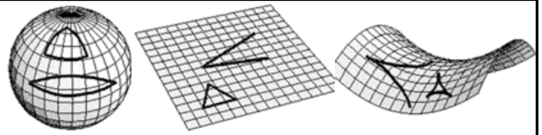
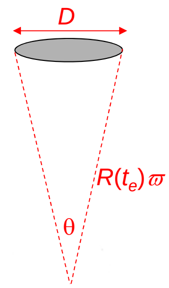
## Angular Diameters



RW metric:

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]$$

What is angular size of galaxy at co-moving distance $\varpi$?




$$dt = d\varpi = d\phi = 0$$

Galaxy's diameter is proper ~~distance~~ linear diameter:

$$D = \int \sqrt{-(ds)^2} = R(t_e)\,\varpi_e\,\theta$$

Using $\varpi$ coordinate
→ Looks like Euclidean result, regardless of curvature of space.

$$\Rightarrow \theta = \frac{D}{R(t_e)\,\varpi}$$

but must use $R(t_e)$

$$= \frac{D(1+z)}{\varpi} \qquad \text{USING} \quad 1+z = \frac{1}{R(t_e)}$$

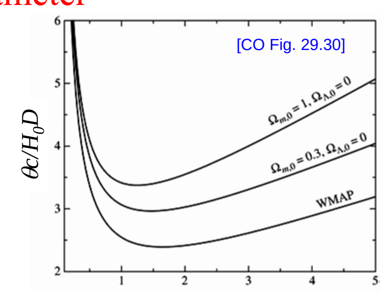
$$\theta = \frac{D(1+z)^2}{d_L} \qquad d_A = \frac{d_L}{(1+z)^2} = \frac{\varpi}{1+z}$$

## More angular diameter

$$\theta = \frac{D(1+z)^2}{d_L}$$

In practice
(because of that @#$% cosmological constant)

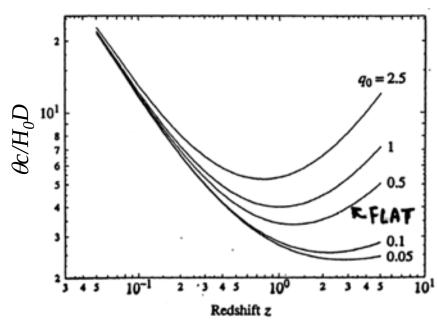
$$\frac{c\theta}{H_0 D} = \frac{(1+z)}{S(z)} \qquad \boxed{29.193}$$



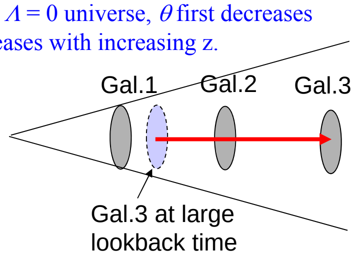

For $\Lambda = 0$:

$$\theta = \frac{H_0 D}{c}\,\frac{q_0^2(1+z)^2}{q_0 z - (1-q_0)(\sqrt{1+2q_0 z}-1)}$$

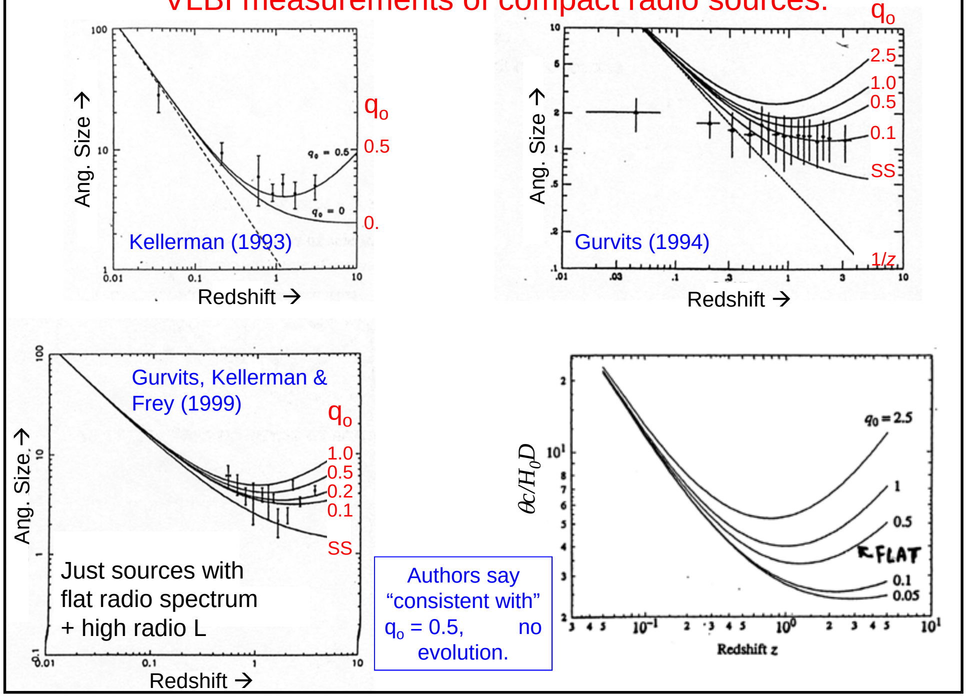
*Surprise!*
Even for flat, $\Lambda = 0$ universe, $\theta$ first decreases but then increases with increasing z.




Competing Effects:
- Distance
- Expansion

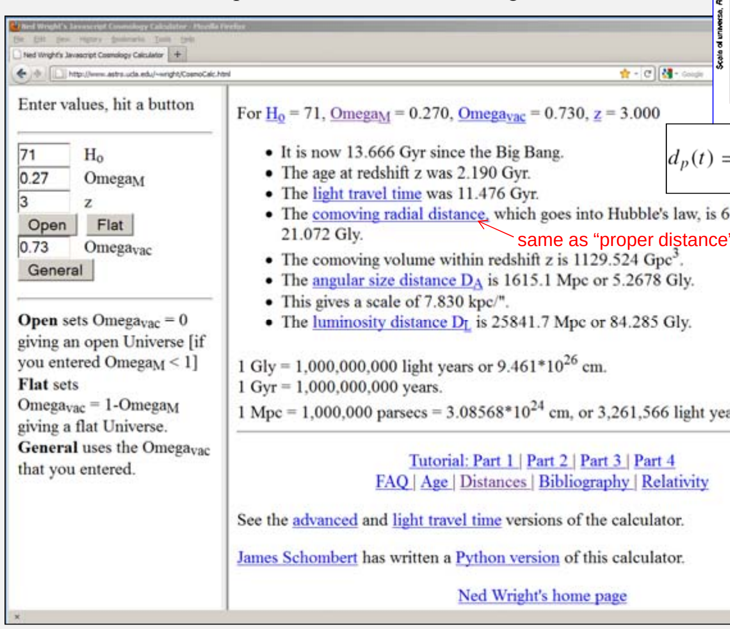


## VLBI measurements of compact radio sources:




## Ned Wright's Javascript Cosmology Calculator

http://www.astro.ucla.edu/~wright/CosmoCalc.html


Enter values, hit a button

71 $H_0$
0.27 $\text{Omega}_M$
3 z
Open Flat
0.73 $\text{Omega}_{vac}$
General

**Open** sets $\text{Omega}_{vac}$ = 0 giving an open Universe [if you entered $\text{Omega}_M$ < 1]
**Flat** sets $\text{Omega}_{vac}$ = 1-$\text{Omega}_M$ giving a flat Universe.
**General** uses the $\text{Omega}_{vac}$ that you entered.

For $H_o$ = 71, $\text{Omega}_M$ = 0.270, $\text{Omega}_{vac}$ = 0.730, z = 3.000

- It is now 13.666 Gyr since the Big Bang.
- The age at redshift z was 2.190 Gyr.
- The light travel time was 11.476 Gyr.
- The comoving radial distance, which goes into Hubble's law, is 6460.6 Mpc or 21.072 Gly.
- The comoving volume within redshift z is 1129.524 $\text{Gpc}^3$.
- The angular size distance $D_A$ is 1615.1 Mpc or 5.2678 Gly.
- This gives a scale of 7.830 kpc/".
- The luminosity distance $D_L$ is 25841.7 Mpc or 84.285 Gly.

1 Gly = 1,000,000,000 light years or $9.461 \times 10^{26}$ cm.
1 Gyr = 1,000,000,000 years.
1 Mpc = 1,000,000 parsecs = $3.08568 \times 10^{24}$ cm, or 3,261,566 light yea

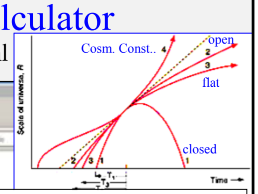
Tutorial: Part 1 | Part 2 | Part 3 | Part 4
FAQ | Age | Distances | Bibliography | Relativity

See the advanced and light travel time versions of the calculator.

James Schombert has written a Python version of this calculator.

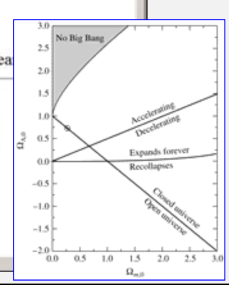
Ned Wright's home page

same as "proper distance"




$$d_p(t) = R(t)\int_0^{\varpi} \frac{d\varpi'}{\sqrt{1-k\varpi'^2}}$$

$$D_A = \frac{\varpi}{1+z}$$
$$D_L = \varpi(1+z)$$



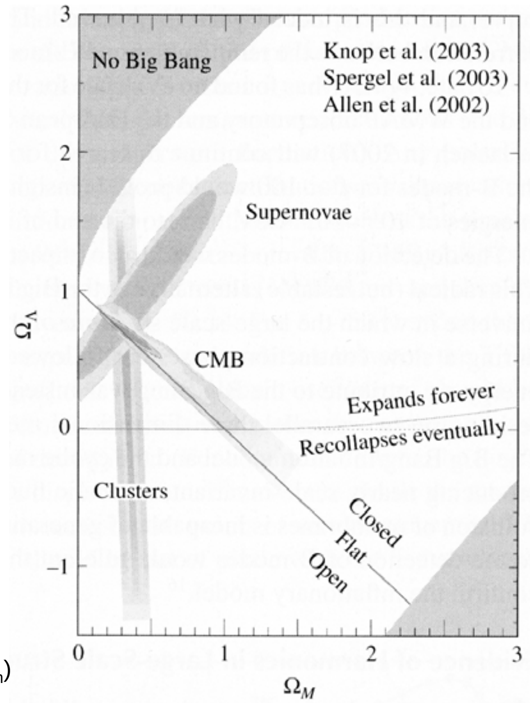
# The Concordance Cosmology

## = ΛCDM = LCDM

Dark Energy (Λ) — Cold Dark Matter (CDM)

Concordance between:
- CMB fluctuations.
- Supernovae.
- Galaxy cluster growth rate

- Globular cluster ages
- Power spectrum of large-scale structure.
- $H_0$ : HST key project vs. WMAP.
- Baryon density:
  primordial nucleosynthesis
  vs. WMAP.
- $\Omega_m$ from $\Omega_{baryon} \times (\rho_{dark\ matter} / \rho_{baryon})$

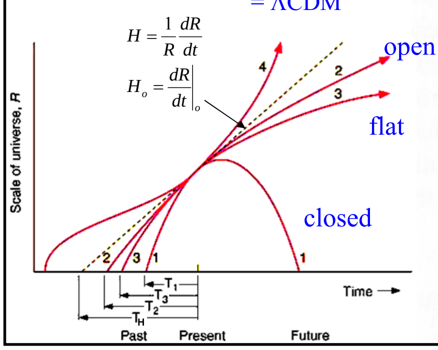


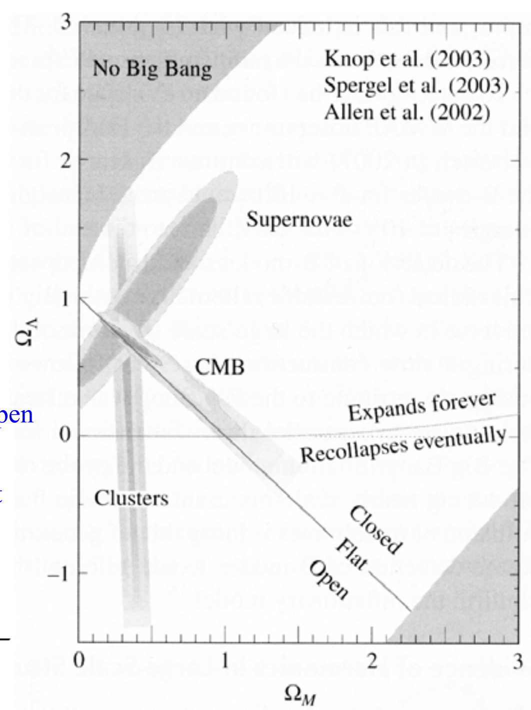

# The Concordance Cosmology

## = ΛCDM = LCDM

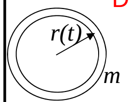
Dark Energy (Λ) — Cold Dark Matter (CDM)

Concordance
= ΛCDM



Definitions, results, etc.




* $r = R(t)\,\varpi$
* $H = \frac{1}{R}\frac{dR}{dt}$

*Densities:*

* Matter: $\rho_m = \rho_{o,m} R^{-3}$
* Radiation: $\rho_r = \rho_{o,r} R^{-4}$
* Dark energy: $\rho_\Lambda = \rho_{o,\Lambda} R^{0}$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

* $\Omega(t) = \frac{\rho(t)}{\rho_c(t)}$
* $\Omega \equiv \Omega_m + \Omega_{\rm rel} + \Omega_\Lambda$

$$\frac{d(R^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(R^3)}{dt}$$

$$q(t) = -\frac{R(t)\left[d^2R(t)/dt^2\right]}{\left[dR(t)/dt\right]^2}$$

* $P = wu = w\rho c^2$

Physics

*Per unit mass:*

K.E. + potential E. = Total Energy

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

$\rho = \frac{u}{c^2}$ *

* Temp. of radiation field: $T_0 = RT(R)$

* $$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]$$

* $$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2$$

Cosmological Constant (a.k.a. *Dark Energy*)

Curvature $k = \frac{1}{\Re^2} \times \begin{matrix} +1 \\ 0 \\ -1 \end{matrix}$

$dU = -PdV$

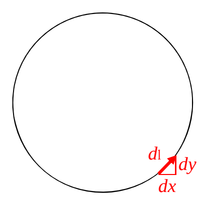
$$\frac{d^2R}{dt^2} = \left\{-\frac{4}{3}\pi G\left[\rho_m + \rho_{\rm rel} + \rho_\Lambda + \frac{3(P_m + P_{\rm rel} + P_\Lambda)}{c^2}\right]\right\}R$$

* = you should be able to write these down from memory.

# Homework 5 Question 1

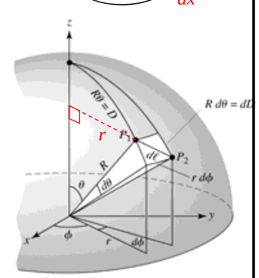
Circle in a 2D space:

$\Re^2 = x^2 + y^2$ $\qquad d\ell^2 = dx^2 + dy^2$




Sphere in a 3D space:

$\Re^2 = x^2 + y^2 + z^2$ $\qquad d\ell^2 = dx^2 + dy^2 + dz^2$




3-Sphere in a 4D space:

$\Re^2 = x^2 + y^2 + z^2 + w^2$ $\qquad d\ell^2 = dx^2 + dy^2 + dz^2 + dw^2$

But we (mysteriously) had to use:

$$r^2 = x^2 + y^2 + z^2 \qquad d\ell^2 = dx^2 + dy^2 + dz^2 = dr^2 + (r\,d\theta)^2 + (r\sin\theta\, d\phi)^2$$

… to derive the R-W metric:

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]$$

## All Universes ~ "flat" ($\rho \sim \rho_c$) at early times.

- Homework problem 29.9 showed:

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} = 1 + \frac{kc^2}{(dR/dt)^2} \qquad (29.194)$$

and that $dR/dt \to \infty$ as $t \to 0$

implying $\rho(t) \to \rho_c(t)$ as $t \to 0$ for all values of $k$.

## Consequences:

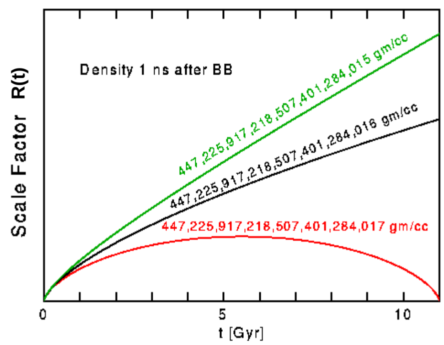
For small $t$, it is OK to use:

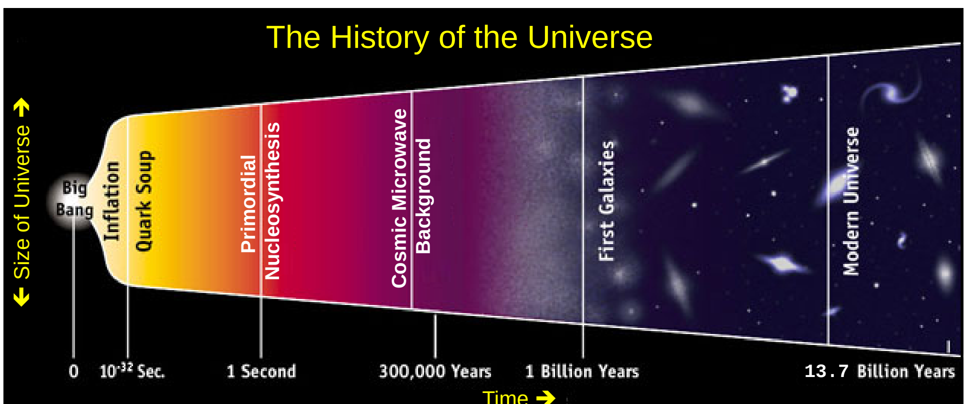
$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = 0$$

To show dr/dt → ∞

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G\rho_0}{3R} - kc^2$$

and R → 0 as t → 0




| Planck time | Inflation | Formation of H, He, Li | Decoupling of CMB | Galaxy Formation | Now |
|---|---|---|---|---|---|

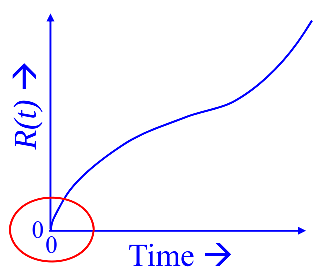
Hot ($10^{32}$ K) → ($10^9$ K) → (3000 K) → Cool (3 K)

High density → Low density

The rest of these slides will probably wait until after Midterm 2

# The Planck Time

[CO pgs 1233-1234]




- Dimensional arguments
  - Planck time $t_P = \sqrt{\frac{\hbar G}{c^5}}$ = 5 x $10^{-44}$ s
  - Planck mass $m_P = \sqrt{\frac{\hbar c}{G}}$ = 2 x $10^{-8}$ kg
  - Planck length $\ell_P = \sqrt{\frac{\hbar G}{c^3}}$ = 2 x $10^{-35}$ m $= ct_P$
- Before this, everything fuzzed out by uncertainty principle.

$$\Delta t\, \Delta E = \hbar$$
$$\Delta x\, \Delta p = \hbar$$
$$\Delta p\, c = \Delta E = \frac{GM^2}{\Delta x}$$

# Some Problems for
# Friedmann-Robertson-Walker Universes

- Causality and the particle horizon
- Flatness

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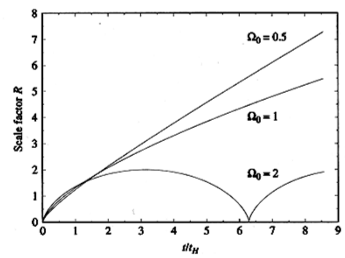
- Absence of magnetic monopoles
- Absence of “Domain Walls”




# The Horizon Problem

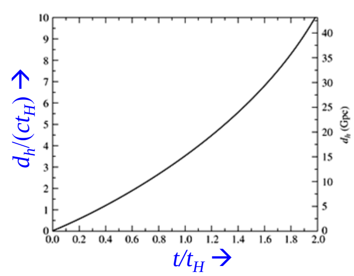



Fig. 29.22
Proper distance from Earth to particle horizon as function of time, including $\Lambda$.

## The Particle Horizon:

For $k = 0$, $\Lambda = 0$, $\Omega = 1$ example:

- Radiation era: $R(t) \sim t^{1/2}$ $\quad d_h(t) = 2ct$ $\quad \varpi_h(t) = d_h(t)/R(t) \sim t^{1/2}$
- Matter Era: $R(t) \sim t^{2/3}$ $\quad d_h(t) = 3ct$ $\quad \varpi_h(t) = d_h(t)/R(t) \sim t^{1/3}$

As time passes, we can see larger and larger fraction of universe.

➔ *causally connected fraction of universe is constantly growing.*

## The Horizon Problem

- Cosmic Microwave Background is smooth to about 1 part in $10^5$
- Yet regions in causal contact at time of decoupling should subtend only ~$2^o$ on sky.
- How do regions $180^o$ apart know about each other?

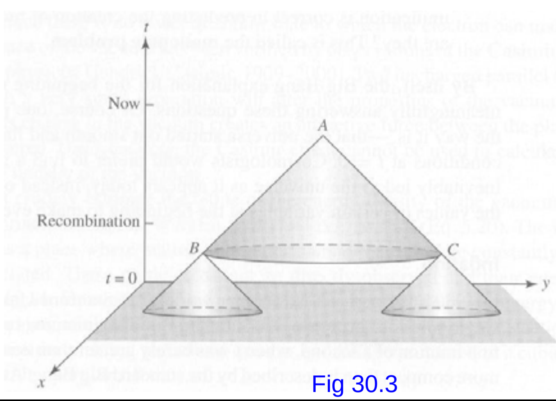
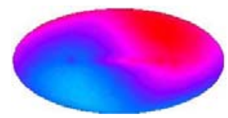
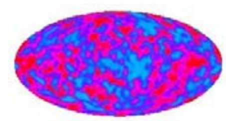



Fig 30.3

Blue = $0^oK$
Red = $4^oK$



Blue = $2.724^oK$
Red = $2.732^oK$
Dipole Anistropy
~ 1 part in 300



After removing dipole
Red – blue = $0.0002^oK$
~ 1 part in $10^5$

## The Flatness Problem

- Tiny departures from ($\rho = \rho_c$) at small $t$ (large $z$) grow into much larger departures than are observed.
- $\Omega_0$ close to 1 at present time.
  - But this requires incredible precision at start (t = 0).
  - ➔ $\Omega_0$ *exactly* = 1

$$\underbrace{\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)}_{}R^2 = -kc^2$$

*Flat* ➔ these add up to zero.

$$\frac{dR}{dt} = \left(\cancelto{0}{\frac{8\pi G\rho_0}{3R}} - kc^2\right)^{1/2}$$

Empty *expanding* U. is not flat (k ≠ 0).