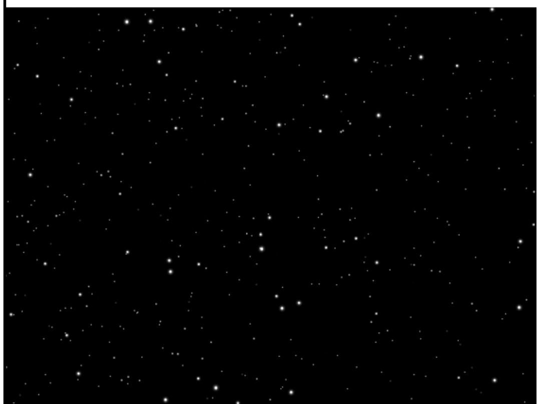
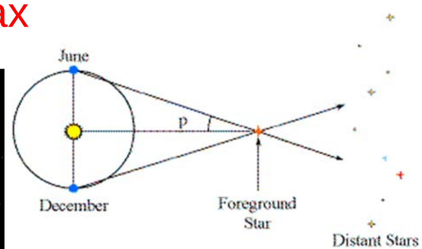
Measuring the Distances to the Stars: Parallax





Parsec (pc)

= distance to star for which angle p = 1 arcsec

= 3.26 LY

Hwk 2 due Sept 23

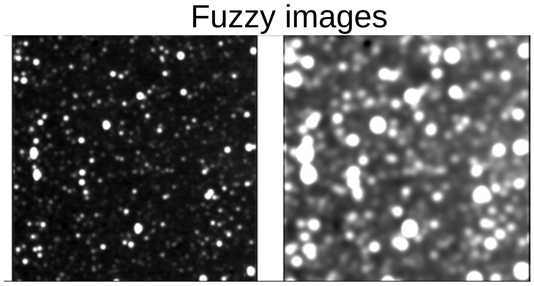
Hwk 3 not yet assigned, but due Sept 30

Midterm 1
Wed, Oct 2

http://sci2.esa.int/interactive/media/html/sec23p1.htm

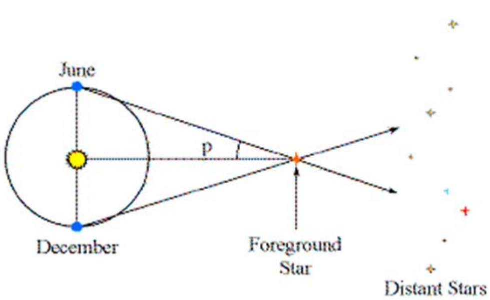
What sets the parallax limit?

Fuzzy images



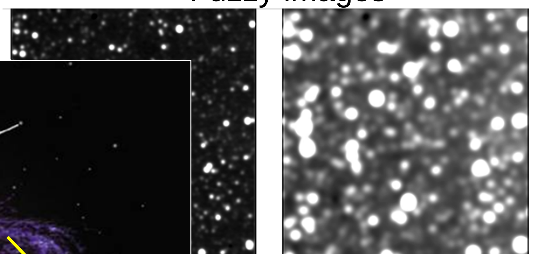
A field of stars …blurred by Earth's atmosphere.

Old limit for parallax distances:
20-50 parsecs

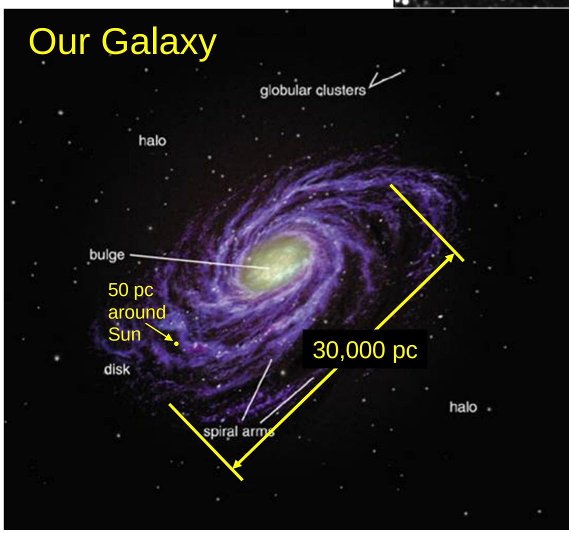


What sets the parallax limit?

Fuzzy images

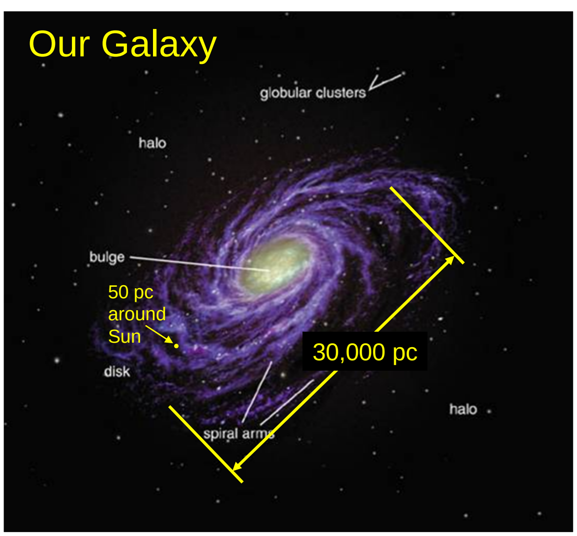


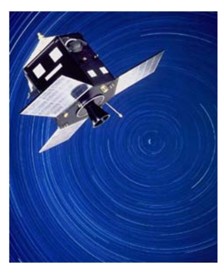
...blurred by Earth's atmosphere.



Old limit for parallax distances:
20-50 parsecs

Sharp Images from Space



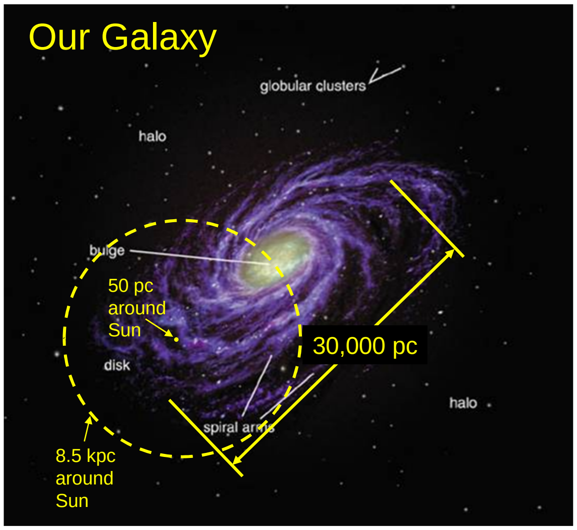


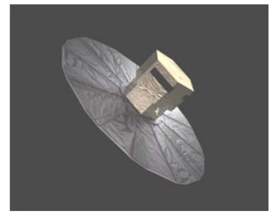
Hipparcos

Old limit for parallax distances:
20-50 parsecs

Hipparcos (1989-1993):
100-200 parsecs
(1σ =1 milliarcsec = 1kpc)

Coming Soon





GAIA spacecraft: Dec 2011 launch

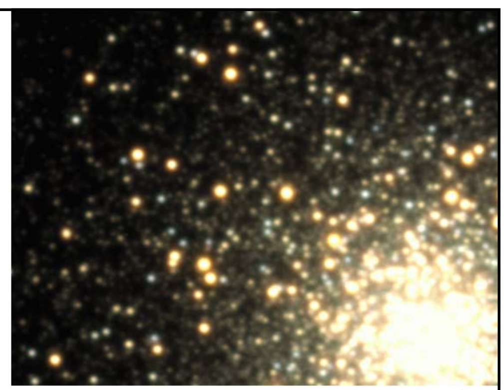
Old limit for parallax distances:
20-50 parsecs

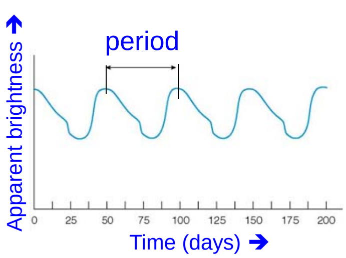
Hipparcos (1989-1993):
100-200 parsecs
(1σ =1 milliarcsec = 1kpc)

GAIA: 8.5 kpc

Pulsating Variable Stars

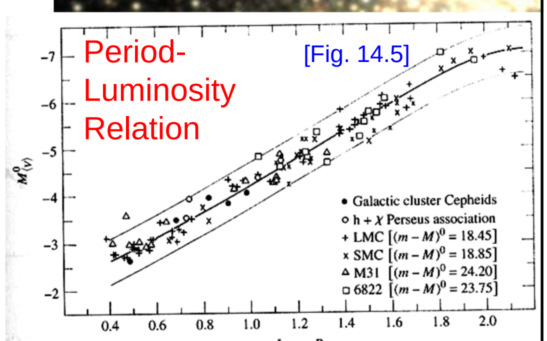
- These stars regularly expand & contract.
- Like a big spring.
- Change in size ➔
  - change in temperature
  - change in luminosity





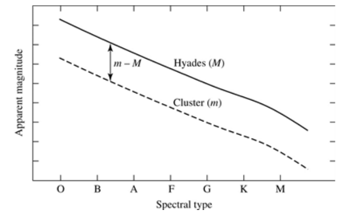
P-L relation

- discovered in Magellanic Clouds
- calibrated locally, using (statistical) parallaxes



Measuring Distances inside the MW

- Parallax
- Pulsating variables
- Main sequence fitting for clusters

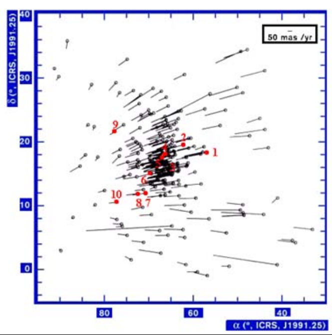


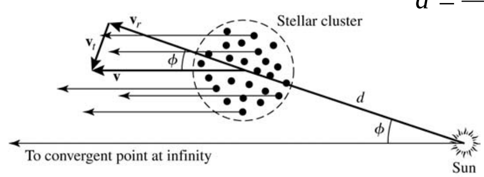
- Calibrate with Hyades (moving cluster method = pp. 919-922)

$$v_t = v_r \tan\phi$$

$$\mu = \frac{v_t}{d}$$

$$d = \frac{\langle v_r \rangle \tan\phi}{\langle \mu \rangle}$$

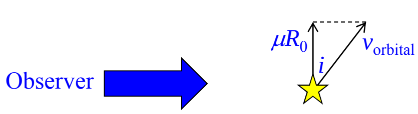


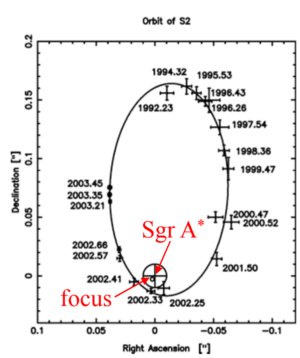


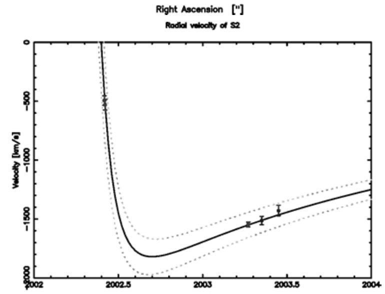
Distance R_0 to the Galactic Center

- Massive (> 10^6 M_{sun}) black hole at G.C.
- Can follow orbits of stars around it.
- Measured radial velocity (v_r) of one star at several points in its orbit, and proper motion μ (arcsec yr^{-1}).
- Orbit fit gives i (inclination), and predicts orbital velocity at each point.

- In concept, solve for R_0 = 7.94 ± 0.38 kpc
  - (actually, it comes out of orbit fit)



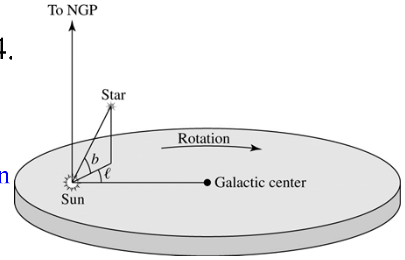




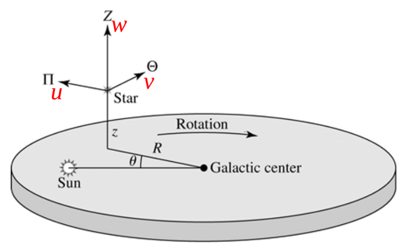
Eisenhauer et al. 2003, ApJ, 597, L121

Kinematics of the Milky Way

- From [CO] 24.3, especially pp 901-14.
- Coordinate systems
  - Galactic latitude (b_{II}) , longitude (l_{II})
    - Spherical coordinates centered on Sun



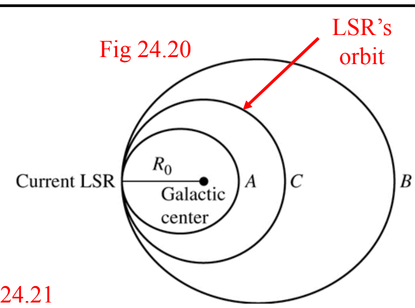
  - R, θ, z
    - Cylindrical coordinate system centered on Galactic Center
  - Π, Θ, Z
    - Velocity components in R, θ, z system.



  - Peculiar velocities u, v, w
    - Π, Θ, Z velocities but relative to *Local Standard of Rest*
    - LSR is point instantaneously centered on Sun, but moving in a perfectly circular orbit.
    - *Solar motion* = motion of sun relative to LSR

- Star density is higher towards GC.
  - Those stars are on orbits that Sun overtakes.

Fig 24.20

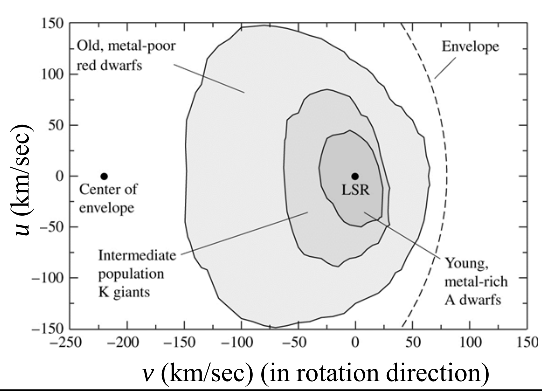


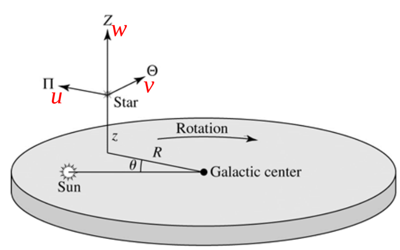
- Velocity ellipsoids and asymmetrical drift.

➔ LSR's orbital velocity

$\Theta_0(R_0) = 220$ km s^{-1}

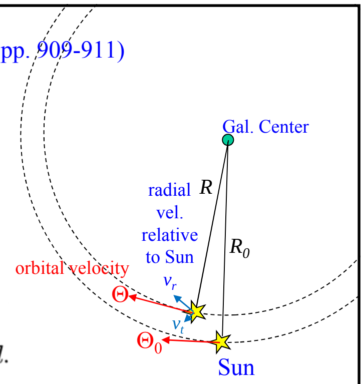
Fig 24.21





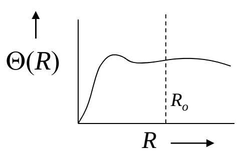
"Envelope" = halo stars
- Net rotation = 0

Differential Rotation (see [CO pp. 909-911)



- From the figure: $v_r = \Theta\cos\alpha - \Theta_0\sin\ell,$ $v_t = \Theta\sin\alpha - \Theta_0\cos\ell,$
- Angular rotation velocity: $\Omega(R) \equiv \dfrac{\Theta(R)}{R}$
- + some geometry ➔ $v_r = (\Omega - \Omega_0)\,R_0\sin\ell,$ $v_t = (\Omega - \Omega_0)\,R_0\cos\ell - \Omega d.$
- Taylor expansion:

$$\Omega(R) = \Omega_0\,(R_0) + \left.\frac{d\Omega}{dR}\right|_{R_0}(R - R_0) + \cdots$$

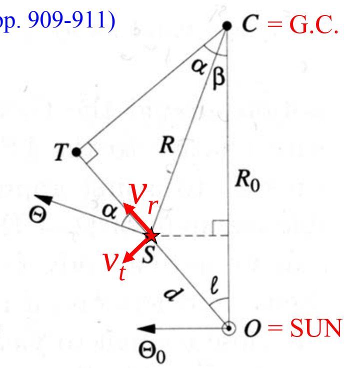


$$v_r \simeq Ad\sin 2\ell,$$
$$v_t \simeq Ad\cos 2\ell + Bd$$

Oort's Constants:

$$A \equiv -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_{R_0} - \frac{\Theta_0}{R_0}\right]$$
$$B \equiv -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_{R_0} + \frac{\Theta_0}{R_0}\right]$$

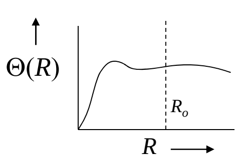
Differential Rotation (see [CO pp. 909-911)



[Fig 24.22]

- From the figure: $v_r = \Theta\cos\alpha - \Theta_0\sin\ell,$ $v_t = \Theta\sin\alpha - \Theta_0\cos\ell,$
- Angular rotation velocity: $\Omega(R) \equiv \dfrac{\Theta(R)}{R}$
- + some geometry ➔ $v_r = (\Omega - \Omega_0)\,R_0\sin\ell,$ $v_t = (\Omega - \Omega_0)\,R_0\cos\ell - \Omega d.$
- Taylor expansion:

$$\Omega(R) = \Omega_0\,(R_0) + \left.\frac{d\Omega}{dR}\right|_{R_0}(R - R_0) + \cdots$$



$$v_r \simeq Ad\sin 2\ell,$$
$$v_t \simeq Ad\cos 2\ell + Bd$$

Oort's Constants:

$$A \equiv -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_{R_0} - \frac{\Theta_0}{R_0}\right]$$
$$B \equiv -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_{R_0} + \frac{\Theta_0}{R_0}\right]$$

Oort's Constants

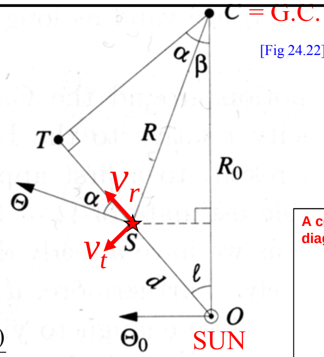
$$A \equiv -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_{R_0} - \frac{\Theta_0}{R_0}\right]$$

$$B \equiv -\frac{1}{2}\left[\left.\frac{d\Theta}{dR}\right|_{R_0} + \frac{\Theta_0}{R_0}\right]$$

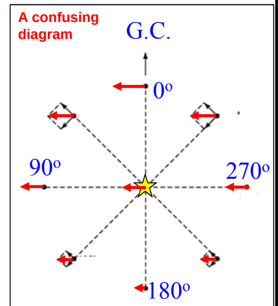
$v_r \simeq Ad\sin 2\ell,$ ← galactic longitude

$v_t \simeq Ad\cos 2\ell + Bd$ ← distance to star

$$\Omega(R) = \frac{\Theta(R)}{R}$$



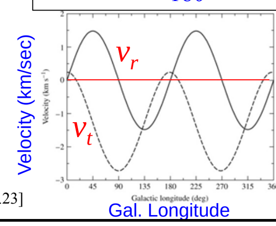
[Fig 24.22]



- Evaluate *A*, *B* from observations:
- Contain important information about Galactic rotation curve.
  - Angular velocity for circular motion at R_o:

    $\Omega_o = A - B \;\; = 27.2$ km s^{-1} kpc^{-1}
  - Gradient of rotation curve at R_o:

    $$\left.\frac{d\Theta}{dR}\right|_{R_0} = -(A+B) \;\; = -2.4 \text{ km s}^{-1}\text{ kpc}^{-1}$$



[CO Fig 24.23]

- Distance to Galactic Center

  $R_0 \sim 8$ kpc

➔ Sun's orbital velocity

  $\Theta_0 = \Omega_0 \times R_0 \sim 220$ km s^{-1}
- Sun's orbital period

  $$P = \frac{R_0}{\Theta_0} \sim 230 \text{ million yrs.}$$
- Approx. mass interior to Sun's orbit:

  $$F_{centrifugal} = F_{grav}$$

  $$\frac{m\Theta_0^{\,2}}{R_0} = \frac{GmM}{R_0^{\,2}}$$

  $$M = \frac{\Theta_0^{\,2} R_0}{G} \sim 9\times10^{10}\, M_\odot$$

Rotation Curves

- Spherical mass shell, uniform density

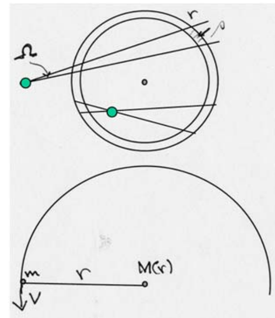
  Inside shell: no effect

  Outside shell:

  acts as if all mass at center

$$\frac{dF_{gravity}}{d\Omega} \propto \frac{\rho \times d(vol)}{r^2}$$

$$d(vol) = d\Omega\, r^2 dr$$



- $F_{centripital} = F_{gravity}$

$$\frac{mv^2}{r} = \frac{GM(r)m}{r^2}$$

$$v = \left(\frac{GM(r)}{r}\right)^{1/2}$$

Where

v = space velocity

= Θ if circular orbit.

- Inside spherical mass distribution

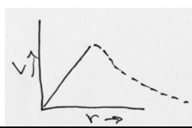
$$M(r) = \int_0^r \rho(r') 4\pi r'^2 dr' = \bar{\rho}\frac{4}{3}\pi r^3$$

$$v \propto r$$

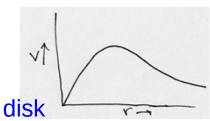
- Outside spherical mass distribution

$$M(r) = const.$$

$$v \propto r^{-1/2}$$



- Spher. Distr. + exponential disk

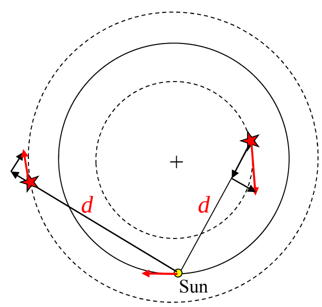


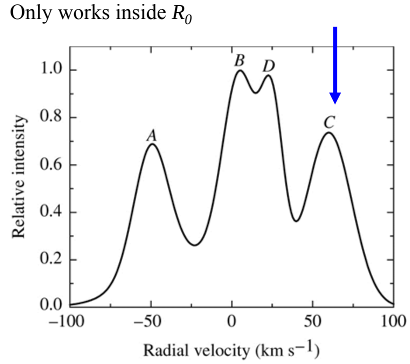
$$V = \left(\frac{G}{r}\right)^{1/2} \left\{ M(r) + 4 m_d s^2 \left[I_0(s)K_0(s) - I_1(s)K_1(s) \right] \right\}^{1/2}$$

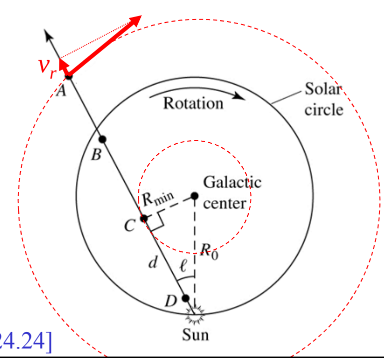
$s = \frac{1}{2} r/h$ $\quad$ I, K = Bessel functions

Measuring the MW Rotation Curve

- In principle, for stars, clusters, etc:
  - measure distance d and v_r
    - v_r = radial velocity w.r.t. Sun
  - assume circular orbit
- For H I 21cm, CO, etc. radio emission:
  - Only can measure v_r
  - Use tangent point method
  - Only works inside R_0



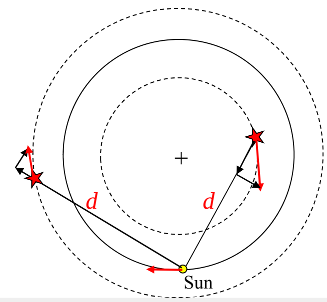


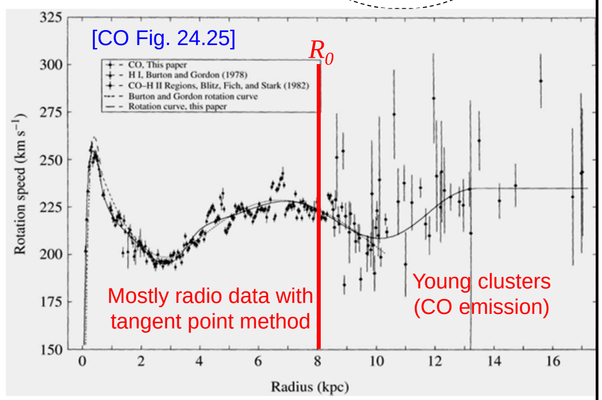


[CO fig 24.24]

Measuring the MW Rotation Curve

- In principle, for stars, clusters, etc:
  - measure distance d and v_r
    - v_r = radial velocity w.r.t. Sun
  - assume circular orbit
- For H I 21cm, CO, etc. radio emission:
  - Only can measure v_r
  - Use tangent point method
  - Only works inside R_0
- Outside R_o
  - Must use other methods.
  - Must know actual distance + velocity.

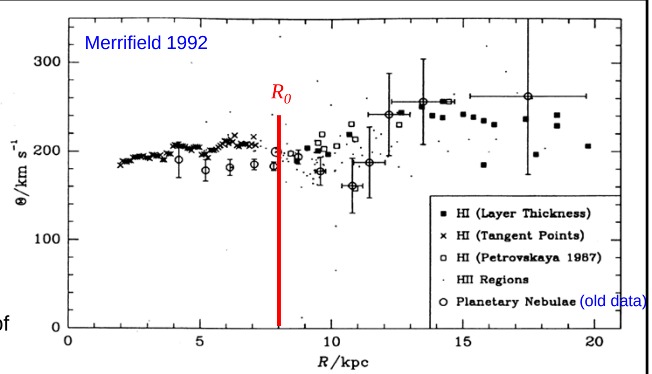


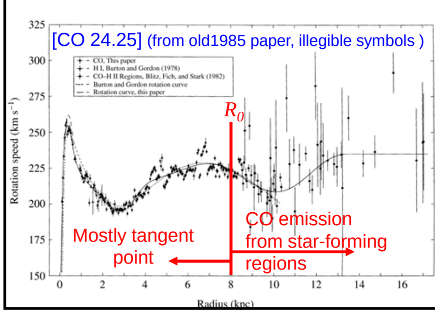


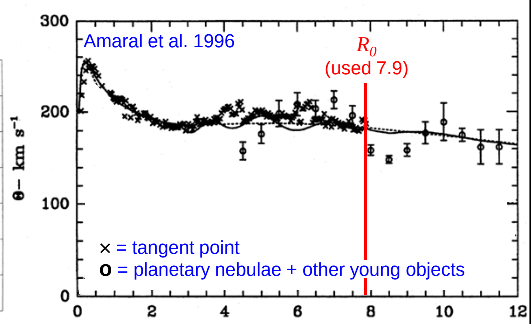
Galactic Rotation Curves

(take your pick)

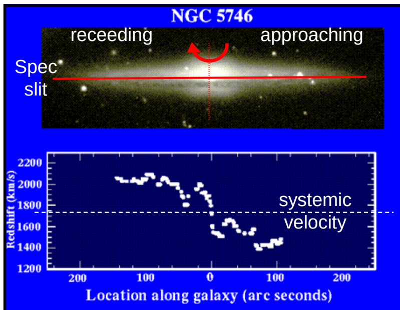
- Most data points interior to R_0 are from tangent point method.
- Beyond R_0 most data points are young objects with known distances & motions.
- But Merrifield used variation of apparent thickness of H I disk with longitude (complicated).



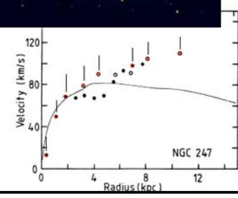




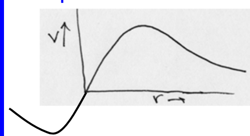
Rotation curves in other galaxies



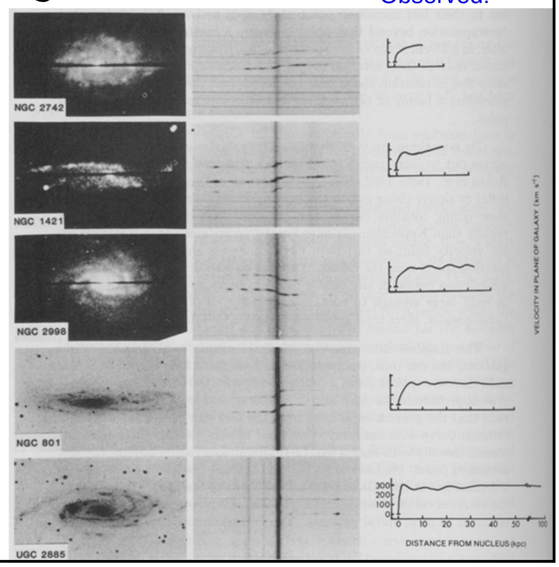
- Vera Rubin & Kent Ford (late 1970's)
- Image tube spectrograph



Expected:



Observed:



[CO pg. 917] Density as shown by flat rotation curves

Back to $F_{\text{centripital}} = F_{\text{gravitational}}$

$$\frac{mv^2}{r} = \frac{GM(r)\,m}{r^2}$$

$$M(r) = \frac{v^2 r}{G}$$

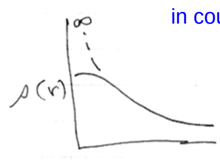
$$\frac{dM(r)}{dr} = \frac{v^2}{G}$$

but also $\dfrac{dM(r)}{dr} = 4\pi r^2 \rho(r)$

$$\Rightarrow \rho(r) = \frac{v^2}{4\pi G r^2}$$

$v \sim \text{constant} \Rightarrow \rho(r) \propto \dfrac{1}{r^2}$

Use $\rho(r) = \dfrac{C_0}{(a^2 + r^2)}$

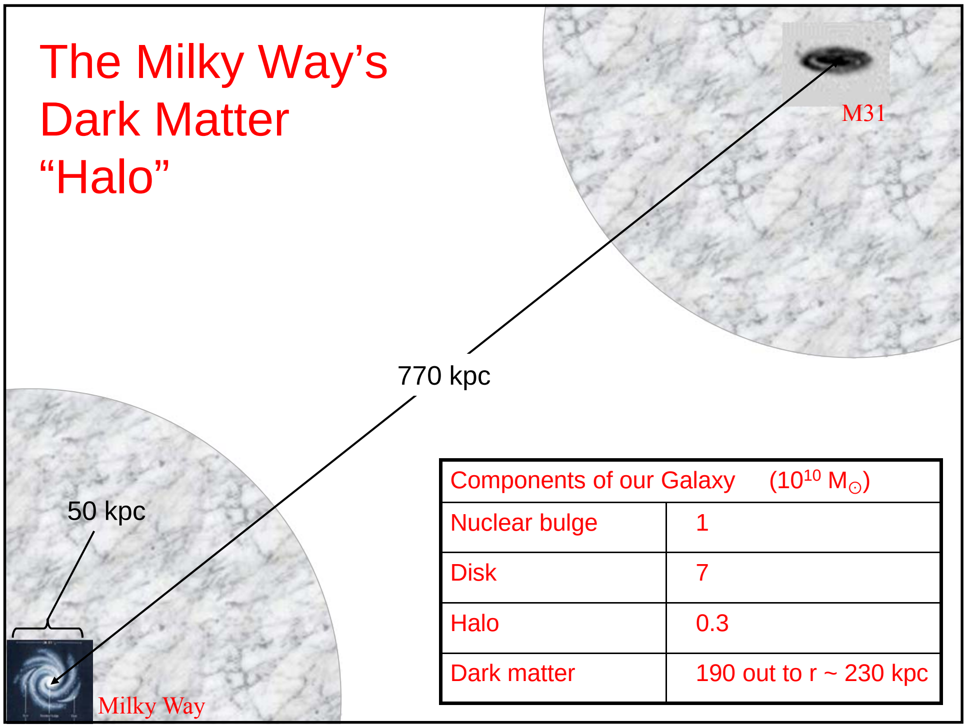


- dM(r)/dr ~ constant
  - Unbounded mass distribution??
- NFW profile

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

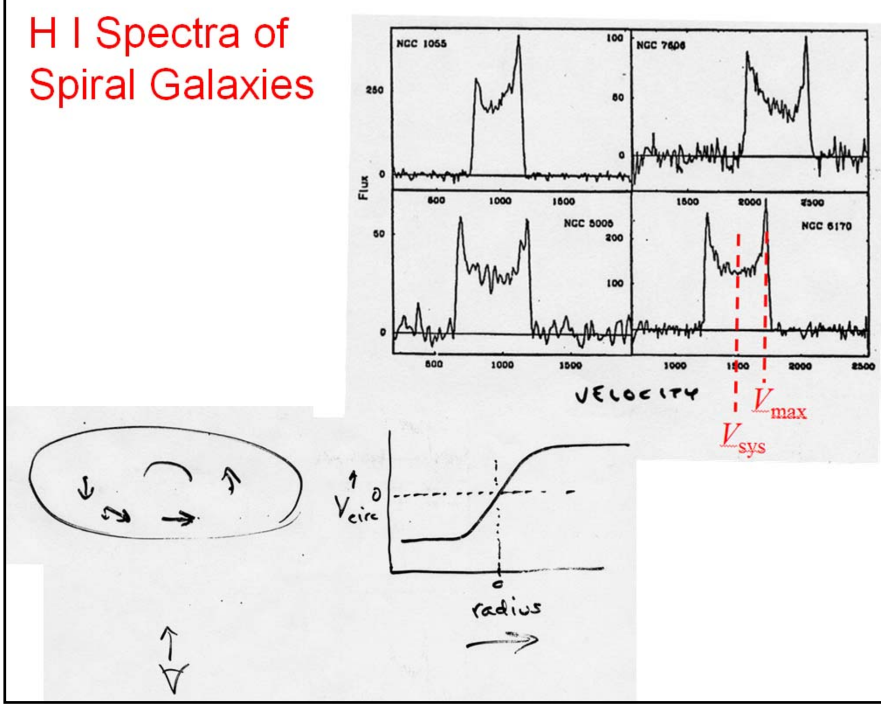
  - Predicted for Cold dark matter (CDM)
  - Actual derived dark matter profiles often slightly different than this
  - What is CDM? Coming later in course.

The Milky Way's Dark Matter "Halo"



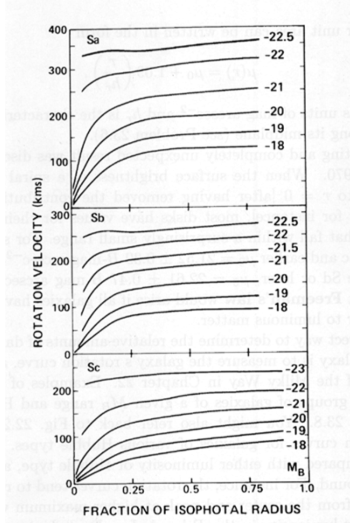
Components of our Galaxy	(10^{10} $M_\odot$)
Nuclear bulge	1
Disk	7
Halo	0.3
Dark matter	190 out to r ~ 230 kpc

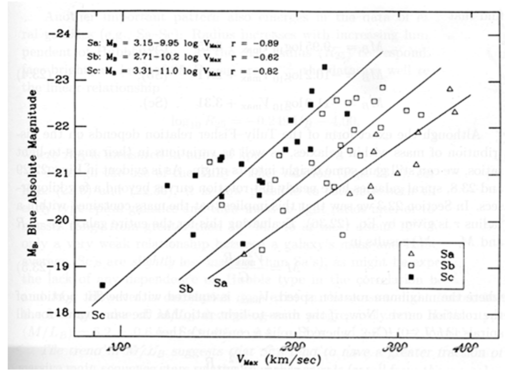
H I Spectra of Spiral Galaxies



Tully-Fisher Relation

Rotation Curves





- Maximum rotation velocity-Luminosity relation [FIG 25.10]
  - **Tully-Fisher** relation
    - $M_B = -9.95 \log_{10} V_{Max} + 3.15$ (Sa)
    - $M_B = -10.2 \log_{10} V_{Max} + 2.71$ (Sb)
    - $M_B = -11.0 \log_{10} V_{Max} + 3.31$ (Sc)

Semi-derivation of Tully-Fisher Relation:

$M_B = -9.95 \log_{10} V_{max} + 3.15$ (Sa)
$M_B = -10.2 \log_{10} V_{max} + 2.71$ (Sb)
$M_B = -11.0 \log_{10} V_{max} + 3.31$ (Sc)

- Mass interior to outermost R where rotation curve can be measured:

$$Mass = \frac{V_{max}^2 R}{G}$$

- Assume $L = Mass / const.$
- "Freeman Law" (observed fact ---maybe):

$$Surf.Bright. = \frac{L}{4\pi R^2} = const.$$

$$L = const \times V_{max}^4$$

Important as a DISTANCE calibrator!

- Convert to Absolute B-band magnitudes:

$$\mathrm{M_B} = \mathrm{M_{sun}} - 2.5\log_{10}\left(\frac{L}{L_{sun}}\right) = -10\log_{10} V_{max} + const.$$

SO FAR:

- Galaxy types
- Ancient history
- Milky Way and spiral galaxy morphology
  - Nuclear bulge
  - Disk
  - Stellar halo
  - Dark matter halo
- Star-forming regions
- Chemical enrichment

Hwk 3 Due Sept 29
CO 24.15 – just part (a).
CO 24.21
CO 24.36 (a),(b)
CO 25.14
CO 25.16

Distance measurements within MW

Kinematics of spiral galaxies

- Rotation curves → mass distribution
- Spiral structure [CO 25.3]

- General properties of S, E, Irr galaxies
- Midterm 1 (Tu. Oct 4)