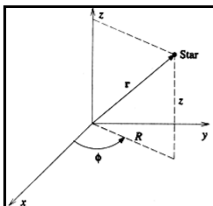
Homework Set 3 Due Sept 29

CO 24.15 – just part (a). (see CO pg. 908)
CO 24.21
CO 24.36 (a),(b)
CO 25.14 (assume that Sun currently has its max. u velocity.)
CO 25.16 (Keplerian orbit = orbit around a point mass)

Coming Attractions:

- Spiral structure [CO 25.3]
- E galaxies
- Midterm 1 (Tu. Oct 4)
- Study guide is on course web site



Epicycles… the short form.

For lurid details, see [CO 25.3]

Ptolemy
Alexandria, 140AD

U = potential energy
$\Phi = U/M$

$$M\frac{d^2\mathbf{r}}{dt^2} = -\nabla U(R,z) = -\frac{\partial U}{\partial R}\hat{\mathbf{e}}_R - \frac{1}{R}\frac{\partial U}{\partial \phi}\hat{\mathbf{e}}_\phi - \frac{\partial U}{\partial z}\hat{\mathbf{e}}_z$$

Circular symmetry ➔ independent of ϕ

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\partial \Phi}{\partial R}\hat{\mathbf{e}}_R - \frac{\partial \Phi}{\partial z}\hat{\mathbf{e}}_z \qquad (25.17)$$

$$\frac{d^2\mathbf{r}}{dt^2} = \left(\ddot{R} - R\dot{\phi}^2\right)\hat{\mathbf{e}}_R + \frac{1}{R}\frac{\partial\left(R^2\dot{\phi}\right)}{\partial t}\hat{\mathbf{e}}_\phi + \ddot{z}\hat{\mathbf{e}}_z$$

Separate d^2r/dt^2 into R, ϕ, z components ➔ 3 equations. (25.19-25.21)

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R},$$

$$\frac{1}{R}\frac{\partial\left(R^2\dot{\phi}\right)}{\partial t} = 0,$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}.$$

Conservation of specific angular momentum $J_Z = R^2\, d\phi/dt$

Define an effective potential:

$$\Phi_{\rm eff}(R,z) \equiv \Phi(R,z) + \frac{J_z^2}{2R^2}$$

$$\ddot{R} = -\frac{\partial \Phi_{\rm eff}}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi_{\rm eff}}{\partial z}$$

Conservation of J_Z ➔ acceleration in ϕ direction when r changes.

Taylor series expansion around position of minimum Φ_{eff} (circular orbit):

$\rho = R - R_m$

$$\Phi_{\rm eff}(R,z) = \Phi_{{\rm eff},m} + \cancel{\left.\frac{\partial \Phi_{\rm eff}}{\partial R}\right|_m}\rho + \cancel{\left.\frac{\partial \Phi_{\rm eff}}{\partial z}\right|_m} z + \frac{1}{2}\cancel{\left.\frac{\partial^2 \Phi_{\rm eff}}{\partial R\,\partial z}\right|_m}\rho z + \frac{1}{2}\left.\frac{\partial^2 \Phi_{\rm eff}}{\partial R^2}\right|_m \rho^2 + \frac{1}{2}\left.\frac{\partial^2 \Phi_{\rm eff}}{\partial z^2}\right|_m z^2 + \cdots$$

$$\Phi_{\rm eff}(R,z) \simeq \Phi_{{\rm eff},m} + \frac{1}{2}\kappa^2\rho^2 + \frac{1}{2}\nu^2 z^2. \qquad (25.31)$$

$$\ddot{\rho} \simeq -\kappa^2\rho \qquad \kappa^2 \equiv \left.\frac{\partial^2 \Phi_{\rm eff}}{\partial R^2}\right|_m$$

$$\ddot{z} \simeq -\nu^2 z \qquad \nu^2 \equiv \left.\frac{\partial^2 \Phi_{\rm eff}}{\partial z^2}\right|_m$$

(25.32-25.33)

Harmonic oscillation in R, ϕ, z about circular orbit (Epicycles)

$$\ddot{\rho} \simeq -\kappa^2 \rho$$

$$\ddot{z} \simeq -\nu^2 z$$

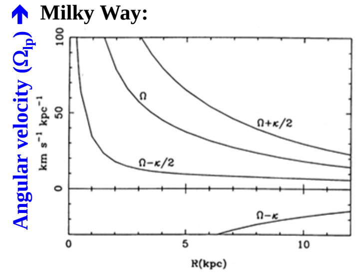
$$\rho(t) = R(t) - R_m = A_R \sin \kappa t$$

$$z(t) = A_z \sin(\nu t + \zeta)$$

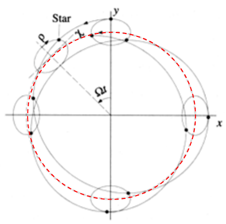
$$\phi(t) = \phi_0 + \frac{J_z}{R_m^2} t + \frac{2J_z}{\kappa R_m^3} A_R \cos \kappa t = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m} A_R \cos \kappa t.$$

R_m = R at min. Φ_{eff}
Ω = circular ang. vel.

Ω = the rotation curve for Milky Way:



In inertial frame:



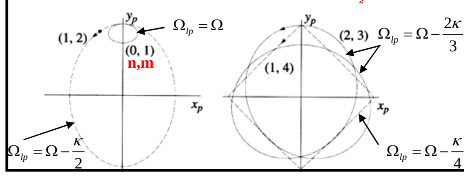
"local pattern speed"

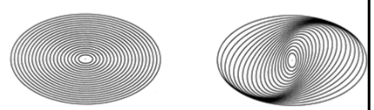
Orbits closed if:

$m(\Omega - \Omega_{lp}) = n\kappa$

$$\Omega_{\ell p}(R) = \Omega(R) - \frac{n}{m}\kappa(R)$$

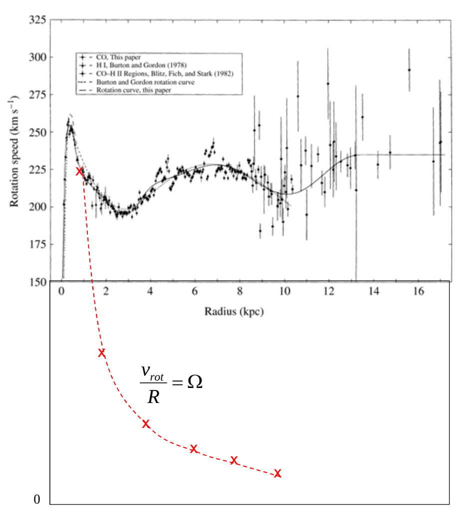
Viewed from frame rotating with Ω_{lp}:





Two ways to line up closed elliptical orbits (as seen from frame rotating with Ω_{lp})

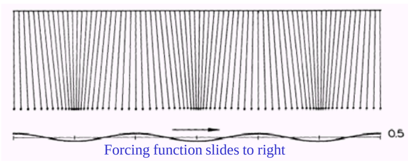
Flat rotation curves



Basic nature of a density wave

From: Toomre, Annual Review of Astronomy & Astrophysics, 1977 Vol. 15, 437.

Pendulum example.
Forced travelling waves.



Rotation is opposite to the example on previous slide:

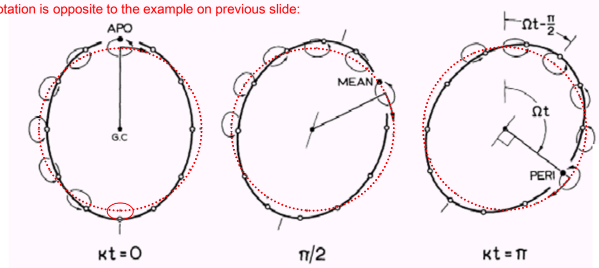
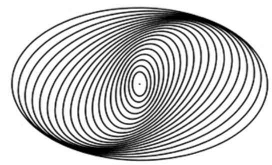


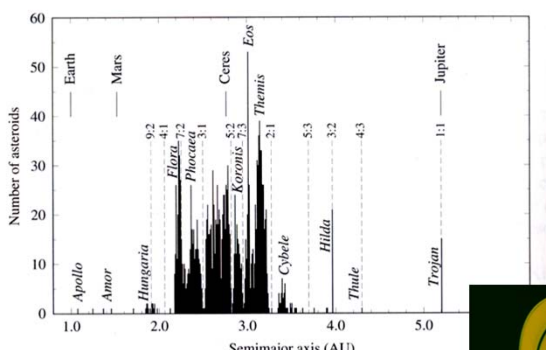
Figure 2 Slow $m = 2$ kinematic wave on a ring of test particles, all revolving clockwise (like the 12 shown) with mean angular speed Ω in strictly similar and nearly circular orbits. The small elliptical "epicycles," traversed counterclockwise in the above sequence of snapshots separated in time by exactly one-quarter of the period $2\pi/\kappa$ of radial travel along each orbit, depict the apparent motions of these particles relative to their mean orbital positions or "guiding centers." Drawn for the case $\kappa = \sqrt{2}\Omega$—or one where the rotation speed $V(r) = r\Omega(r) =$ const at neighboring radii—the diagram emphasizes that the oval *locus* of such independent orbiters advances in longitude considerably more slowly than the particles themselves. That precession rate equals $\Omega - \kappa/2$, as one can verify at once by comparing the last frame with the first.

- At each R_m , stars' positions in epicycles are forced into a specific pattern by gravitational potential of spiral arm.
- Sum of positions of stars at this R_m forms an ellipse rotating at pattern speed.

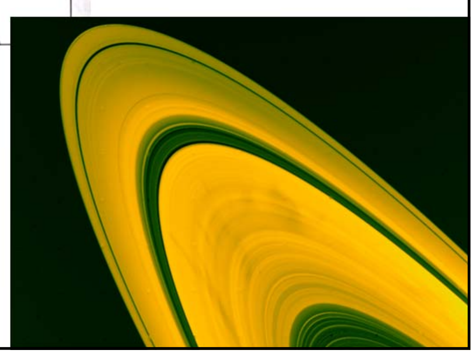


- Spiral density pattern is sum of many ellipses, all rotating at same pattern speed.

Some Solar System Resonance Phenomena



Asteroid orbits



Gaps in Saturn's rings

Lin & Shu's theory

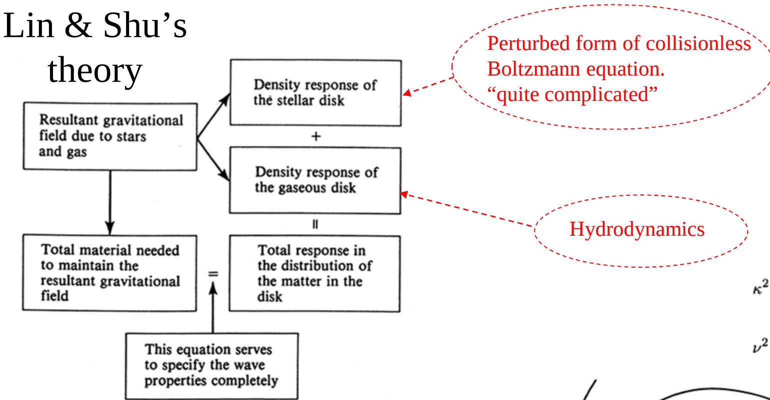
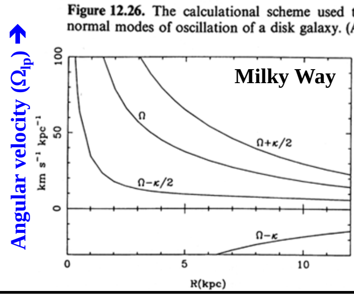


Figure 12.26. The calculational scheme used to calculate the normal modes of oscillation of a disk galaxy. (After C. C. Lin.)

$$\kappa^2 \equiv \left.\frac{\partial^2 \Phi_{\rm eff}}{\partial R^2}\right|_m$$

$$\nu^2 \equiv \left.\frac{\partial^2 \Phi_{\rm eff}}{\partial z^2}\right|_m$$



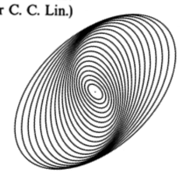
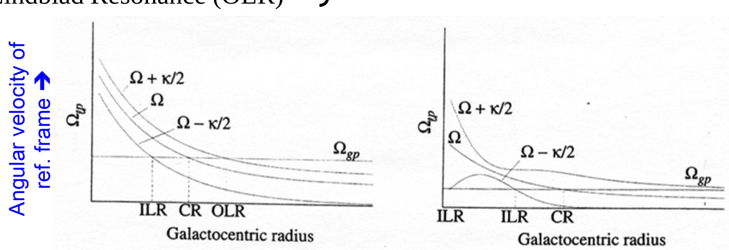


Figure 12.27. Contours of equal density excesses above the average value around a circle in a typical spiral mode. The dashed circle gives the radius where the material rotates at the same speed as the wave pattern.

Inner Lindblad Resonance (ILR)
Co-rotation Radius
Outer Lindblad Resonance (OLR)

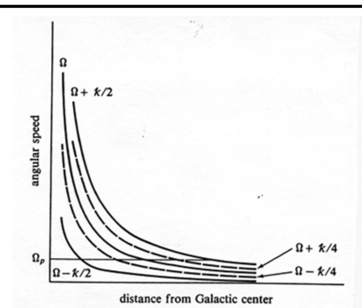
} Important in all disk galaxies



Density waves cannot propagate across ILR or OLR

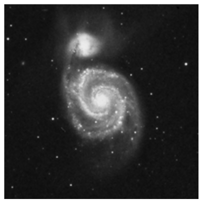
Density wave theory interprets most spirals as 2-armed

- 4-armed pattern is n / m = 1 / 4
- exists over a narrow range of radius.
  - ➔ less likely to be seen.



Spiral Structure

[CO 25.3]

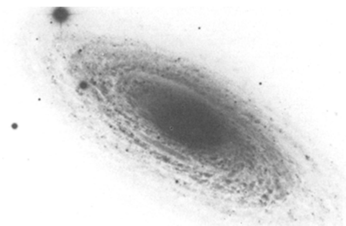


Grand design (10%)

M51

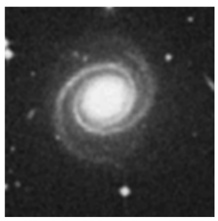
Multi-arm (60%)

M101



Flocculent (30%)

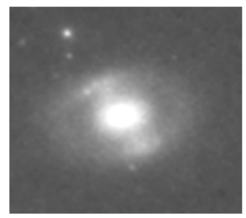
NGC 2841



Inner rings

NGC 7096

M81

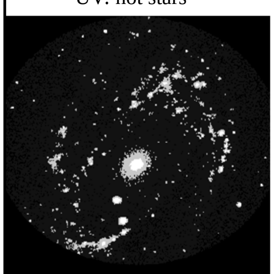


Outer Ring

NGC 4340

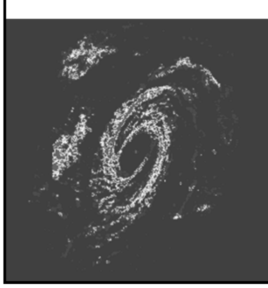
M81 spiral structure at different wavelengths

UV: hot stars



Visible: stars + obscuration

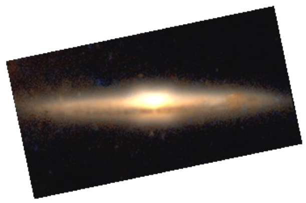
Near IR: late-type stars

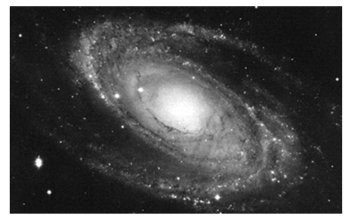


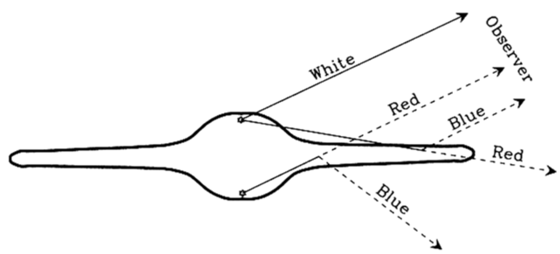
21 cm: HI

Old red population shows small but real spiral density enhancement.

Trailing vs. leading spirals
Which is the near side of the galaxy?







Passage of gas through spiral arms

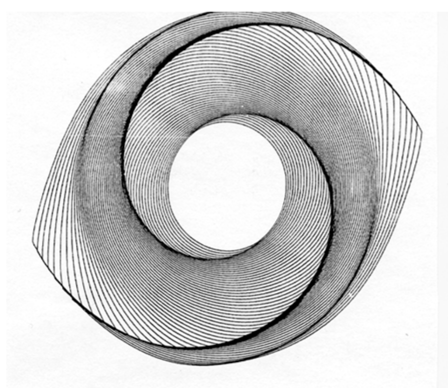
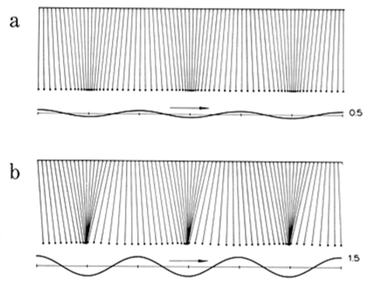


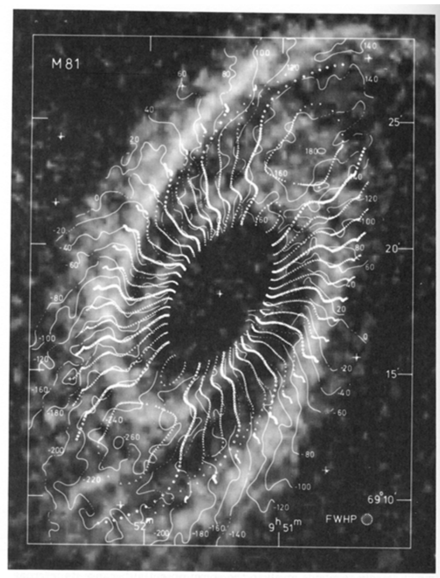
Figure 12.33. The streamlines of gas from a theoretical model of the spiral galaxy M81 (NGC3031, type Sb). (From H. Visser, *Astr. Ap.*, **88**, 1980, 159.)

Calculated streamlines for gas

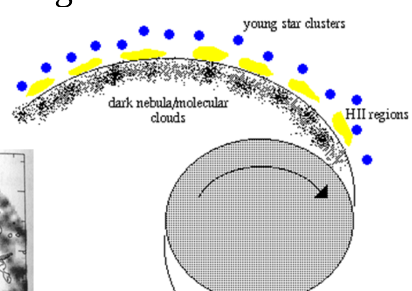
Response of gas to density waves

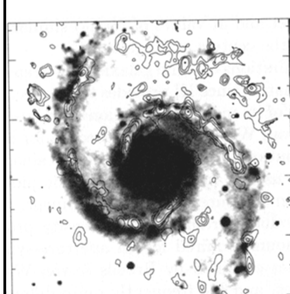


- Simple pendulum model
  - Each pendulum = 1 gas cloud
  - For large amplitude forcing, pendulums collide.
  - ➔ shock fronts in spiral arms
- HI map (right) shows velocity jumps at spiral arms.

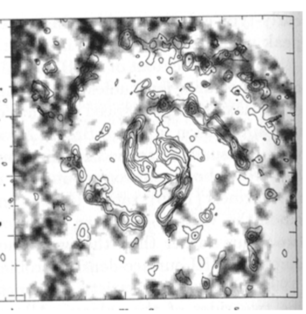


Molecular clouds on inner edges of arms

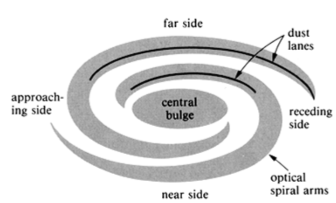




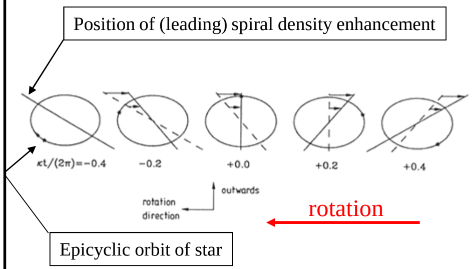
CO contours over red image



CO contours over 21 cm map

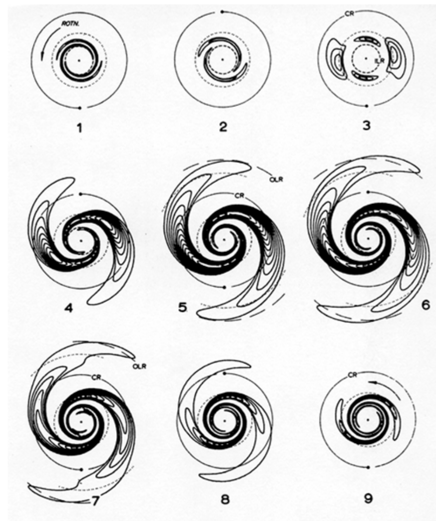


Swing Amplification



While it is swinging around, the spiral arm moves at about same angular speed as star.

Automatically converts any leading spirals into much stronger trailing spirals.

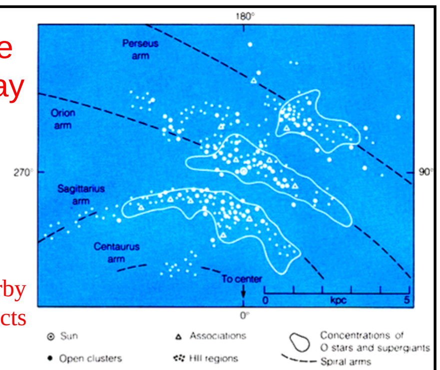


Time steps = ½ of co-rotation period at CR.

Spiral Structure of the Milky Way

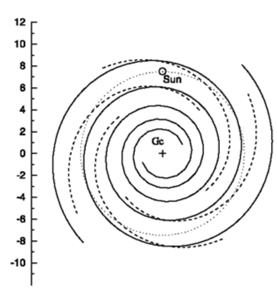
From HI (21 cm observations) assuming circular rotation.

Hard to measure, because we are inside it.



Map of nearby young objects

- Recent model
  - Lepine et al (2001) ApJ 546, 234.
- ➔ mix of
  - 2-armed mode
  - 4-armed mode
- Sun at ~ co-rotation radius.



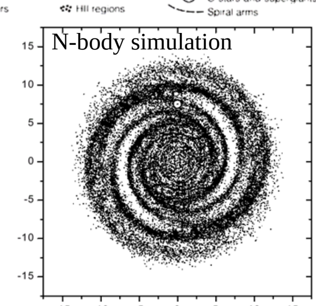


FIG. 3.—Visible structure of the Galaxy derived for the best model (superposition of 2 + 4 self-sustained wave harmonics) by means of cloud-particle simulation. The scale is indicated in kpc. Note that the model is not valid for radii smaller than about 2.5 kpc.

Summary: Density Waves?

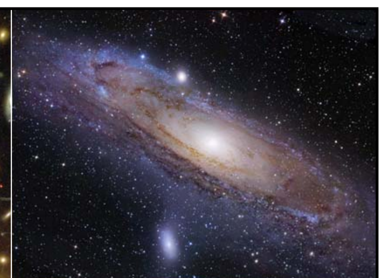
- Evidence showing density waves *do* occur.
  - Old, red stars show spiral density perturbation.
  - Molecular clouds form on inner edges of spiral arms.
  - HI gas flow shows discontinuity due to shocks at inner edges of spiral arms.
  - Bright young stars also in narrow arms.
    - Observed width $\Delta\theta \sim t_*(\Omega - \Omega_p)$, as predicted.
- Are these waves self-sustaining over 10^{10} years? Problems:
  - Lin-Shu theory is linear; does not predict whether waves will grow or decay.
  - How are density waves initially formed?
- The usual interpretation
  - Density waves need a driving force
    - Satellite galaxy at co-rotation radius (M51)
    - Bars
  - Otherwise, act to prolong life of transitory phenomena.
  - Other mechanisms probably also important.
    - Swing-amplification efficiently builds up temporary trailing spirals.

Ellipticals

Huge mass range:

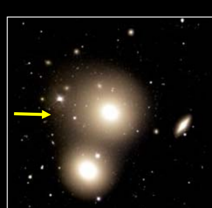
- Dwarf spheroidals: 10^7-10^8 $M_\odot$
- Blue compact dwarfs: ~10^9 $M_\odot$
- Dwarf ellipticals: 10^7-10^9 $M_\odot$
- Normal (giant) ellipticals: 10^8-10^{13} $M_\odot$
- cD galaxies in cluster centers: 10^{13}-10^{14} $M_\odot$

Dwarf spheroidal (Leo I)



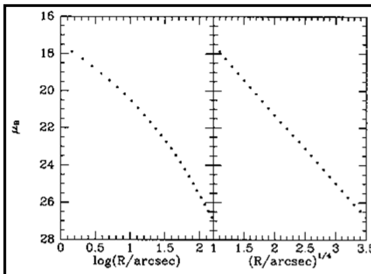
Dwarf ellipticals M32, NGC 205

Galaxy Cluster Abell 1689

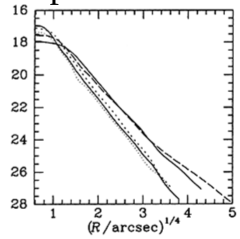


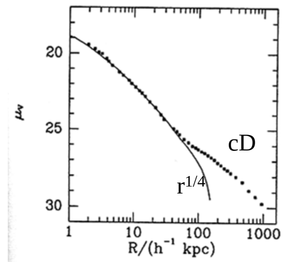
cD (NGC 3311)

Giant E (NGC 1407)



Ellipticals





Good

- Hubble's law $I(R) = \frac{I_o}{\left(1+\frac{R}{R_o}\right)^2}$

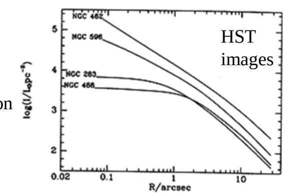
Better

- Modified Hubble's law

$$I(R) = \frac{I_o}{1+\left(R/R_o\right)^2}$$

Diverges, but at least is projection of simple 3D distribution:

$$I(r) = \frac{I_o}{\left[1+\left(r/R_o\right)^2\right]^{3/2}}$$



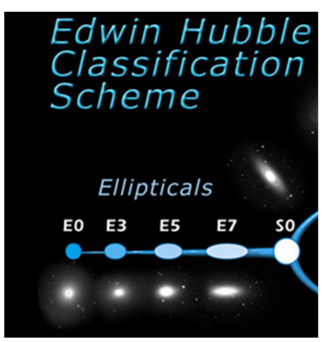
Best

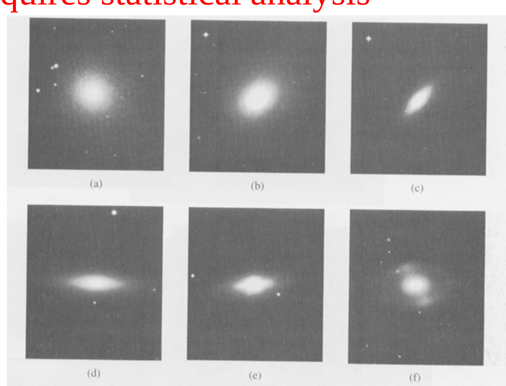
- deVaucouleurs' $R^{1/4}$ law usually fits radial surface brightness distribution

$$I(R) = I_e 10^{-3.33\left[\left(\frac{R}{R_e}\right)^{1/4}-1\right]}$$

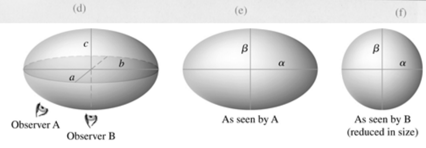
- + others

True shapes requires statistical analysis

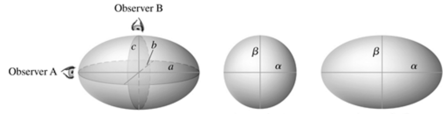




- *Oblate* = pancakes



- *Prolate* = footballs



True shapes requires statistical analysis

- Lower luminosity ➔ rotationally supported
  - $(V_{rot} / \sigma) \sim \sqrt{\varepsilon/(1-\varepsilon)}$ ← Ellipticity = 1 – *b*/*a*
- Higher L ➔ pressure supported
  - $(V_{rot} / \sigma) << 1$

CO pgs. 988-989

Curve expected for galaxies that are flattened by rotation (i.e. have isotropic random velocity dispersions)

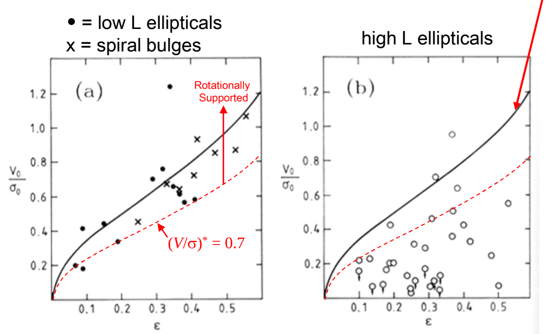
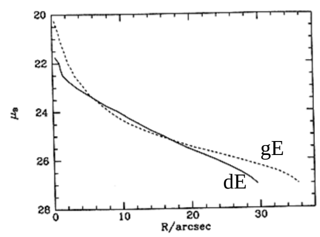
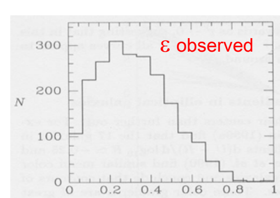


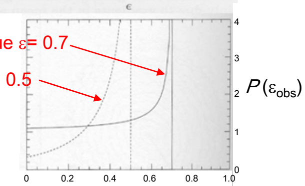
Figure 4-6. (a) The positions in the $(v/\sigma, \epsilon)$ plane of elliptical galaxies (dots), and of spheroids (crosses), that have luminosities smaller than $L = 2.5 \times 10^{10} L_\odot$. (b) The same as (a) but for elliptical galaxies brighter than $L = 2.5 \times 10^{10} L_\odot$. (After Davies et al. 1983.) From Binney & Tremaine, *Galactic Dynamics*

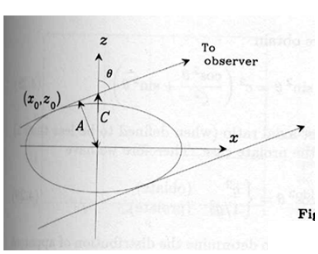


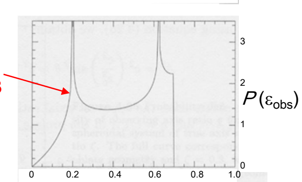
Statistics of $\varepsilon = (1- b/a)$

- Oblate, prolate spheroids can't fit the observed distribution.
  - Summing over wide range of true values of ε would fill in the dip at $\varepsilon_{obs} = 0$.
- Triaxial spheroids can fit.
  - Nearly oblate triaxial spheroids seem best.









From Binney & Merrifield, *Galactic Astronomy*

Other evidence for triaxial systems

- Isophotal twists
- Kinematics (star motions)

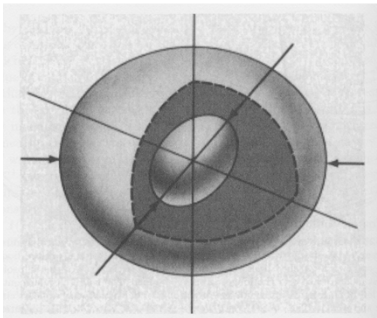
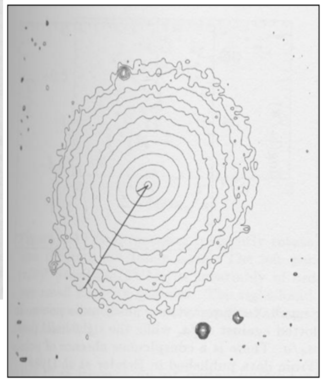


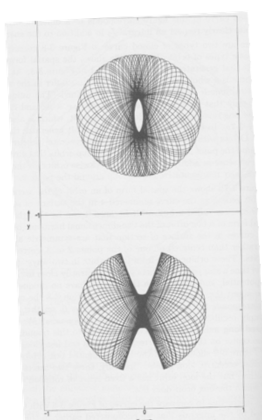
Figure 4.24 Isophotal twist as a consequence of triaxiality. Two concentric, coaxial ellipsoids are shown. The dashed lines mark the intersections of the ellipsoids with the coordinate planes, while the solid lines show their outlines to the observer. The arrows mark the directions of their apparent principal axes.



From Binney & Merrifield, *Galactic Astronomy*

Orbits in E galaxies

- Some families of non-closed orbits in a mildly triaxial potential.



3.4 Orbits in Three-Dimensional Triaxial Potentials 155

From Binney & Tremaine, *Galactic Dynamics*

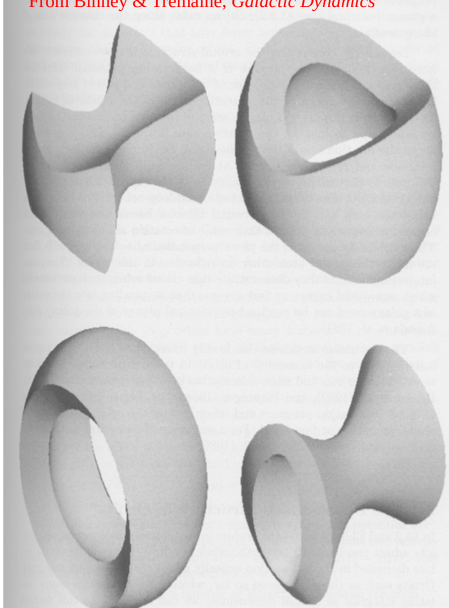


Figure 3-20. Orbits in a nonrotating triaxial potential. Clockwise from top left: (a) box orbit; (b) short-axis tube orbit; (c) inner long-axis tube orbit; (d) outer long-axis tube orbit. [Courtesy of T. Statler; see Statler (1986).]