Dark Matter
so far

Mass/Luminosity

- Local stellar luminosity function: $M/L = 0.67$
- Our Galaxy, at larger scales:
  - Local motions $\perp$ disk (Oort limit): $M/L \sim 3\text{-}5$
  - MW Rotation curve > 30
  - Escape speed > 30
  - Pop II dynamics (glob. clusters, etc.) ~ 27
  - Flat rotation curves in other spirals > 20
  - E galaxy virial theorem 9

Virial Theorem for Clusters

- Galaxy clusters – "fair samples" of the universe.
- Coma is closest relaxed cluster
- Original mass measurement was by Zwicky (1933).

$$M_{virial} = \frac{5R\sigma_r^2}{G} = 3\times10^{15}$$

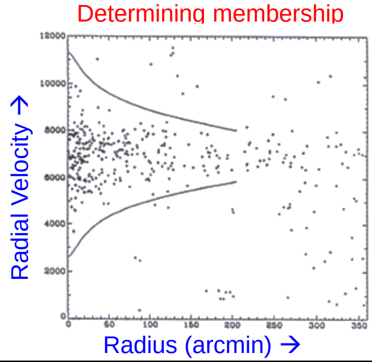
Virial Theorem
$2K = -U$
[CO 2.4], and pp. 959-962

- Measure $n(r)$, $\sigma_v(r)$
  $n(r)$ = # of galaxies,
  $\sigma_v(r)$ = vel. Dispersion

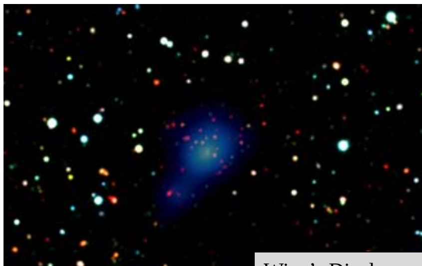
- Fit to models based on
  collisionless Boltzmann eq.
  ~ isothermal, non-spherical.

- Coma: $M = 2\text{x}10^{15}\, M_\odot$
  $M/L = 360h$ (+0, -180h)
- Perseus: $M/L = 600h$

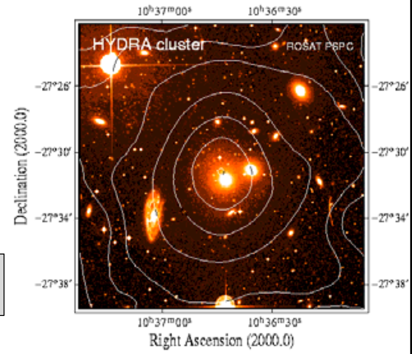
Determining membership



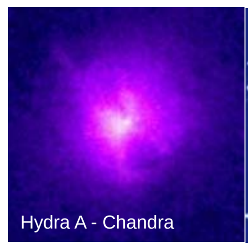
X-ray emitting gas in clusters

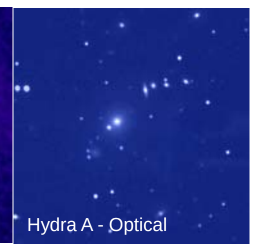


Wien's Displacement Law
$\lambda_{max} T = .003$ m K [CO eq. 3.15]



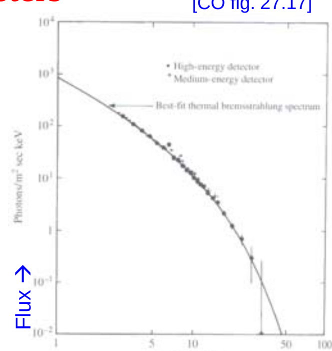
10^{-10}m ➔ 3×10^{7}K
(x-ray)





X-ray emitting gas in clusters

[CO fig. 27.17]



T ~ 10^7K gas is important mass component of cluster

- emission by thermal bremsstrahlung (free-free).
- $L_X \sim 10^{43}$ - 10^{45} erg/s (5x10^{44} erg/s for Coma)

amplitude, freq. distr. [CO eq. 27.18]

$$\ell_\nu \, d\nu = 5.44 \times 10^{-52} \underbrace{(4\pi n_e^2)}_{\text{amplitude}} T^{-1/2} \underbrace{e^{-h\nu/kT}}_{\text{freq. distr.}} \, d\nu \ \text{W m}^{-3}$$

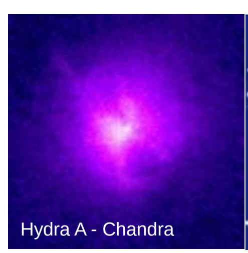
$$L_{total} = \frac{4}{3}\pi R^3 \int \ell_\nu d\nu = \frac{4}{3}\pi R^3 \times 1.42 \times 10^{-40} n_e^2 T^{1/2} \text{ W}$$

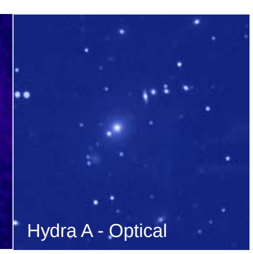
[CO eq. 27.19]

Measure L_X, R, T
Solve for n_e = electron density (electrons m^{-3})
= H nuclei m^{-3}
Mass = $n_e \times m_H \times$ *volume*

- M_{gas} = $(4/3)\,\pi R^3 n_e m_H$ = 1×10^{14} $M_\odot$
- M_{stars} = $(M/L)_{Local} L_V$ = 1.5×10^{13} $M_\odot$

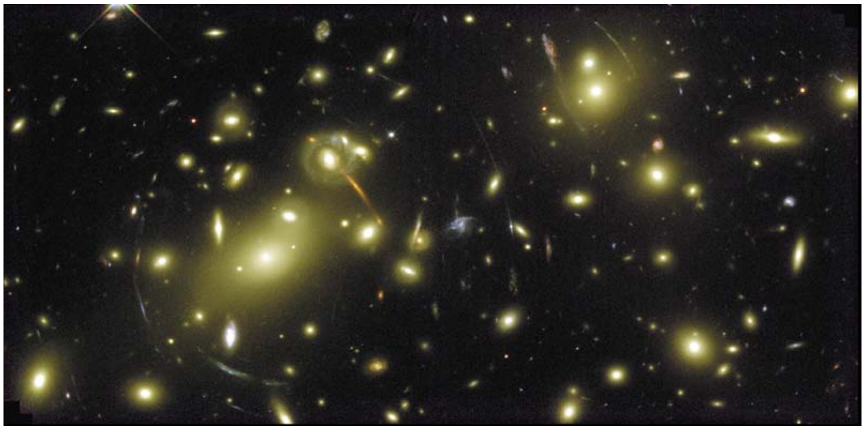
10x more baryons in hot intergalactic gas than in stars





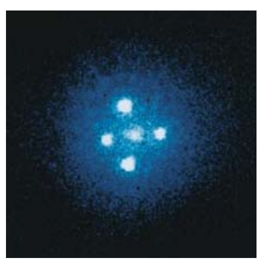
But still factor of ~10 short…

Gravitational Lensing



- Foreground cluster distorts images of numerous background galaxies.
- Use to determine total mass of foreground cluster.
- Shows that 85% of mass is Dark Matter.

Gravitational Lensing



Robert Frost

TWINKLE, TWINKLE LITTLE STAR
HOW I WONDER WHERE YOU ARE.
"1.75 SECONDS OF ARC FROM WHERE I SEEM TO BE
FOR $ds^2 \simeq (1 - 2GM/r^2)dt^2 - (1 + 2GM/r)dr^2 - r^2 d\theta^2 - r^2 sin^2\, \theta\, d\phi^2$."

Source unknown

Gravitational Lensing

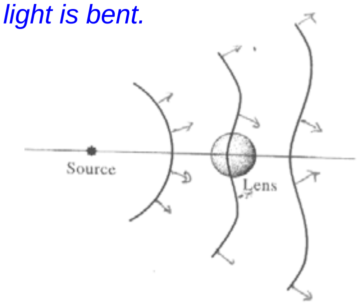
The Schwarzschild metric:

$$(ds)^2 = \left(c\,dt\sqrt{1-2GM/rc^2}\right)^2 - \left(\frac{dr}{\sqrt{1-2GM/rc^2}}\right)^2 - (r\,d\theta)^2 - (r\sin\theta\,d\phi)^2$$

For light: $ds = 0$

$$\frac{dr}{dt} = c\left(1-\frac{2GM}{rc^2}\right) \quad \text{[CO 17.28]}$$

➔ *Wavefront is retarded near a massive object.*
➔ *path of light is bent.*



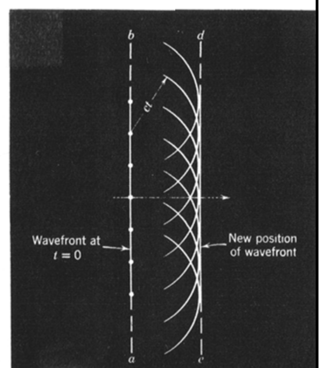
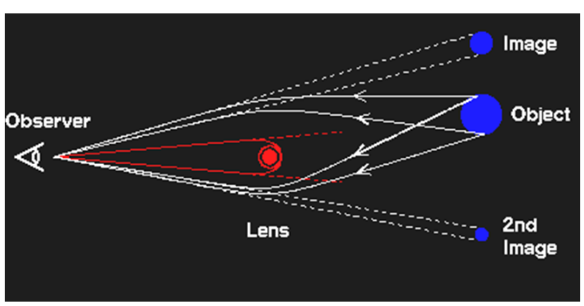
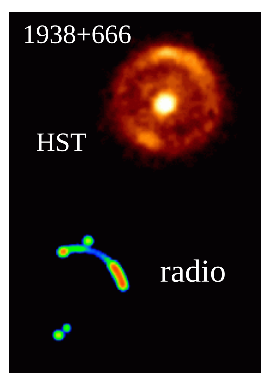
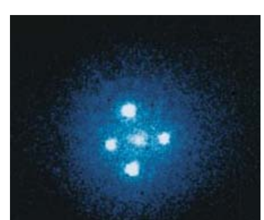


Fig. 41–7 The propagation of a plane wave in free space is described by the Huygens construction. Note that the ray (horizontal arrow) representing the wave is perpendicular to the wavefronts.

Gravitational Lenses



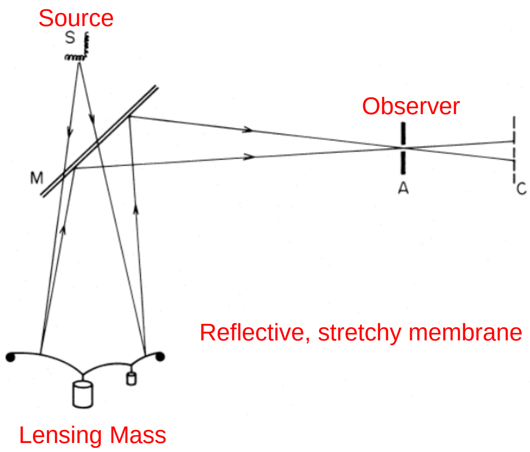




Galaxy at center causes 4 images of same quasar.

Gravitational Lens Simulator

Blandford & Narayan 1986 ApJ, 310, 568



Gravitational Lensing by a Point Mass

[CO Sect. 28.4]

Angle of deflection of photon: $\phi = \frac{4GM}{r_0 c^2}$ (28.20)

From Schw. Metric.

$$\Rightarrow \theta^2 - \beta\theta - \frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right) = 0 \quad \text{(from trig)} \quad (28.21)$$

The Quadratic Eqn.
$ax^2 + bx + c = 0$
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic eq. in θ $\Rightarrow$ 2 solutions θ_1, θ_2

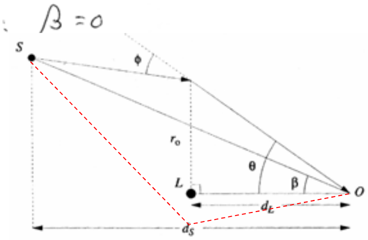
$$\beta = \theta_1 + \theta_2$$

$$M = -\frac{\theta_1 \theta_2 c^2}{4G}\left(\frac{d_S d_L}{d_S - d_L}\right)$$

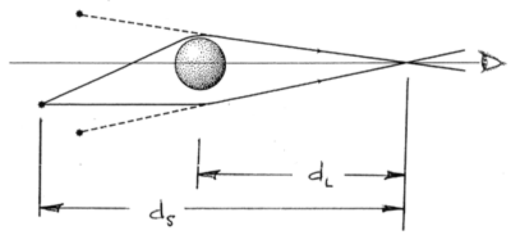
If lens is exactly on line of sight to source: $\beta = 0$

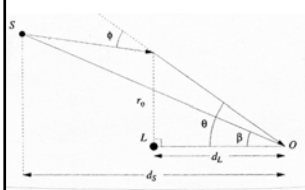
image is Einstein Ring

(28.24) with $\theta_E = \sqrt{\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)}$



Point mass forms two images (or ring)



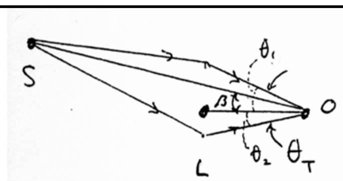


- For sun: rays intersect at $d_L \sim 50$ ly
- For $d_S >> d_L$

$$\theta_E = \sqrt{\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)} = \sqrt{\frac{4GM}{c^2 d_L}}$$

- For stars in Milky Way:
  - $M = 1\ M_{sun}$, $d_L = 10^4$ ly ==> $\theta_E \sim 2 \times 10^{-3}$ arcsec
- For external galaxies
  - $M = 10^{11}\ M_{sun}$, $d_L = 10^{10}$ ly ==> $\theta_E \sim 1$ arcsec
- Need $\beta < \theta_E$ to see multiple images (strong lensing)

Effect of Lensing on Flux



See Refsdal (1964) MNRAS 128, 295

Use [CO] notation, but also define

$\theta_T = |\theta_1| + |\theta_2|$ = total separation between images

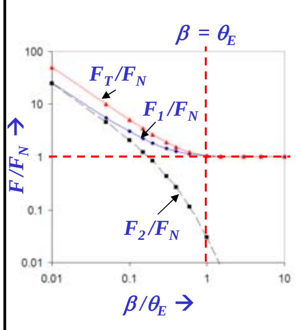
$$\theta_T = \sqrt{\theta_E^2 + \beta^2}$$

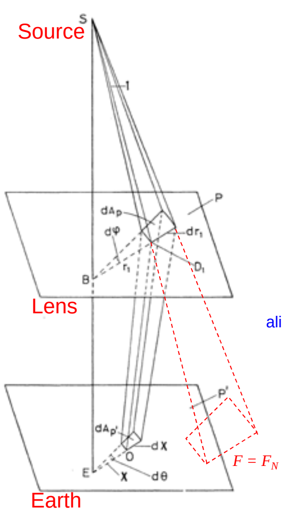
F_n = no lensing

$$F_1 = \frac{1}{4}\left(2 + \frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right) F_N$$

$$F_2 = \frac{1}{4}\left(-2 + \frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right) F_N$$

$$F_{TOTAL} = F_1 + F_2 = \frac{1}{2}\left[\frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right] F_N$$

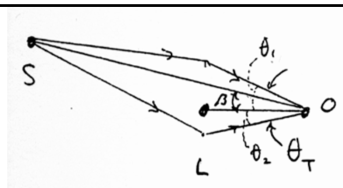




	β/θ_E	F_1/F_N	F_2/F_N	F_T/F_N
Not aligned	10	1·0000063	0·0000063	1·000013
	5	1·0001	0·0001	1·0002
	3	1·0014	0·0014	1·0028
	1·5	1·0084	0·0084	1·017
$\beta = \theta_E$	1	1·030	0·030	1·06
	0·6	1·116	0·116	1·23
	0·4	1·27	0·27	1·54
	0·3	1·44	0·44	1·88
	0·2	1·83	0·83	2·66
	0·15	2·23	1·23	3·46
	0·1	3·04	2·04	5·08
Close	0·05	5·52	4·52	10·0
alignment	0·01	25·5	24·5	50·0

Effect of Lensing on Flux

See Refsdal (1964) MNRAS 128, 295



Use [CO] notation, but also define

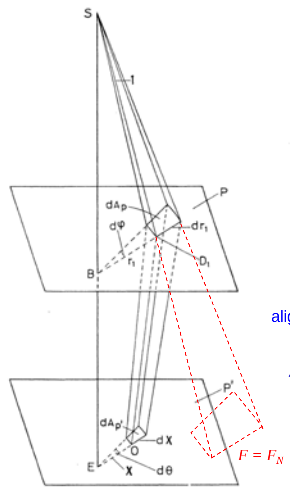
$\theta_T = |\theta_1| + |\theta_2| = $ total separation between images

$$\theta_T = \sqrt{\theta_E^2 + \beta^2}$$

$$F_1 = \frac{1}{4}\left(2 + \frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right) F_N$$

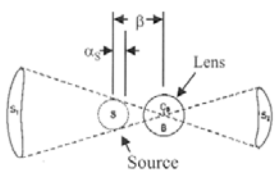
$$F_2 = \frac{1}{4}\left(-2 + \frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right) F_N$$

$$F_{TOTAL} = F_1 + F_2 = \frac{1}{2}\left[\frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right] F_N$$



Lensing of Extended Sources

- Image has same surface brightness as unlensed image, but more area.
- Ring if $\beta < \alpha_S$
- Arcs if $\beta > \alpha_S$
- Max amplification when $\beta = 0$ $\sim \theta_E/\alpha_S$



	β/θ_E	F_1/F_N	F_2/F_N	F_T/F_N
Not aligned	10	1·0000063	0·0000063	1·000013
	5	1·0001	0·0001	1·0002
	3	1·0014	0·0014	1·0028
	1·5	1·0084	0·0084	1·017
$\beta = \theta_E$	1	1·030	0·030	1·06
	0·6	1·116	0·116	1·23
	0·4	1·27	0·27	1·54
	0·3	1·44	0·44	1·88
	0·2	1·83	0·83	2·66
	0·15	2·23	1·23	3·46
	0·1	3·04	2·04	5·08
Close	0·05	5·52	4·52	10·0
alignment	0·01	25·5	24·5	50·0

Caustics & Catastrophes

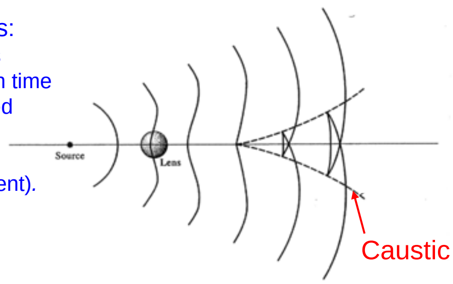
Lensing by a Transparent Mass Distribution

$$(ds)^2 = \left(c\,dt\sqrt{1-2GM/rc^2}\right)^2 - \left(\frac{dr}{\sqrt{1-2GM/rc^2}}\right)^2 - (r\,d\theta)^2 - (r\sin\theta\,d\phi)^2$$

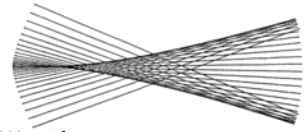
Wavefront retarded by gravitational field: $\frac{dr}{dt} = c\left(1 - \frac{2GM}{rc^2}\right)$

For a transparent mass distribution with Grav Pot = Φ: $(ds)^2 \approx \left(c\,dt\sqrt{1+2\Phi/c^2}\right)^2 - \left(\sqrt{1-2\Phi/c^2}\right)^2\left(dx^2+dy^2+dz^2\right)$

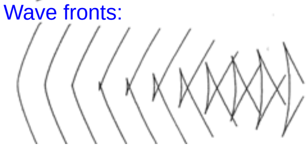
Caustic Surfaces:
Number of images changes by 2 each time a caustic is crossed
➔always an odd number
(*if* lens is transparent).



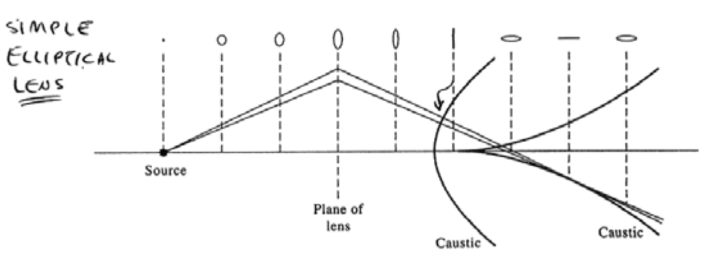
Rays:



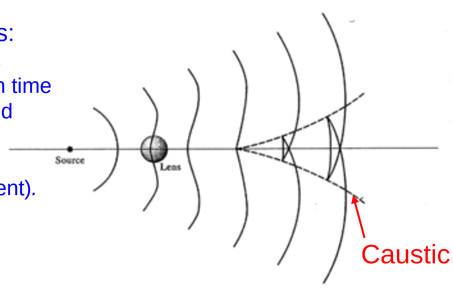
Wave fronts:



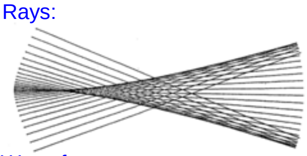
Lensing by a Transparent Mass Distribution



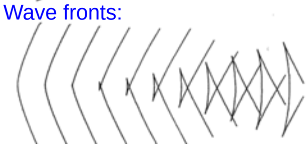
Caustic Surfaces:
Number of images changes by 2 each time a caustic is crossed
➔always an odd number
(*if* lens is transparent).

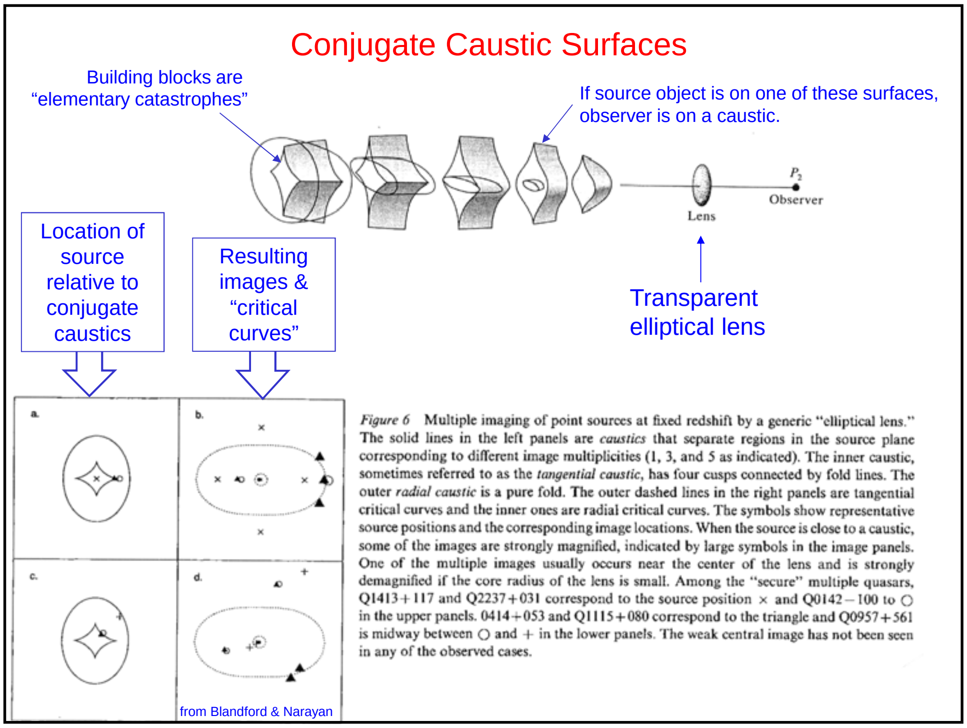


Rays:

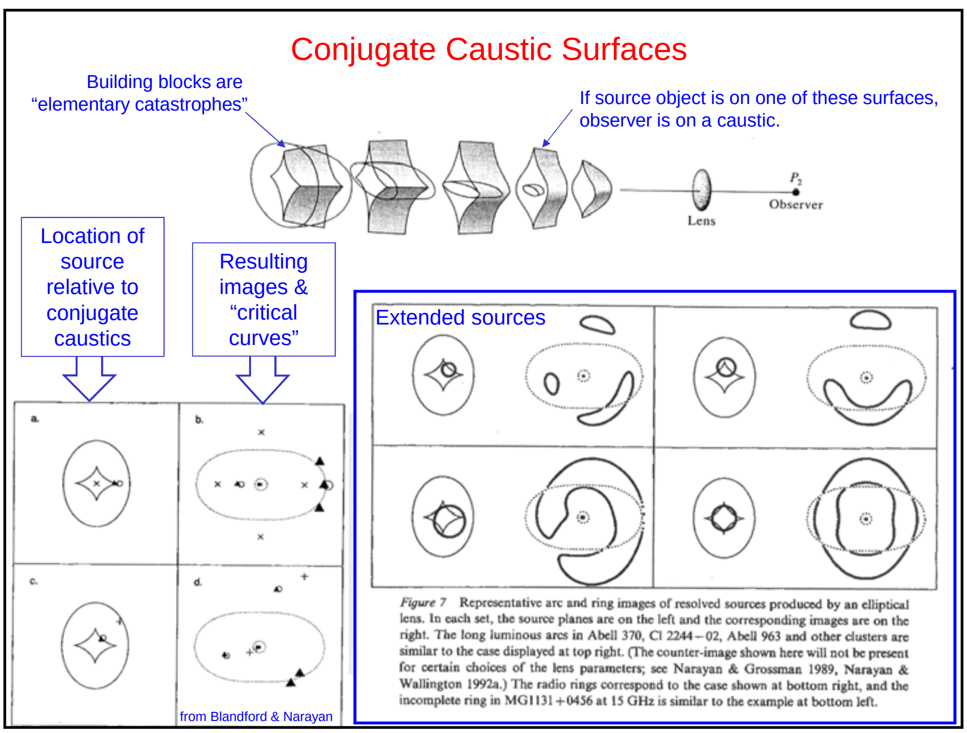


Wave fronts:

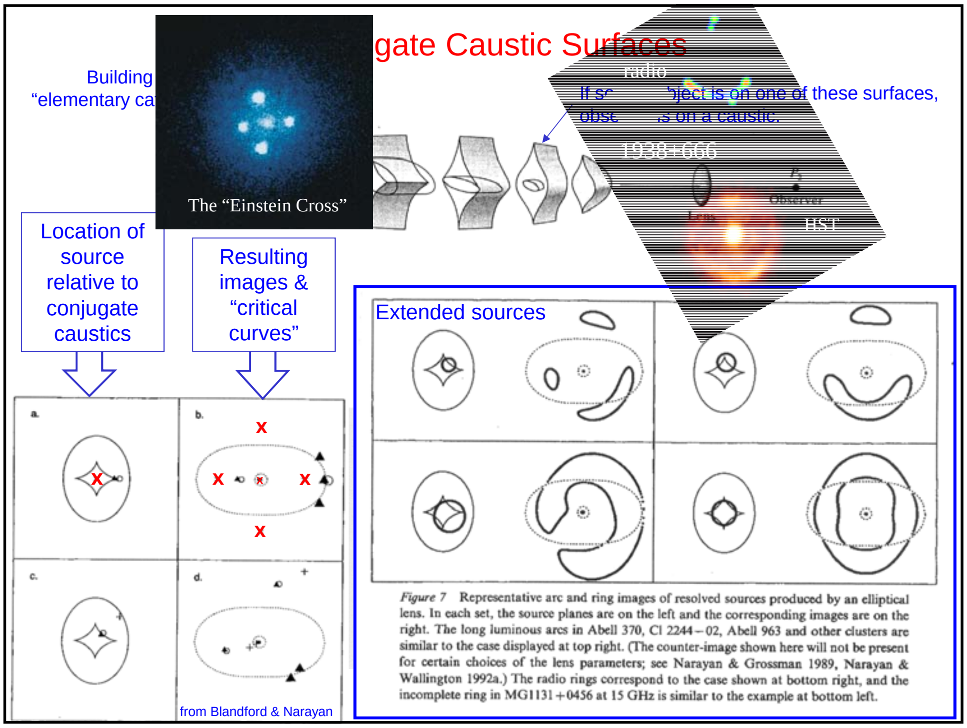




Conjugate Caustic Surfaces
Building blocks are “elementary catastrophes”
If source object is on one of these surfaces, observer is on a caustic.
Lens
Observer
Transparent elliptical lens
Location of source relative to conjugate caustics
Resulting images & “critical curves”
a.
b.
c.
d.
from Blandford & Narayan
Figure 6 Multiple imaging of point sources at fixed redshift by a generic "elliptical lens." The solid lines in the left panels are caustics that separate regions in the source plane corresponding to different image multiplicities (1, 3, and 5 as indicated). The inner caustic, sometimes referred to as the tangential caustic, has four cusps connected by fold lines. The outer radial caustic is a pure fold. The outer dashed lines in the right panels are tangential critical curves and the inner ones are radial critical curves. The symbols show representative source positions and the corresponding image locations. When the source is close to a caustic, some of the images are strongly magnified, indicated by large symbols in the image panels. One of the multiple images usually occurs near the center of the lens and is strongly demagnified if the core radius of the lens is small. Among the "secure" multiple quasars, Q1413+117 and Q2237+031 correspond to the source position × and Q0142−100 to ○ in the upper panels. 0414+053 and Q1115+080 correspond to the triangle and Q0957+561 is midway between ○ and + in the lower panels. The weak central image has not been seen in any of the observed cases.



Conjugate Caustic Surfaces
Building blocks are “elementary catastrophes”
If source object is on one of these surfaces, observer is on a caustic.
Lens
Observer
Location of source relative to conjugate caustics
Resulting images & “critical curves”
Extended sources
a.
b.
c.
d.
from Blandford & Narayan
Figure 7 Representative arc and ring images of resolved sources produced by an elliptical lens. In each set, the source planes are on the left and the corresponding images are on the right. The long luminous arcs in Abell 370, Cl 2244−02, Abell 963 and other clusters are similar to the case displayed at top right. (The counter-image shown here will not be present for certain choices of the lens parameters; see Narayan & Grossman 1989, Narayan & Wallington 1992a.) The radio rings correspond to the case shown at bottom right, and the incomplete ring in MG1131+0456 at 15 GHz is similar to the example at bottom left.



Observations of lensed objects

- Extended background source (e.g. a galaxy)
  - ==> arcs or rings
- Weak lensing: $\theta >> \theta_E$
  - images slightly extended
  - currently being exploited to look for cluster halos, dark galaxies, etc.
- Strong lensing: $\theta < \theta_E$
  - multiple images formed
  - weak central image usually not seen