*$ = 30,000

Planetary Nebula



Heating and cooling

- Heating
  - Photoionization followed by recombination ➔ temperature input

$$G(H) = N_{H^0} \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} h(\nu - \nu_0)\, a_\nu(H^0)\, d\nu = N_e N_{H^+} \alpha(H^0, T_e) \frac{3}{2} k T_{\text{initial}}$$

$$j_{\text{RECOMB}}(H) = N_e N_{H^+} k T_e \beta_A(H_0, T_e)$$

$\beta \propto v^{-2} \Rightarrow$ slower electrons preferentially captured

$\Rightarrow$ net energy gain

  - ➔ Recombination lines of H, He
- Cooling
  - Free-free emission

$$j_{FF} = 1.42 \times 10^{-27} \underset{\substack{\uparrow \\ \text{charge}}}{Z^2} T_e^{1/2} N_e N_+ \text{ erg cm}^{-3}\text{s}^{-1}$$

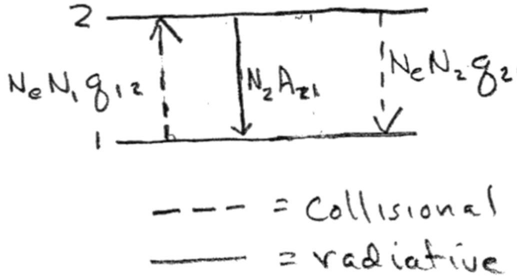
where $N_+ = N_{H^+} + N_{He^+}$

  - Collisional excitation of low-lying levels
    - Permitted lines are important coolants in UV, IR.
    - Strongest optical lines are *forbidden*
      - [OIII], [OII], [OI], [NeIII]

$$j_c = N_2 A_{21} h\nu_{21}$$

$$= N_e N_1 q_{12} h\nu_{21}$$

(simplest case:
1 level, only
rad. de-excitation)



If collisional de-excitation:

$$\frac{N_2}{N_1} = \frac{N_e q_{12}}{A_{21}} \left[\frac{1}{1 + \frac{N_e q_{12}}{A_{21}}} \right]$$

Plus must allow for multiple cascade paths.

Temperature Balance

Heating(T_e) = Cooling(T_e)

- Sets $T_e \sim 10^4$K in most cases.

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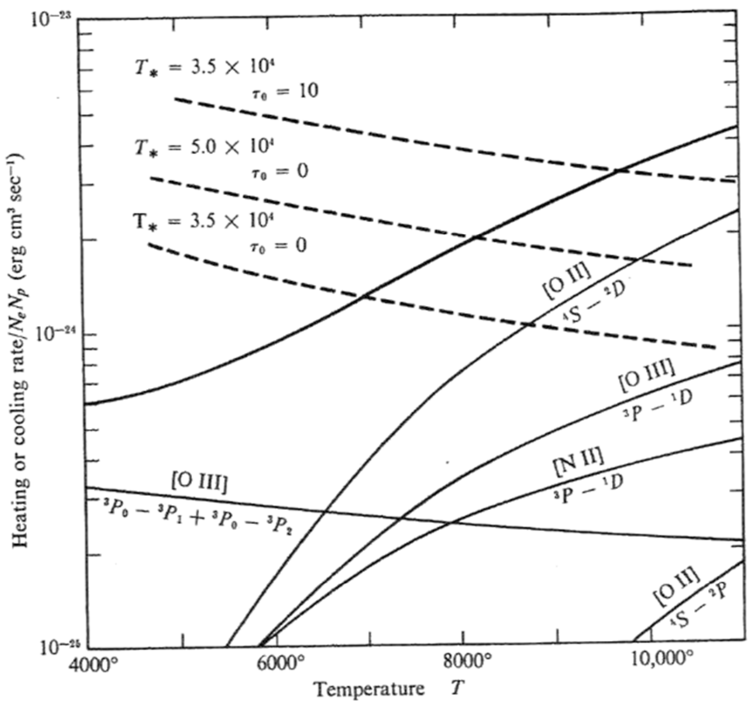

FIGURE 3.2
Net effective heating rates ($G - L_R$) for various stellar input spectra, shown as dashed curves. Total radiative cooling rate ($L_{FF} + L_C$) for the simple approximation to the H II region described in the text is shown as heavy solid black curve, and the most important individual contributors to radiative cooling are shown by lighter solid curves. The equilibrium temperature is given by the intersection of a dashed curve and the heavy solid curve. Note how the increased optical depth τ_0 or increased stellar temperature T_* increases T by increasing G.

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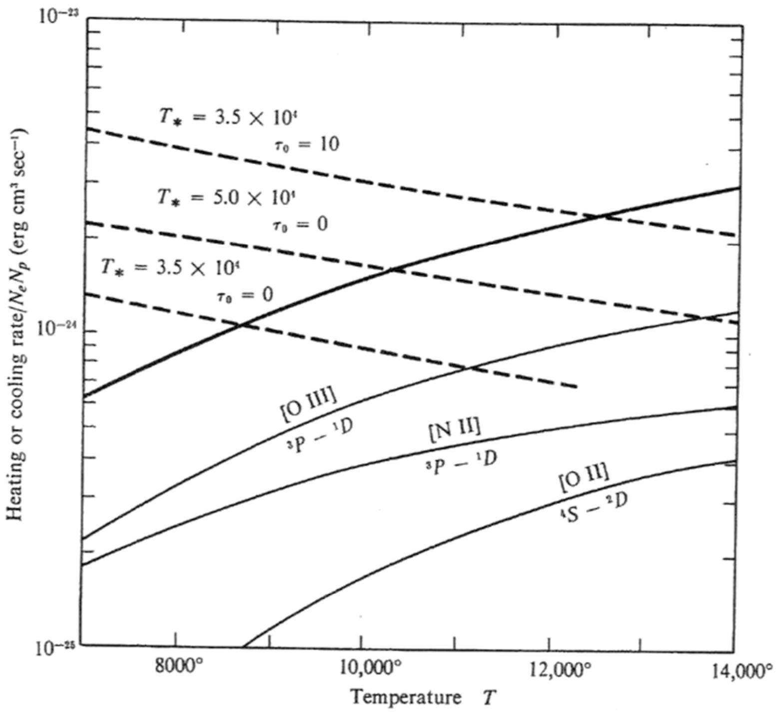


FIGURE 3.3
Same description as for Figure 3.2, except that collisional de-excitation at $N_e = 10^4$ cm^{-3} has been approximately taken into account in the radiative cooling rates.

Including collision de-excitation (denser gas)

MEASURING DENSITY

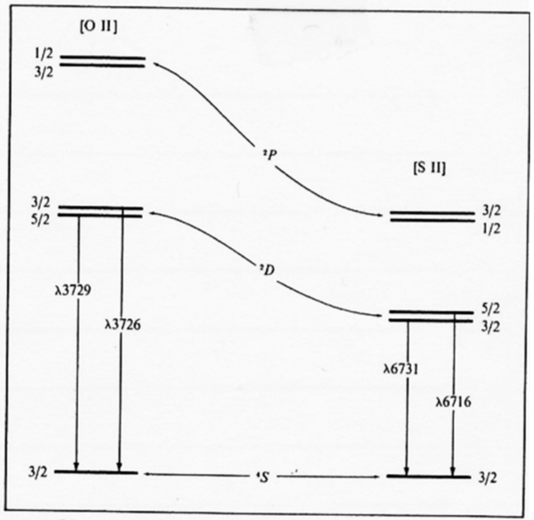


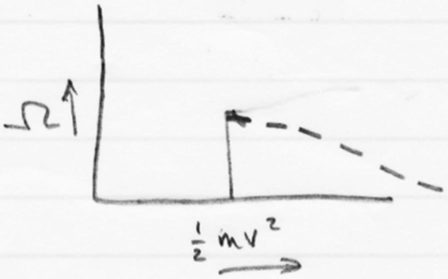
FIGURE 5.2
Energy-level diagrams of the $2p^3$ ground configuration of [O II] and $3p^3$ ground configuration of [S II].

Use competition between radiative & collisional de-excitation of metastable levels:

Collisional excitation rate

$= N_e N_1 q_{12}$

$$= N_e N_1 \int_0^\infty \frac{\pi \hbar^2}{m^2 \omega_1} \frac{\Omega(1,2)}{v^2} dv$$



For ${}^2D_{5/2}$, ${}^2D_{3/2}$

ratio of $\Omega \propto$ ratio of statistical weights $= \frac{6}{4}$

For pure radiative de-excitation:

$j(3729)/j(3726) = \frac{6}{4} = 1.5$

But if collisional mixing constantly redistributes level population

$$\frac{j(3729)}{j(3726)} \propto \frac{\omega_{5/2}}{\omega_{3/2}} \frac{A_{\lambda 3729}}{A_{\lambda 3726}}$$

$$= \frac{6}{4} \cdot \frac{4.2 \times 10^{-5}}{1.8 \times 10^{-4}} = 0.35$$

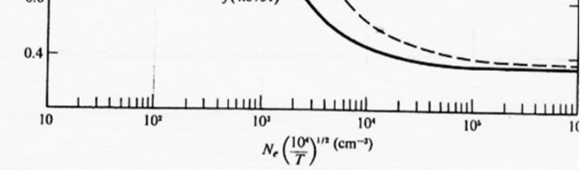
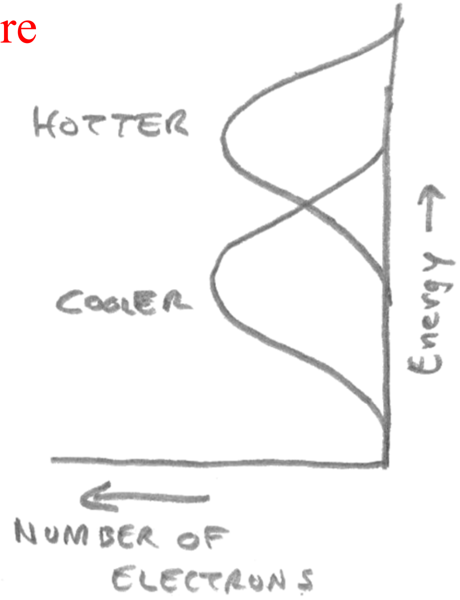


FIGURE 5.3
Calculated variation of [O II] (*solid line*) and [S II] (*dashed line*) intensity ratios as function of $N_e(10^4/T)^{1/2}$. Note that the electron density can be read directly from the horizontal scale if $T = 10{,}000$°K.

Measuring Temperature



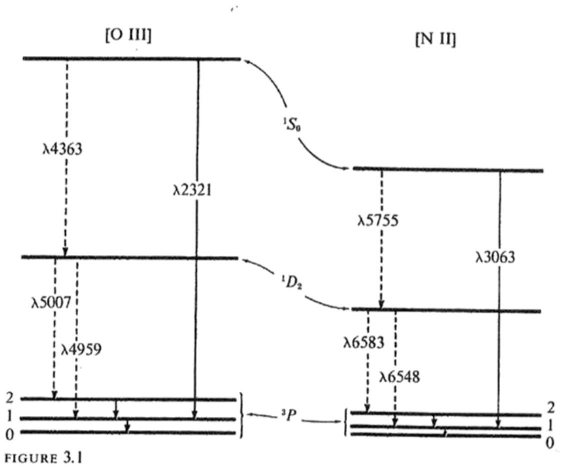


FIGURE 3.1
Energy-level diagram for lowest terms of [O III], all from ground $2p^2$ configuration, and for [N II], of the same isoelectronic sequence. Splitting of the ground ^3P term has been exaggerated for clarity. Emission lines in the optical region are indicated by dashed lines and by solid lines in the infrared and ultraviolet. Only the strongest transitions are indicated.

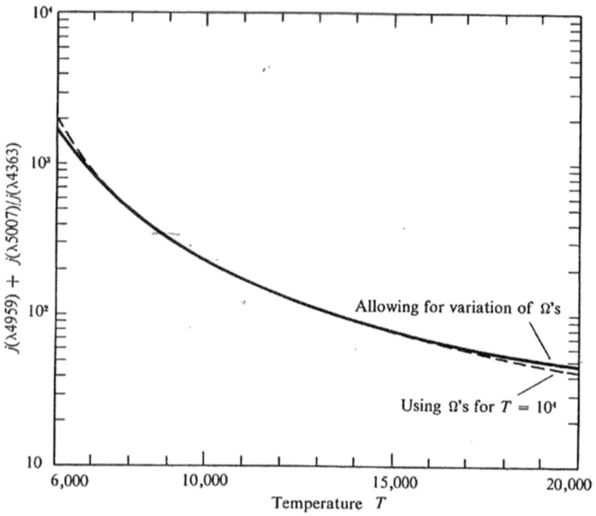


FIGURE 5.1
[O III] (λ4959 + λ5007)/λ4363 intensity ratio (in low-density limit $N_e \to 0$) as a function of temperature. Solid line shows accurately calculated value; dashed line shows approximation of equation (5.4) using mean values of Ω.

Determine N_e, T_e simultaneously

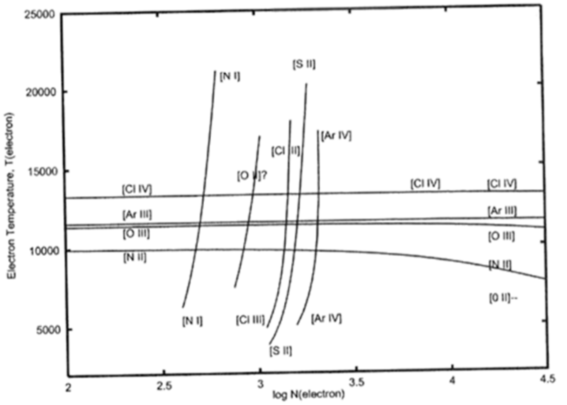


FIG. 2.—Diagnostic diagram for NGC 6818: T_e vs. log N_e. All values refer to the data taken at the 4″–5″ east position of the central star. [O II]? diagnostic is from the redshifted line profile component, λ3729/λ3726 (see Fig. 5). The [N I] clearly originates in a partially ionized zone in a blob. Note the [Cl IV] result is uncertain because diagnostic lines are weak. Compare with Figs. 4 and 5.

Finding abundances

- We measure total flux in emission lines from different ions.
  - Recombination lines (H, He)

N_x = number density of ionization state x.

$$j_{\mathrm{RECOMBO}} = N_e N_{x+1} \, k T_e \, \beta_A(H_0, T_e)$$

  - Collisionally excited lines (strong lines of heavy elements)

$$j_c = N_2 A_{21} h\nu_{21} = N_e \underset{\uparrow}{N_{1,x}} f(T_e, N_e)$$

↳ ground state pop. of particular ion

- Use knowledge of T_e, N_e to calculate coefficients, etc
- Solve for ratios of ion abundances
  - O^{++}/H^{+} , O^{+}/H^{+} , O^{0}/H^{+} , Ne^{++}/H^{+} , N^{+}/H^{+} , N^{+4}/H^{+} , S^{+}/H^{+} etc.
  - He^{++}/H^{+} , He^{+}/H^{+}
- Use models or empirical interpolations to correct for un-observed ions of element of interest.
- Get total element abundance ratios.
  - $N(H) = N(H^{+})$
  - $N(O)/N(H) = N(O^{++})/N(H^{+}) + N(O^{+})/N(H^{+}) + N(O^{0})/N(H^{+})$ + higher ionization states.

CONTINUUM EMISSION

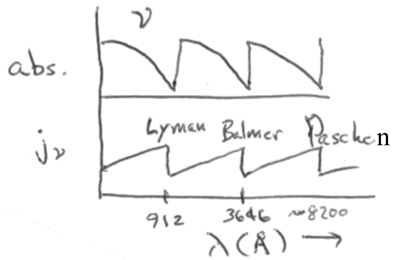
FREE-FREE (BREMSSTRAHLUNG):

$$j_\nu \propto N_e^2 Z^2 T_e^{-1/2} e^{-h\nu/kT_e}$$

[see CO eqn 25.16]

FREE - BOUND

$$j_\nu \propto \frac{N_e^2 Z^4 e^{-k(\nu-\nu_n)/kT_e}}{T_e^{3/2} n^3}$$



TOTAL MASS

Mass = N_H × Volume.

$N_H \sim N_{H^+} \sim N_e$

Integrated over nebula:

$$L(H\beta) = N_e N_{H^+} \alpha(H^0, T_e) \times \text{Volume}$$

$$= N_e \alpha \cdot \text{mass}$$

Emission Measure

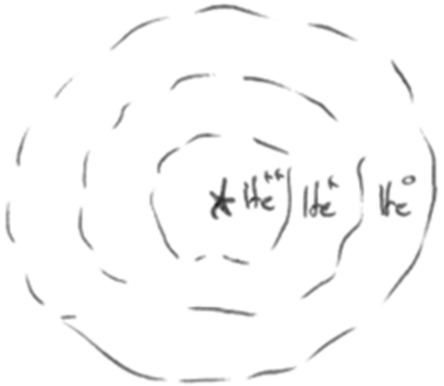
Surface brightness $S = \int_0^d N_e^2 \alpha \, dr = (N_e^2 d) \cdot \alpha$

But denser clumps radiate more per unit mass

$$S = \int_0^d N_e \alpha \left(\frac{dM}{dr}\right) dr$$

Absorption within nebula also matters.

MODEL NEBULA



Calculate ionization structure.

- ionizing spectrum
- gas density distribution
- gas abundances

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Homework Assignment 2 – Due Tuesday Sept 20
An Atypical HII Region

The Textbook Nebula is a perfectly spherically symmetric ball of gas of absolutely uniform density which is photoionized by a hot star located precisely at its center. There is too much gas for the star to completely ionize, so a sharply-bounded Stromgren sphere is formed with highly ionized gas in the center and neutral gas surrounding it on all sides.

Surprisingly (?), there is no dust in this nebula or along the line of sight to it. We even know the distance to this nebula: 437 pc.

From the following observations, find the mass of ionized hydrogen. Express this in the conventional units of solar masses. Show the values you used for each term in the eqn..

Warning… Pay close attention to units. Cgs units are used here.

Find T_e from [F(4959)+F(5007)]/(F4363), then interpolate to get α.

T_e (K)	5,000	10000	20,000
$\alpha_{H\beta}$ (cm^{3}s^{-1})	5.37 E-14	3.02E-14	1.61E-14

$\alpha_{H\beta}$ is the hydrogen recombination coefficient taking into account the fraction of recombinations that lead to the production of an Hβ photon (the n=4 to n=2 transition).

$$L(H\beta) = h\nu_{H\beta}\int_{vol} N_e N_{H+}\alpha_{H\beta}\, dvol = h\nu_{H\beta} N_e \alpha_{H\beta} \bullet \mathrm{mass}/m_H$$

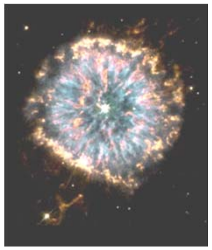
In cgs units: h = 6.63E-27 erg s
1 pc = 3.09E18 cm; 1 Å = 1.0E-8 cm

Measured flux received at Earth from whole visible surface of nebula. (Flux ∝ j)

Emission Line (λ in Å)	Flux (erg cm^{-2} s^{-1})
[OIII] λ4363	1.07E-10
Hβ λ4861	1.00E-8
[OIII] λ4959	1.06E-8
[OIII] λ5007	3.42E-8
[SII] λ6716	3.74E-9
[SII] λ6731	3.56E-9

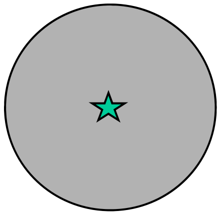

Not the Textbook Nebula

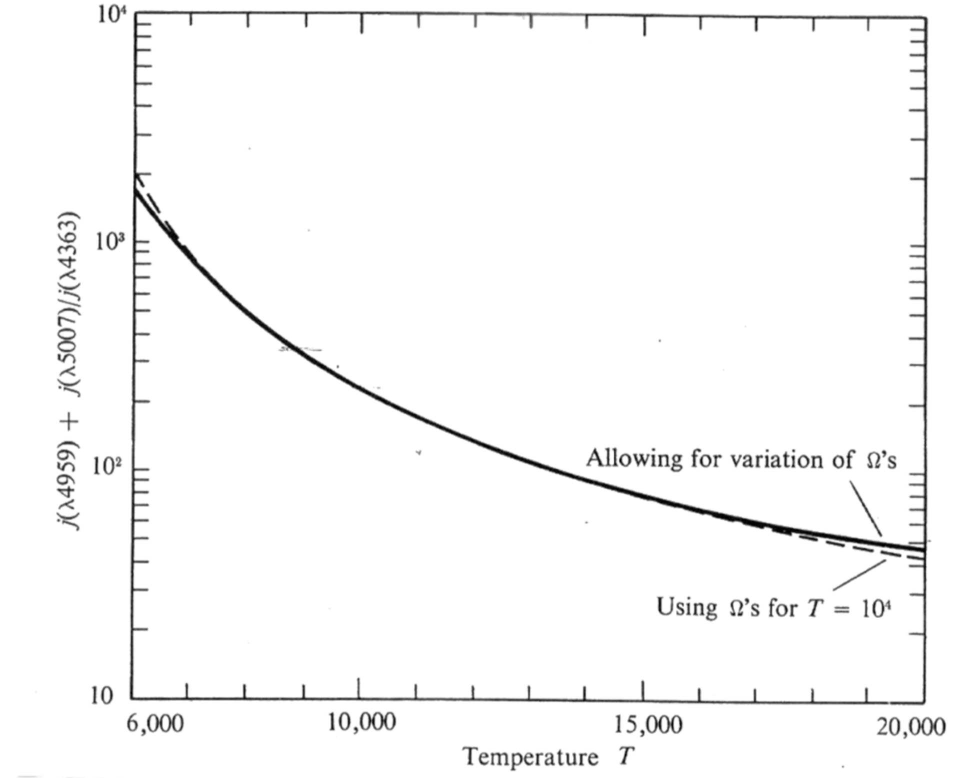

Not the Textbook Nebula

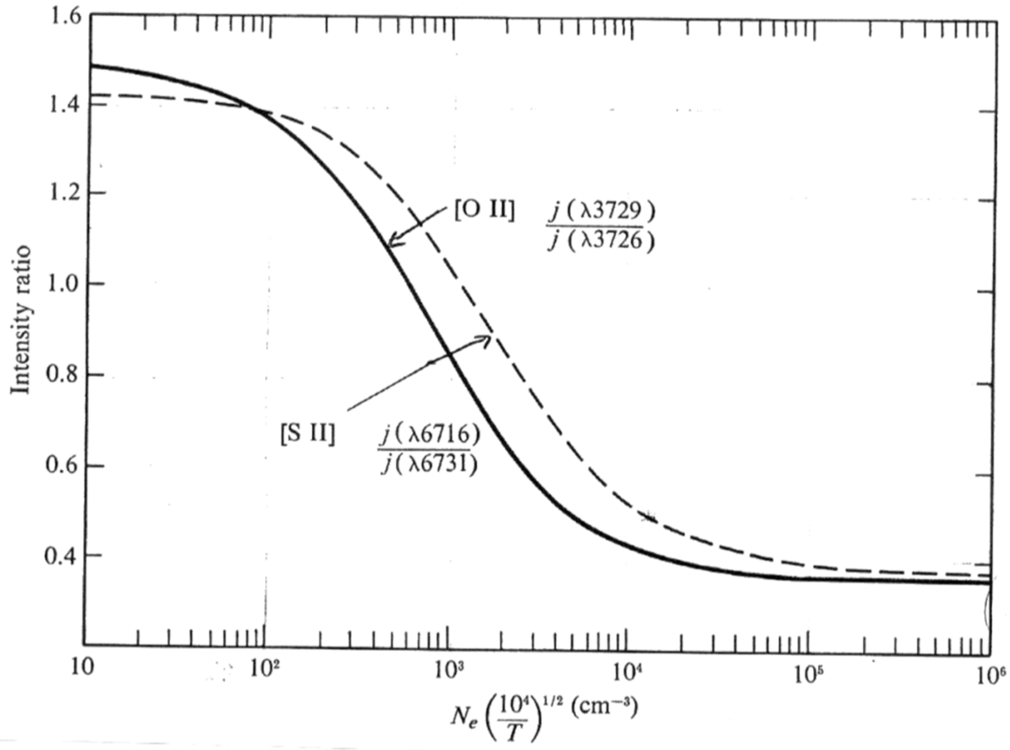

Not the Textbook Nebula


Not the Textbook Nebula


Not the Textbook Nebula


The Textbook Nebula


10^4
10^3
10^2
10
$j(\lambda4959) + j(\lambda5007)/j(\lambda4363)$
6,000
10,000
15,000
20,000
Temperature T
Allowing for variation of Ω's
Using Ω's for $T = 10^4$


1.6
1.4
1.2
1.0
0.8
0.6
0.4
Intensity ratio
[O II] $\frac{j(\lambda3729)}{j(\lambda3726)}$
[S II] $\frac{j(\lambda6716)}{j(\lambda6731)}$
10
10^2
10^3
10^4
10^5
10^6
$N_e\left(\frac{10^4}{T}\right)^{1/2}$ (cm^{-3})