Some Universes

Open *vs.* Closed:

$$\left(H^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

$$H^2(1-\Omega)R^2 = -kc^2$$

$$H_0^2(1-\Omega_0) = -kc^2$$

$$k = 0 \Rightarrow \quad \Omega_0 = \Omega_{m,0} + \overset{\sim 0}{\cancel{\Omega_{\text{rel},0}}} + \Omega_{\Lambda,0} = 1$$

Accelerating *vs.* Decelerating:

Deceleration parameter:
$$q(t) = -\frac{R(t)\left[d^2R(t)/dt^2\right]}{\left[dR(t)/dt\right]^2}$$

+ accel. eqn:

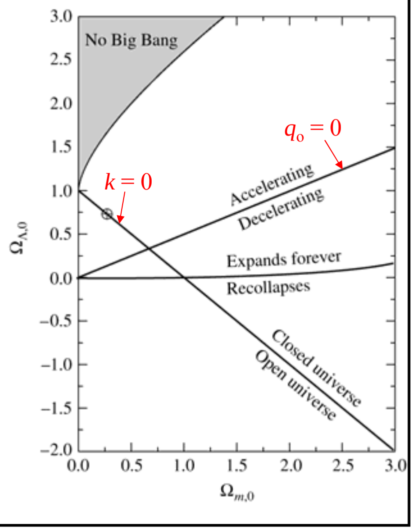
$$\frac{d^2R}{dt^2} = -\frac{H^2}{2\rho_c}\left[\rho_m + 2\rho_{rel} - 2\rho_\Lambda\right] R$$

(Camouflaged accel. eqn.)

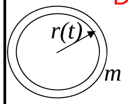
$$q(t) = \frac{1}{2}\Omega_m(t) + \overset{\sim 0}{\cancel{\Omega_{\text{rel}}}}(t) - \Omega_\Lambda(t)$$

Expands Forever *vs.* Recollapses:

Does *dR/dt* ever = 0? See [29.135]



Definitions, results, etc.



* $r = R(t)\,\varpi$

* $H = \frac{1}{R}\frac{dR}{dt}$

Densities:

* Matter: $\rho_m = \rho_{o,m}R^{-3}$

* Radiation: $\rho_r = \rho_{o,r}R^{-4}$

* Dark energy: $\rho_\Lambda = \rho_{o,\Lambda}R^0$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

* $\Omega(t) = \frac{\rho(t)}{\rho_c(t)}$

* $\Omega \equiv \Omega_m + \Omega_{\text{rel}} + \Omega_\Lambda$

$$\frac{d(R^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(R^3)}{dt}$$

$$q(t) = -\frac{R(t)\left[d^2R(t)/dt^2\right]}{\left[dR(t)/dt\right]^2}$$

* $P = wu = w\rho c^2$

Physics

Per unit mass:

K.E. + potential E. = Total Energy

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

$\rho = \frac{u}{c^2}$ *

* Temp. of radiation field: $T_0 = RT(R)$

*

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\,d\theta)^2 + (\varpi\sin\theta\,d\phi)^2\right]$$

*

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2$$

Cosmological Constant (a.k.a. *Dark Energy*)

Curvature $k = \frac{1}{\Re^2} \times \begin{matrix} +1 \\ 0 \\ -1 \end{matrix}$

$dU = -PdV$

$$\frac{d^2R}{dt^2} = \left\{-\frac{4}{3}\pi G\left[\rho_m + \rho_{\text{rel}} + \rho_\Lambda + \frac{3(P_m + P_{\text{rel}} + P_\Lambda)}{c^2}\right]\right\}R$$

* = you should be able to write these down from memory.

[29.4] Observational Cosmology

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\,d\theta)^2 + (\varpi\sin\theta\,d\phi)^2\right]$$

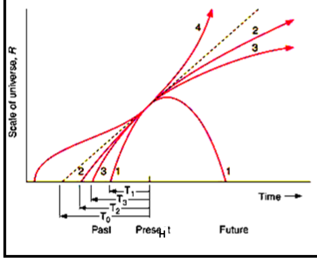
$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2$$

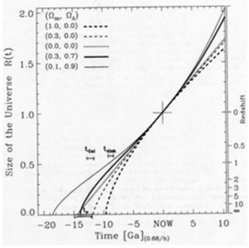
$$H_0^2(1-\Omega_0) = -kc^2$$

- Some theoretical parameter sets:
  - *R(t)* vs. *t*
  - $\Omega_{\Lambda,o}$ vs. $\Omega_{m,o}$
  - Curvature *k*, dR/dt, d^2R/dt^2

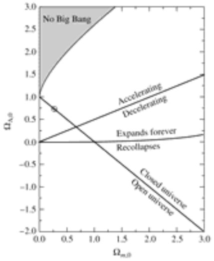
- But what can we actually *measure* that will tell us which universe we live in?

  As a function of *z*:
  - Apparent mag. of standard candles.
  - Angular sizes.
  - Space density of galaxies.

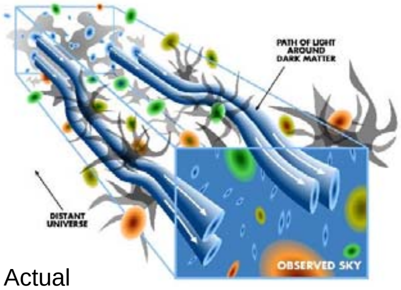




Are two numbers enough?



The paths of photons through space-time



Actual
(sketched in 3D space)

Idealized
(R-W metric)

$$\overset{0}{\cancel{(ds)^2}} = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + \overset{0}{\cancel{(\varpi\,d\theta)^2}} + \overset{0}{\cancel{(\varpi\sin\theta\,d\phi)^2}}\right]$$

$$\int_{t1}^{t2}\frac{cdt}{R(t)} = -\int_{\varpi 1}^{\varpi 2}\frac{d\varpi}{\sqrt{1-k\varpi^2}} = \int_{\varpi 2}^{\varpi 1}\frac{d\varpi}{\sqrt{1-k\varpi^2}}$$

Take neg. square root so that
ϖ will decrease with increasing *t*

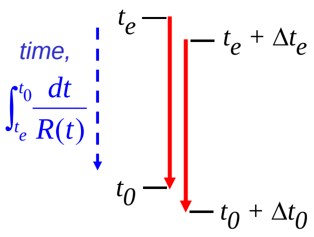
Redshift and Cosmological Time Dilation

(See pg. 1200)

$$(ds)^2 = 0 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + \cdots\right.$$

For two radially travelling wave crests:

$$\int_{t_e}^{t_0} \frac{dt}{R(t)} = \frac{1}{c}\int_0^{\varpi_e} \frac{d\varpi}{\sqrt{1-k\varpi^2}} = \int_{t_e+\Delta t_e}^{t_0+\Delta t_0} \frac{dt}{R(t)}$$



Both red paths are
comoving distance ϖ_e long.

$$\int_{t_0}^{t_0+\Delta t_0} \frac{dt}{R(t)} = \int_{t_e}^{t_e+\Delta t_e} \frac{dt}{R(t)}$$

$$\frac{\Delta t_e}{R(t_e)} = \frac{\Delta t_0}{R(t_0)}$$

$$\frac{R(t_0)}{R(t_e)} = \frac{\Delta t_0}{\Delta t_e}$$

$$\frac{1}{R(t_e)} = \frac{\lambda_0}{\lambda_e} = 1 + z$$

[CO 29.142]

Proper distance

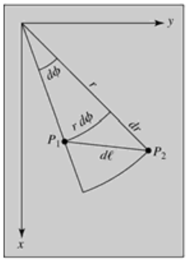
= the *current* distance to a distant object.

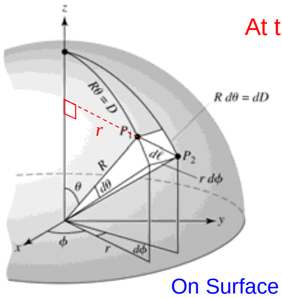
$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]$$

$dt = 0$, proper distance $d_p(t) = \mathrm{sqrt}(-ds^2)$

$$d_p(t) = R(t)\int_0^{\varpi} \frac{d\varpi'}{\sqrt{1-k\varpi'^2}}$$

On Flat Surface





On Surface of Ball

At the current time (using $R(t_0) = 1$):

Flat: $d_{p,0} = \varpi$

Closed: $d_{p,0} = \frac{1}{\sqrt{k}}\sin^{-1}(\varpi\sqrt{k})$

Open: $d_{p,0} = \frac{1}{\sqrt{|k|}}\sinh^{-1}(\varpi\sqrt{|k|})$

The particle horizon

Horizon distance = distance a photon has traveled since $t = 0$.

$$\int_{t1}^{t2} \frac{cdt}{R(t)} = -\int_{\varpi 1}^{\varpi 2} \frac{d\varpi}{\sqrt{1-k\varpi^2}} = \int_{\varpi 2}^{\varpi 1} \frac{d\varpi}{\sqrt{1-k\varpi^2}}$$ ← For a *photon*

$$d_p(t) = R(t) \int_0^{\varpi} \frac{d\varpi'}{\sqrt{1-k\varpi'^2}} = R(t) \int_{t_e}^{t_0} \frac{c\,dt'}{R(t')}$$

$$\boxed{d_h(t) = R(t) \int_0^t \frac{c\,dt'}{R(t')}}$$

Radiation dominated flat universe: $R \propto t^{1/2}$ → $d_h(t) = 2ct$

Matter dominated flat universe: $R \propto t^{2/3}$ → $d_h(t) = 3ct$

Matter dominated flat universe in terms of *redshift* → $d_h(z) = \frac{2c}{H_0\sqrt{\Omega_{m,0}}} \frac{1}{(1+z)^{3/2}}$

Including Ω_Λ → $d_h(t) = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2} H_0 t \sqrt{\Omega_{\Lambda,0}}\right) \int_0^t \frac{c\,dt'}{\left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3} \sinh^{2/3}\left(\frac{3}{2} H_0 t' \sqrt{\Omega_{\Lambda,0}}\right)}$

= 14.6 Gpc (WMAP)

[29.158]

The paths of photons in terms of proper distance.

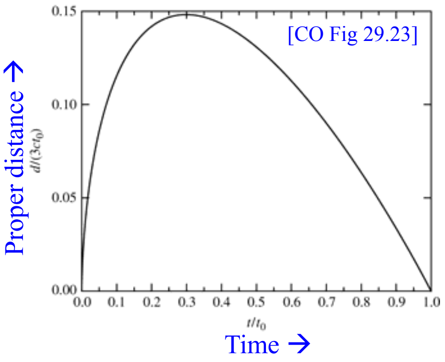
$$\cancelto{0}{(ds)^2} = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + \cancelto{0}{(\varpi\, d\theta)^2} + \cancelto{0}{(\varpi \sin\theta\, d\phi)^2}\right]$$

Matter dominated flat universe:

$$\int_0^t \frac{c\,dt'}{R(t')} = \int_{\varpi}^{\varpi_e} d\varpi'$$

$$R(t) = \left(\frac{3}{2}\right)^{\frac{2}{3}}\left(\frac{t}{t_H}\right)^{\frac{2}{3}} = \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

$$\varpi = \varpi_e - 3ct_0\left(\frac{t}{t_0}\right)^{1/3}.$$



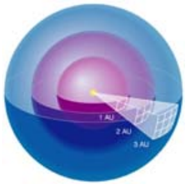
At $t = t_o$, $\varpi = 0$ → $\varpi_e = 3ct_0$

Proper distance:

$$R(t)\varpi = d_p(t) = 3ct_0\left[\left(\frac{t}{t_0}\right)^{2/3} - \left(\frac{t}{t_0}\right)\right]$$

[29.165]

Luminosity Distance



$$F = \frac{L}{4\pi d^2}$$

$$F = \frac{L}{4\pi \varpi^2 (1+z)^2}$$

Redshift → (1+z)
Time dilation → (1+z)

$$d_L = \varpi(1+z)$$

We need a ϖ - z relation.

In principle:

- R-W metric:

$$\int_{t_e}^{t_0} \frac{c\,dt}{R(t)} = \int_0^{\varpi_e} \frac{d\varpi}{\sqrt{1-k\varpi^2}}$$

$$\int_{R(t_e)}^{R(t_0)} \frac{c\,dR}{R \times {dR}/{dt}} = \int_0^{\varpi_e} \frac{d\varpi}{\sqrt{1-k\varpi^2}}$$

- Use Friedmann Eqn to find dR/dt
- Then: $R(t) = \frac{1}{1+z}$

In practice:
[CO pg. 1209]

$$I(z) = H_0 \int_{\frac{1}{1+z}}^{1} \frac{dR}{R(dR/dt)} = H_0 \int_0^z \frac{dz'}{H(z')}$$

$$I(z) \equiv \int_0^z \frac{dz'}{\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{\text{rel},0}(1+z')^4 + \Omega_{\Lambda,0} + (1-\Omega_0)(1+z')^2}}$$

$$d_{p,0}(z) = \frac{c}{H_0} I(z)$$

$$\varpi(z) = \frac{c}{H_0} S(z)$$

$$S(z) \equiv I(z) \qquad (\Omega_0 = 1)$$

$$\equiv \frac{1}{\sqrt{\Omega_0 - 1}} \sin\left[I(z)\sqrt{\Omega_0 - 1}\right] \qquad (\Omega_0 > 1)$$

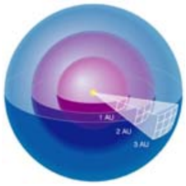
$$\equiv \frac{1}{\sqrt{1 - \Omega_0}} \sinh\left[I(z)\sqrt{1 - \Omega_0}\right] \qquad (\Omega_0 < 1),$$

$$I(z) = \int_0^z \left\{1 - (1+q_0)z' + \left[\frac{1}{2} + 2q_0 + \frac{3}{2}q_0^2 + \frac{1}{2}(1-\Omega_0)\right]z'^2 + \cdots\right\} dz'$$

$$I(z) = z - \frac{1}{2}(1+q_0)z^2 + \left[\frac{1}{6} + \frac{2}{3}q_0 + \frac{1}{2}q_0^2 + \frac{1}{6}(1-\Omega_0)\right]z^3 + \cdots$$

$$\boxed{\varpi \simeq \frac{cz}{H_0}\left[1 - \frac{1}{2}(1+q_0)z\right] \qquad (\text{for } z \ll 1).}$$

Luminosity Distance



$$F = \frac{L}{4\pi d^2}$$

$$F = \frac{L}{4\pi \varpi^2 (1+z)^2}$$

Redshift → (1+z)
Time dilation → (1+z)

$$d_L = \varpi(1+z)$$

About right…

From previous slide:

$$\varpi \simeq \frac{cz}{H_0}\left[1 - \frac{1}{2}(1+q_0)z\right] \qquad (\text{for } z \ll 1).$$

$$d_L(z) \simeq \frac{cz}{H_0}\left[1 + \frac{1}{2}(1-q_0)z\right] \qquad (\text{for } z \ll 1).$$

In practice
(because of that @#$% cosmological constant)

$$d_L(z) = \frac{c}{H_0}(1+z)S(z)$$

$$S(z) \equiv I(z) \qquad (\Omega_0 = 1)$$

$$\equiv \frac{1}{\sqrt{\Omega_0 - 1}} \sin\left[I(z)\sqrt{\Omega_0 - 1}\right] \qquad (\Omega_0 > 1)$$

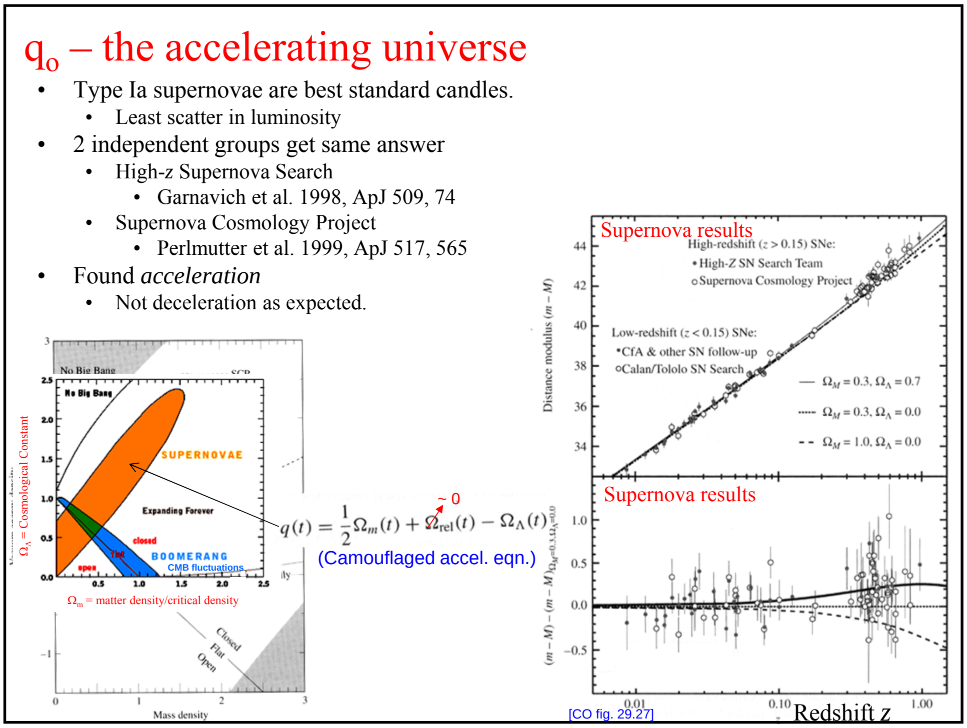
$$\equiv \frac{1}{\sqrt{1 - \Omega_0}} \sinh\left[I(z)\sqrt{1 - \Omega_0}\right] \qquad (\Omega_0 < 1),$$

$$\boxed{m - M = 5\log_{10}(d_L/10\text{ pc})}$$

$$m - M \simeq 5\log_{10}\left[\frac{c}{(100\text{ km s}^{-1}\text{ Mpc}^{-1})(10\text{ pc})}\right] - 5\log_{10}(h) + 5\log_{10}(z) + 5\log_{10}\left[1 + \frac{1}{2}(1-q_0)z\right] \qquad (\text{for } z \ll 1)$$

$$m - M = 5\log_{10}\left[\frac{c}{(100\text{ km s}^{-1}\text{ Mpc}^{-1})(10\text{ pc})}\right] - 5\log_{10}(h) + 5\log_{10}(1+z) + 5\log_{10}[S(z)]$$
$$= 42.38 - 5\log_{10}(h) + 5\log_{10}(1+z) + 5\log_{10}[S(z)].$$

$$\boxed{m - M \simeq 42.38 - 5\log_{10}(h) + 5\log_{10}(z) + 1.086(1-q_0)z \qquad (\text{for } z \ll 1).}$$ [CO 29.188]



q_o – the accelerating universe

- Type Ia supernovae are best standard candles.
  - Least scatter in luminosity
- 2 independent groups get same answer
  - High-*z* Supernova Search
    - Garnavich et al. 1998, ApJ 509, 74
  - Supernova Cosmology Project
    - Perlmutter et al. 1999, ApJ 517, 565
- Found *acceleration*
  - Not deceleration as expected.

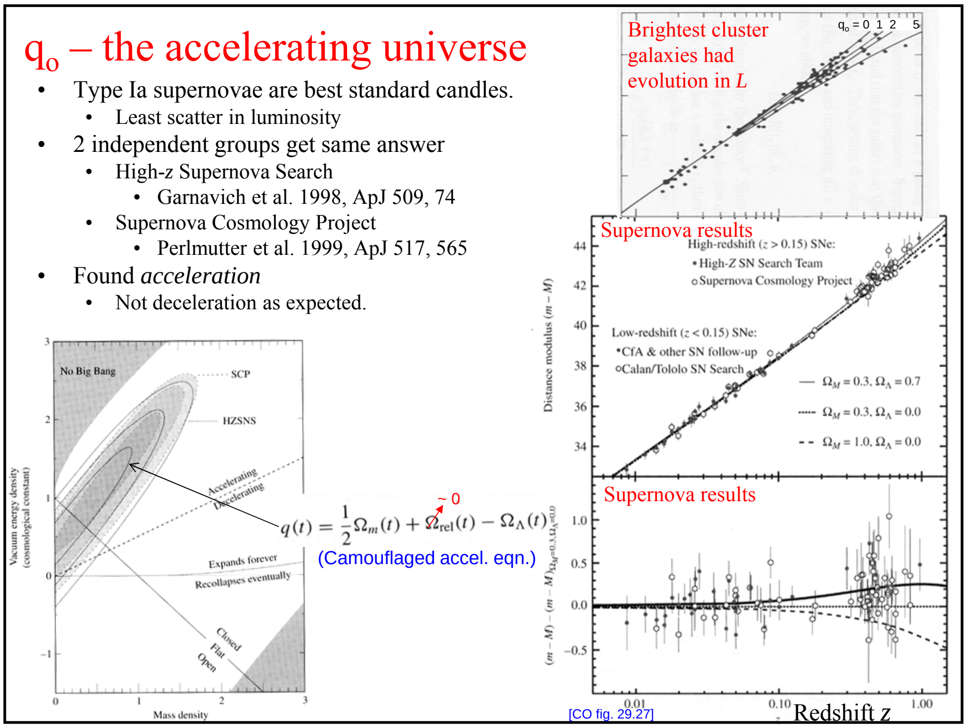
$$q(t) = \frac{1}{2}\Omega_m(t) + \overset{\sim 0}{\cancel{\Omega_{\rm rel}}}(t) - \Omega_\Lambda(t)$$

(Camouflaged accel. eqn.)

Supernova results

Supernova results

[CO fig. 29.27]



q_o – the accelerating universe

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Brightest cluster galaxies had evolution in *L*

$$q(t) = \frac{1}{2}\Omega_m(t) + \overset{\sim 0}{\cancel{\Omega_{\rm rel}}}(t) - \Omega_\Lambda(t)$$

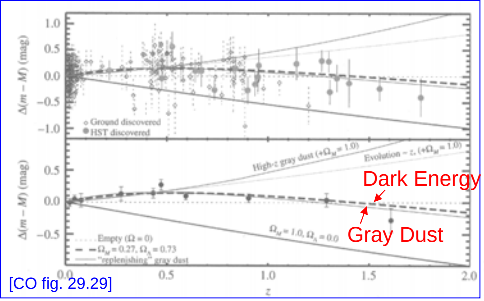
(Camouflaged accel. eqn.)

Supernova results

Supernova results

[CO fig. 29.27]

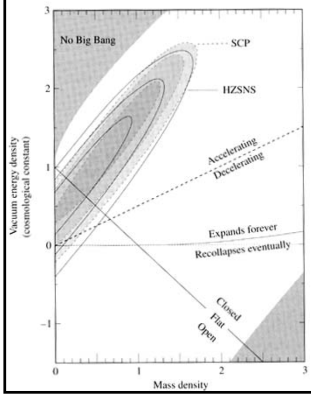
q_o



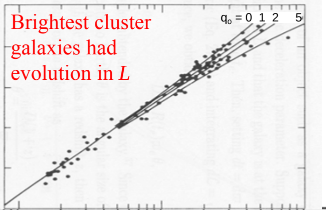
Dark Energy

Gray Dust

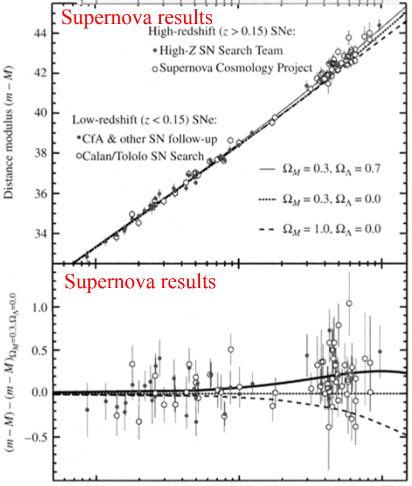
[CO fig. 29.29]



Joint Dark Energy Mission (JDEM)



Brightest cluster galaxies had evolution in *L*



Supernova results

Supernova results

[CO fig. 29.27]

Redshift *z*