

True shapes requires statistical analysis


- Oblate = pancakes

- Prolate = footballs



## True shapes requires statistical analysis

- Lower luminosity $\rightarrow$ rotationally supported
- $\left(V_{\text {rot }} / \sigma\right) \sim \sqrt{\varepsilon /(1-\varepsilon)}$.

Ellipticity $=1-b / a$

- Higher L $\rightarrow$ pressure supported
- $\left(V_{\text {rot }} / \sigma\right) \ll 1$
- = low L ellipticals

Curve expected for galaxies
that are flattened by rotation
(i.e. have isotropic random velocity dispersions)


Figure 4-6. (a) The positions in the ( $v / \sigma, c$ ) plane of elliptical galaxies dots), and
of spheroids (crosses), that have luminosities smaller than $L=2.5 \times 10^{10} \mathrm{~L} \odot$. (b) Davies et al. 1983.) From Binney \& Tremaine, Galactic Dynamics

## Statistics of $\varepsilon=(1-b / a)$

- Oblate, prolate spheroids can't fit the observed distribution.
- Summing over wide range of true values of $\varepsilon$ would fill in the dip at $\varepsilon_{o b s}=0$.
- Triaxial spheroids can fit.
- Nearly oblate triaxial spheroids seem best.





## Other evidence for triaxial systems



- Isophotal twists
- Kinematics (star motions)



## Orbits in E galaxies

- Some families of non-closed orbits in a mildly triaxial potential.



## E galaxies are transparent, but $40 \%$ still have some dust lanes

- Even if complete star formation at $\mathrm{t}=0$, stars must subsequently have lost gas.
- Detected by:
- X-rays (Brehmsstrahlung): $10^{8}-10^{10} \mathrm{M}_{\odot}$
- H I emission lines: $10^{7}-10^{9} \mathrm{M}_{\odot}$
- H II emission lines: $10^{4}-10^{5} \mathrm{M}_{\odot}$
- But gas can be lost by
- Supernova-driven winds
- Ram pressure stripping



## The Virial Theorem [CO 2.4]

- For gravitationally bound systems in equilibrium
- Time-averaged kinetic energy $=-1 / 2$ time-averaged potential energy.

$$
\begin{aligned}
& E=\text { total energy } \\
& U=\text { potential energy. } \\
& K=\text { kinetic energy. } \\
& E=K+U
\end{aligned}
$$

- Can show from Newton’s 3 laws + law of gravity:
- $1 / 2\left(d^{2} I / d t^{2}\right)-2 K=U$ where $I=\Sigma m_{i} r_{i}^{2}=$ moment of inertia.
- Time average $\left\langle d^{2} I / d t^{2}\right\rangle=0$, or at least $\sim 0$.
- Virial theorem $\rightarrow-2\langle K\rangle=\langle U\rangle$

$$
\begin{aligned}
&\langle K\rangle=-1 / 2\langle U\rangle \\
&\langle E\rangle=\langle K\rangle+\langle U\rangle \rightarrow \\
&<E\rangle=1 / 2\langle U\rangle
\end{aligned}
$$

Sorry... I had left out
the $1 / 2$ when I showed
this slide in class.

## Mass determinations from absorption line widths

- Virial Theorem
$2 K=-U$
$U=-\frac{3}{5} \frac{G M^{2}}{R}$
$K=1 / 2 M<v^{2}>=3 / 2 M<\sigma_{r}^{2}>$
$\rightarrow M_{\text {virial }}=\frac{5 R \sigma_{r}^{2}}{G}$
- See pp. 959-962, + Sect. 2.4
- Applied to nuclei of spirals $\rightarrow$ presence of massive black holes


M32

- Also often applied to
- E galaxies
- Galaxy clusters


## Mass determinations from absorption line widths

- Virial Theorem
$2 K=-U$
$U=-\frac{3}{5} \frac{G M^{2}}{R}$
$\left\langle v^{2}\right\rangle=3\left\langle v_{r}^{2}\right\rangle$
$K=1 / 2 M<v^{2}>=3 / 2 M<\sigma_{r}^{2}>$
$\rightarrow M_{\text {virial }}=\frac{5 R \sigma_{r}^{2}}{G}$

- See pp. 959-962, + Sect. 2.4
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- E galaxies
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Gaussian fit:

- Convolution turns into multiplication in F.T. space.
- F.T. of a Gaussian is a Gaussian.
$I(R)=I_{e} 10^{-3.33\left[\left(\frac{R}{R_{e}}\right)^{1 / 4}-1\right]}$
$I_{e}=$ surface brightness at $R_{e}$ $L_{e}=$ luminosity within $R_{e}$

Faber-Jackson relation: $L_{e} \sim \sigma_{0}^{4}$

(Absolute magnitude)

## Mass-Luminosity relationships

- Faber-Jackson relation: $L_{e} \sim \sigma_{0}^{4}$
- $D_{n}-\sigma_{0}$ correlation.
mag/arcsec ${ }^{2}$
- $D_{n}=$ diameter within which $\langle I\rangle=20.75 \mu_{B}$

$$
I(R)=I_{e} 10^{-3.33\left[\left(\frac{R}{R_{e}}\right)^{1 / 4}-1\right]}
$$

- Fundamental plane in $\log R_{e},\langle I\rangle_{e}, \log \sigma_{0}$ space
- $R_{e}=$ scale factor in $R^{1 / 4}$ law
- $\langle I\rangle_{e}=$ mean surface brightness within $R_{e}$ Different from $I_{e}$ ! CO give different coefficients???
- Intro. to Principle Component Analysis: astro-ph/9905079




## Distribution of galaxy types

- Dense regions (cluster centers) predominantly ellipticals.
- Field galaxies predominantly spirals.
- On average, roughly even split between E and S.



Dressler 1980 Log (Projected surface density of galaxies) $\boldsymbol{\rightarrow}$

$$
\begin{aligned}
& \text { Schechter Luminosity Function } \\
& \qquad \begin{aligned}
\phi(L) d L & =L^{\alpha} e^{-L / L^{*}} d L \\
\phi(M) d M & =10^{-0.4(\alpha+1) M} e^{-100.4\left(M^{*}-M\right)} d M
\end{aligned}
\end{aligned}
$$

- The Milky Way is an $L^{*}$ galaxy.



Problem 24.15: "Assuming that the highest velocity stars are near the escape speed, estimate the mass of the M.W."
Correct: $v_{\text {esc }}=v_{\text {circ }}+\max v_{\text {pec }}=220+65 \sim 300 \mathrm{~km} \mathrm{~s}^{-1}$.
K.E. $=$ Potential Energy $\rightarrow \frac{m v_{e s c}^{2}}{2}=\frac{G m M}{R_{0}} \rightarrow M=\frac{R_{0} v_{e s c}^{2}}{2 G}$

Wrong: follow example 24.3.1 and calculate mass required to
hold star in circular orbit with $v=300 \mathrm{~km} \mathrm{~s}^{-1}$

$$
M=\frac{R_{0} v_{e s c}^{2}}{G}
$$

Problem 24.36: "Point mass $M_{0}$ at center of MW + mass distributed with density $\rho(r) \propto 1 / r^{2}$. (a) Show that $M_{r}=k r+M_{0}$."
Correct:

$$
\begin{aligned}
& M_{r}=\underset{\substack{M_{0}}}{ }+\int_{0}^{r} \rho\left(r^{\prime}\right) d \operatorname{vol}\left(r^{\prime}\right)=M_{0}+\int_{0}^{r} \frac{C}{r^{\prime 2}} 4 \pi r^{\prime 2} d r^{\prime}=M_{0}+C 4 \pi r \\
& \quad \text { Black Hole }
\end{aligned}
$$

Wrong: anything that does not show that you realized that you
need to integrate over $\quad \rho\left(r^{\prime}\right) d v o l\left(r^{\prime}\right)$

