Homework Set 3 Due Sept 29

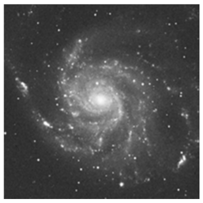
CO 24.15 – just part (a).
CO 24.21
CO 24.36 (a),(b)
CO 25.14
CO 25.16

Coming Attractions:

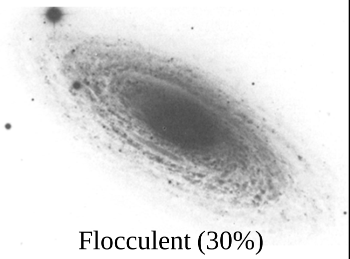
- Spiral structure [CO 25.3]
- E galaxies
- Midterm 1 (Tu. Oct 4)

Spiral Structure
[CO 25.3]

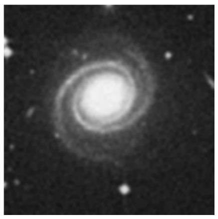
Grand design (10%)
M51



Multi-arm (60%)
M101



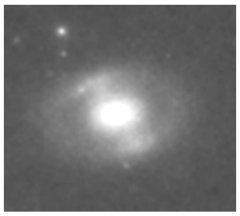
Flocculent (30%)
NGC 2841



NGC 7096

Inner rings

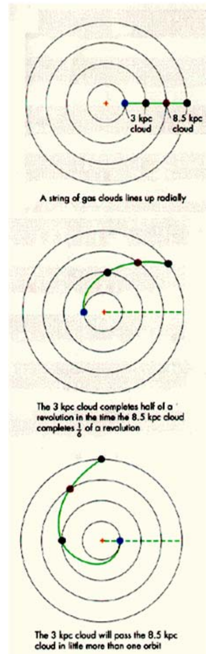
M81

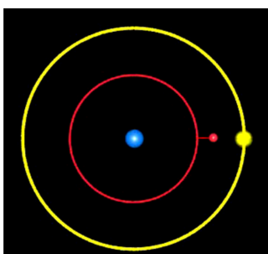


Outer Ring
NGC 4340

What causes spiral structure?

- Winding up of arms
  - Due to differential rotation
- Stochastic, Self-Propagating Star Formation
  - Chain-reaction star formation
  - SN shells ➔ shock fronts ➔ density enhancements ➔ star formation ➔more SN
  - Differential rotation then winds these regions up into spiral patterns
- Density Waves
  - Wave in gravitational potential
  - Orbital velocity of stars different than pattern speed
    - Stars, gas bunch up at position of spiral arms
    - Causes higher grav. potential
  - Unclear if self-sustaining or forced.

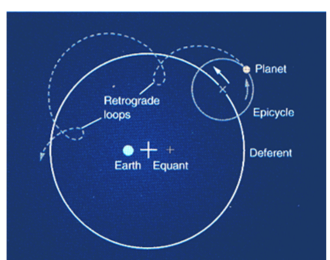


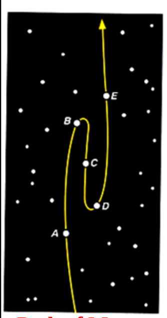


Retrograde Motion & Ptolemy's Epicycles

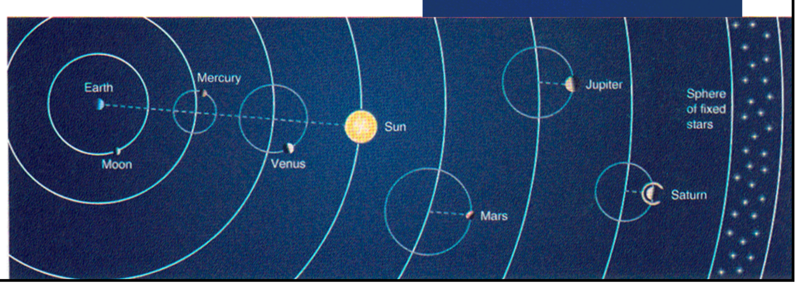
Ptolemy
Alexandria, 140AD

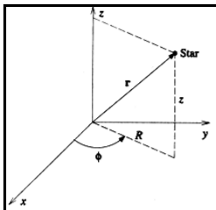
- Trying to place *Earth* at center.
- Using only circular motions.
- Led to very complicated system.





Path of Mars, etc. as seen from Earth





Epicycles… the short form.

For lurid details, see [CO 25.3]

Ptolemy
Alexandria, 140AD

U = potential energy
$\Phi = U/M$

$$M\frac{d^2\mathbf{r}}{dt^2} = -\nabla U(R,z) = -\frac{\partial U}{\partial R}\hat{\mathbf{e}}_R - \frac{1}{R}\frac{\partial U}{\partial \phi}\hat{\mathbf{e}}_\phi - \frac{\partial U}{\partial z}\hat{\mathbf{e}}_z$$

Circular symmetry ➔ independent of ϕ

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\partial \Phi}{\partial R}\hat{\mathbf{e}}_R - \frac{\partial \Phi}{\partial z}\hat{\mathbf{e}}_z \qquad (25.17)$$

$$\frac{d^2\mathbf{r}}{dt^2} = \left(\ddot{R} - R\dot{\phi}^2\right)\hat{\mathbf{e}}_R + \frac{1}{R}\frac{\partial\left(R^2\dot{\phi}\right)}{\partial t}\hat{\mathbf{e}}_\phi + \ddot{z}\hat{\mathbf{e}}_z$$

Separate d^2r/dt^2 into R, ϕ, z components
➔ 3 equations. (25.19-25.21)

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R},$$

$$\frac{1}{R}\frac{\partial\left(R^2\dot{\phi}\right)}{\partial t} = 0,$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}.$$

Conservation of specific angular momentum
$J_Z = R^2\, d\phi/dt$

Define an effective potential:

$$\Phi_{\rm eff}(R,z) \equiv \Phi(R,z) + \frac{J_z^2}{2R^2}$$

$$\ddot{R} = -\frac{\partial \Phi_{\rm eff}}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi_{\rm eff}}{\partial z}$$

Conservation of J_Z ➔ acceleration in ϕ direction when r changes.

Taylor series expansion around position of minimum $\Phi_{\rm eff}$ (circular orbit):

$\rho = R - R_m$

$$\Phi_{\rm eff}(R,z) = \Phi_{{\rm eff},m} + \cancel{\left.\frac{\partial \Phi_{\rm eff}}{\partial R}\right|_m \rho} + \cancel{\left.\frac{\partial \Phi_{\rm eff}}{\partial z}\right|_m z}$$
$$+\cancel{\frac{1}{2}\left.\frac{\partial^2 \Phi_{\rm eff}}{\partial R\partial z}\right|_m \rho z} + \frac{1}{2}\left.\frac{\partial^2 \Phi_{\rm eff}}{\partial R^2}\right|_m \rho^2 + \frac{1}{2}\left.\frac{\partial^2 \Phi_{\rm eff}}{\partial z^2}\right|_m z^2 + \cdots$$

$$\Phi_{\rm eff}(R,z) \simeq \Phi_{{\rm eff},m} + \frac{1}{2}\kappa^2\rho^2 + \frac{1}{2}\nu^2 z^2. \qquad (25.31)$$

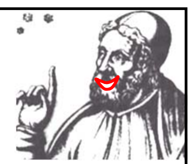
$$\ddot{\rho} \simeq -\kappa^2\rho \qquad \kappa^2 \equiv \left.\frac{\partial^2 \Phi_{\rm eff}}{\partial R^2}\right|_m$$

$$\ddot{z} \simeq -\nu^2 z \qquad \nu^2 \equiv \left.\frac{\partial^2 \Phi_{\rm eff}}{\partial z^2}\right|_m$$

(25.32-25.33)

Harmonic oscillation in R, ϕ, z about circular orbit (Epicycles)

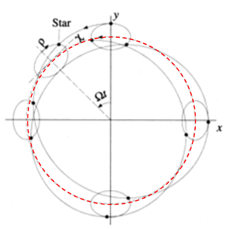
$$\ddot{\rho} \simeq -\kappa^2\rho$$
$$\ddot{z} \simeq -\nu^2 z$$



$$\rho(t) = R(t) - R_m = A_R \sin\kappa t$$
$$z(t) = A_z \sin(\nu t + \zeta)$$
$$\phi(t) = \phi_0 + \frac{J_z}{R_m^2}t + \frac{2J_z}{\kappa R_m^3}A_R\cos\kappa t = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m}A_R\cos\kappa t.$$

R_m = R at min. $\Phi_{\rm eff}$
Ω = circular ang. vel.

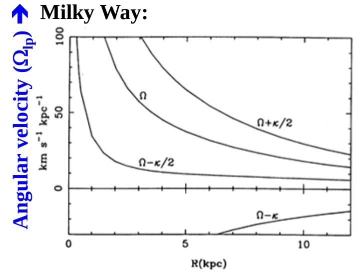
In inertial frame:



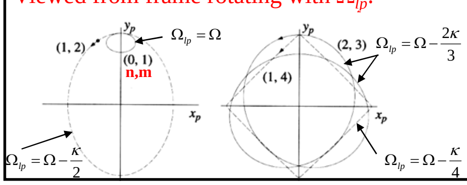
"local pattern speed"

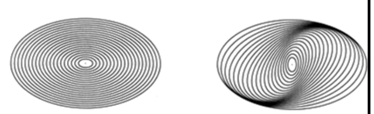
Orbits closed if:
$m\,(\Omega - \Omega_{lp}) = n\kappa$
$$\Omega_{\ell p}(R) = \Omega(R) - \frac{n}{m}\kappa(R)$$

Ω = the rotation curve for Milky Way:



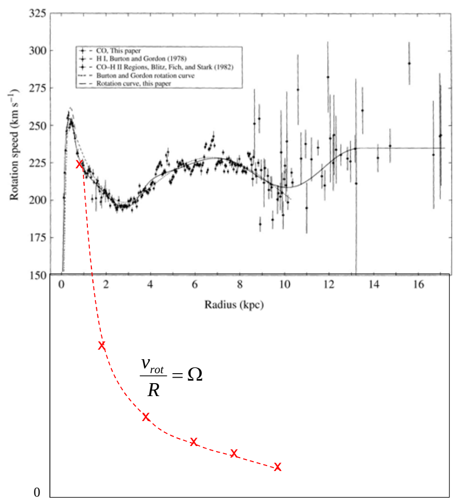
Viewed from frame rotating with Ω_{lp}:





Two ways to line up closed elliptical orbits
(as seen from frame rotating with Ω_{lp})

Flat rotation curves

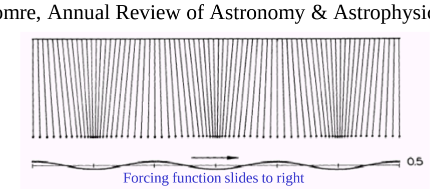


Basic nature of a density wave

From: Toomre, Annual Review of Astronomy & Astrophysics, 1977 Vol. 15, 437.

Pendulum example.

Forced travelling waves.



Rotation is opposite to the example on previous slide:

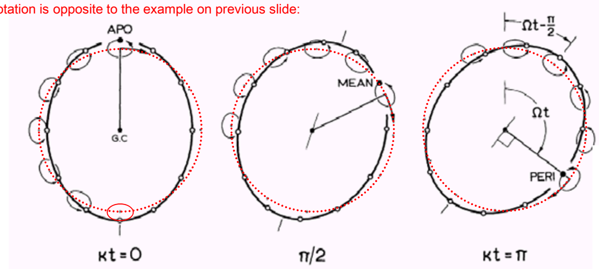
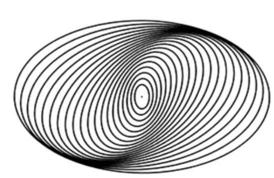


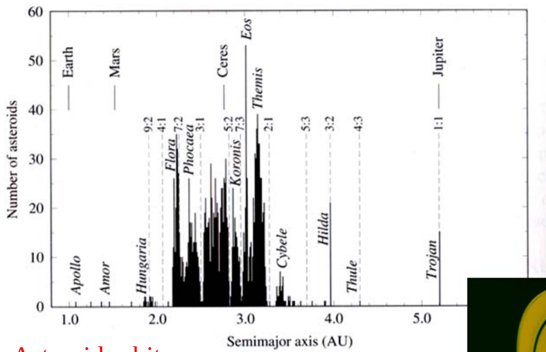
Figure 2 Slow $m = 2$ kinematic wave on a ring of test particles, all revolving clockwis (like the 12 shown) with mean angular speed Ω in strictly similar and nearly circular orbits The small elliptical "epicycles," traversed counterclockwise in the above sequence of snap shots separated in time by exactly one-quarter of the period $2\pi/\kappa$ of radial travel along each orbit, depict the apparent motions of these particles relative to their mean orbital positions or "guiding centers." Drawn for the case $\kappa = \sqrt{2}\Omega$—or one where the rotation speed $V(r) = r\Omega(r) =$ const at neighboring radii—the diagram emphasizes that the oval *locus* of such independent orbiters advances in longitude considerably more slowly than the particles themselves. That precession rate equals $\Omega - \kappa/2$, as one can verify at once by comparing the last frame with the first.

- At each R_m , stars' positions in epicycles are forced into a specific pattern by gravitational potential of spiral arm.
- Sum of positions of stars at this R_m forms an ellipse rotating at pattern speed.

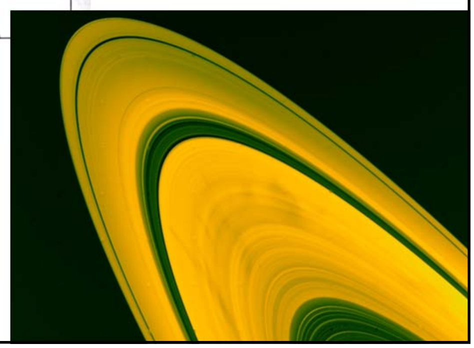


- Spiral density pattern is sum of many ellipses, all rotating at same pattern speed.

Some Solar System Resonance Phenomena



Asteroid orbits



Gaps in Saturn's rings

Lin & Shu's theory

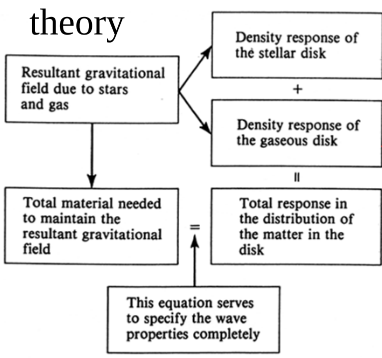


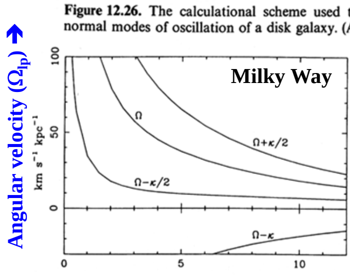
Figure 12.26. The calculational scheme used to calculate the normal modes of oscillation of a disk galaxy. (After C. C. Lin.)

Perturbed form of collisionless Boltzmann equation. "quite complicated"

Hydrodynamics

$$\kappa^2 \equiv \left.\frac{\partial^2 \Phi_{\rm eff}}{\partial R^2}\right|_m$$

$$\nu^2 \equiv \left.\frac{\partial^2 \Phi_{\rm eff}}{\partial z^2}\right|_m$$



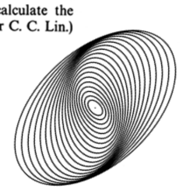
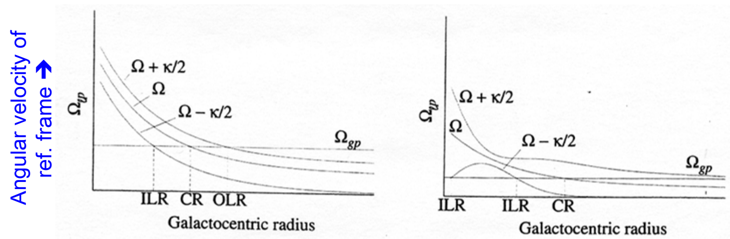


Figure 12.27. Contours of equal density excesses above the average value around a circle in a typical spiral mode. The dashed circle gives the radius where the material rotates at the same speed as the wave pattern.

Inner Lindblad Resonance (ILR)
Co-rotation Radius
Outer Lindblad Resonance (OLR)

Important in all disk galaxies



Density waves cannot propagate across ILR or OLR

Density wave theory interprets most spirals as 2-armed

- 4-armed pattern is n / m = 1 / 4
- exists over a narrow range of radius.
  ➔ less likely to be seen.