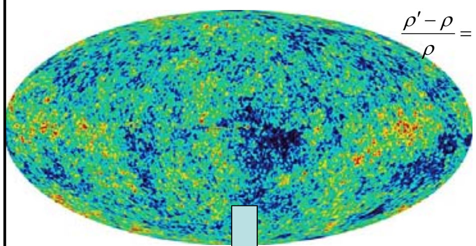
The Growth of Structure

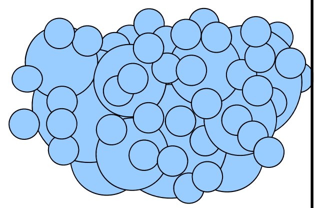
Read [CO 30.2]

WMAP

Density fluctuations at t = 379,000 yr



$$\rho = 1.0000$$
$$\rho' = 1.0001$$
$$\frac{\rho' - \rho}{\rho} = \frac{\delta\rho}{\rho} = 0.0001 = 10^{-4}$$

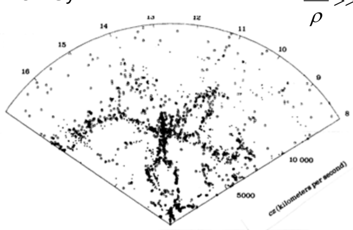


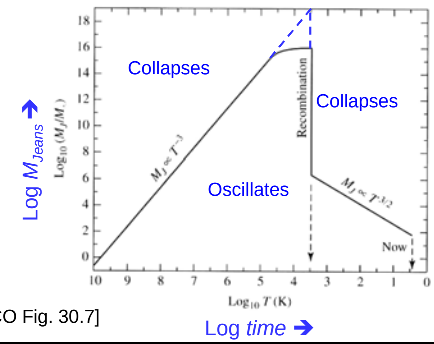
Early U. contained condensations of many different sizes.

Current large-scale structure

$t = t_0$ = 13.7 Gyr

$$\frac{\delta\rho}{\rho} >> 1$$





[CO Fig. 30.7]

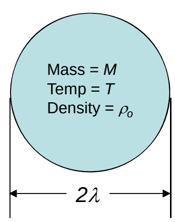
The Simplest Picture of Galaxy Formation and Why It Fails

(chapter title from Longair, "Galaxy Formation")

Will a condensation collapse?

The Jeans criterion:

(see [CO Sect. 12.2 and pg. 1250]



Unstable to collapse if

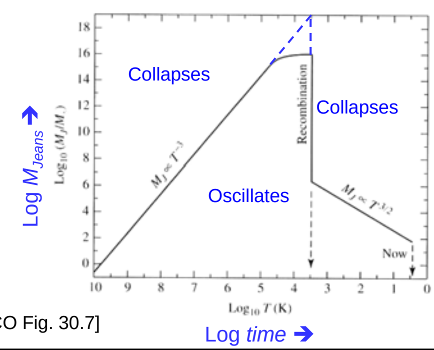
$$2K < -U$$

Pressure < Gravity

$$\frac{3MkT}{\mu m_H} < \frac{3}{5}\frac{GM^2}{\lambda}$$

$$M = \frac{4}{3}\pi\lambda^3\rho_o$$

$$M > \underbrace{\left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_o}\right)^{1/2}}_{\text{Jeans mass } M_J}$$



[CO Fig. 30.7]

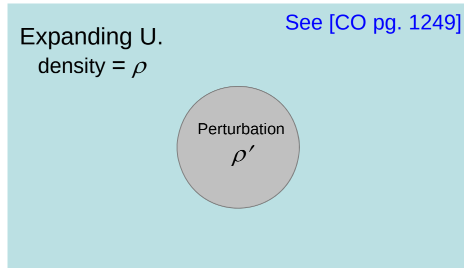
How fast will it collapse?

In a static medium (e.g. star formation):

Perturbation analysis shows

$M < M_J$
$\delta\rho/\rho \propto \exp(-ir/\lambda - i\omega t)$ ➔ Oscillations

$M > M_J$
$\delta\rho/\rho \propto \exp(-ir/\lambda + Kt)$ ➔ Exponential growth

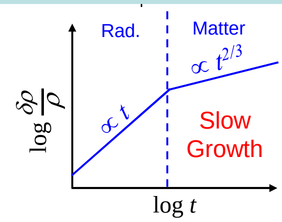


In an expanding medium (e.g. the universe):

Outside the perturbation (flat universe): $$H^2R^2 - \frac{8}{3}\pi G\rho R^2 = 0$$ (Friedman Eqn)

Inside the perturbation (closed mini-universe): $$H^2R^2 - \frac{8}{3}\pi G\rho' R^2 = -kc^2$$

$$\frac{\delta\rho}{\rho} = \frac{\rho' - \rho}{\rho} = \frac{3kc^2}{8\pi G\rho R^2}$$



Radiation era: $\rho = \rho_0 R(t)^{-4}$ $\quad R(t) \propto t^{1/2}$ ➔ $$\frac{\delta\rho}{\rho} = \left(\frac{\delta\rho}{\rho}\right)_i \left(\frac{t}{t_i}\right)$$

Matter era: $\rho = \rho_0 R(t)^{-3}$ $\quad R(t) \propto t^{2/3}$ ➔ $$\frac{\delta\rho}{\rho} = \left(\frac{\delta\rho}{\rho}\right)_i \left(\frac{t}{t_i}\right)^{2/3}$$

The Simplest Picture of Galaxy Formation and Why It Fails

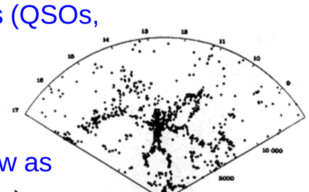
- Cosmic Microwave Background is smooth to a few parts in 10^5

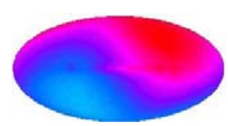
  $\delta\rho/\rho \sim 10^{-4}$

Blue = 0°K
Red = 4°K

- Yet high contrast structures (QSOs, galaxies) by $z \sim 6$.

  $\delta\rho/\rho >> 1$





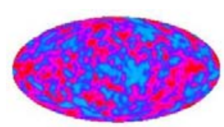
Blue = 2.724°K
Red = 2.732°K
Dipole Anistropy
~ 1 part in 300

- Adiabatic perturbations grow as

  $\delta\rho/\rho \propto t^{2/3} \propto R(t) \propto 1/(1+z)$

- Expect only

$$\left(\frac{\delta\rho}{\rho}\right)_{QSO} = \frac{(1+z)_{CMB}}{(1+z)_{QSO}}\left(\frac{\delta\rho}{\rho}\right)_{CMB} = \frac{1100}{7}\times 10^{-4} = 0.01$$



After removing dipole
Red – blue = 0.0002°K
~ 1 part in 10^4

So where did galaxies and clusters come from?

In an expanding universe, will a cloud collapse?

The Jeans criterion Version 2:

$2K < -U$

Pressure support < gravity

Collapse if $2K < -U$

$$2\left(\frac{1}{2}M_T v_s^2\right) < \frac{3}{5}\frac{GM_T^2}{\lambda}$$

v_s = sound speed

$$v_s^2 < \frac{3}{5}\frac{GM_T}{\lambda} = \frac{3G}{5}\frac{(4/3)\pi\lambda^3\rho_T}{\lambda} = \frac{4}{5}\pi G\lambda^2\rho_T$$

$$\left(\frac{3M_b}{4\pi\rho_b}\right)^{2/3} = \lambda^2 > \frac{5v_s^2}{4\pi G\rho_T}$$

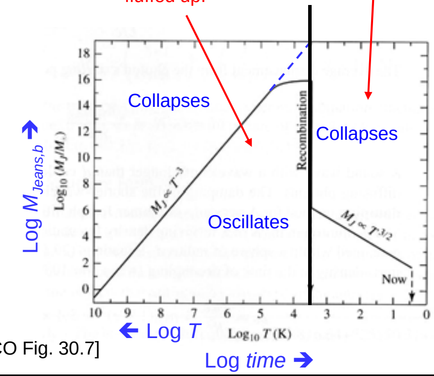
$$M_{J,b} > const.\times\frac{\rho_b v_s^3}{\rho_T^{3/2}}$$ = [CO eq. 30.27]

Radiation era

$$v_s = \frac{c}{\sqrt{3}}$$
$$\rho_b \propto R(t)^{-3} \propto T^3$$
$$\rho_T \propto R(t)^{-4} \propto T^4$$
$$M_{J,b} \propto T^{-3}$$

After decoupling

$$v_s = \sqrt{\frac{5kT}{3\mu m_H}}$$
$$\rho_b \propto T^0$$
$$\rho_T \approx \rho_b \propto T^0$$
$$M_{J,b} \propto T^{3/2}$$



[CO Fig. 30.7]

Q.When do oscillations start?

$2K < -U$

Pressure support < gravity

When

Particle horizon = λ_M

Size scale for mass M

Before decoupling:

- Particle Horizon

  $d_h = 2ct \propto R(t)^2 \propto T^{-2}$ (radiation era)

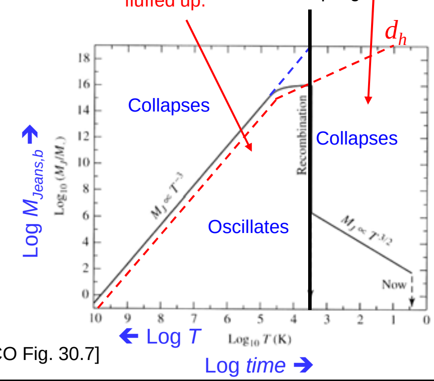
- Proper distance containing mass M

  $\lambda = (M_b/\rho_b)^{1/3} \propto M_b^{1/3}R(t) \propto M_b^{1/3}T^{-1}$

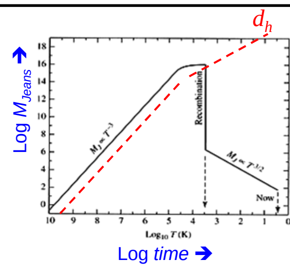
- Mass for which $\lambda = d_h$

  $M_b \propto T^{-3} \propto R^3 \propto t^{3/2}$ (radiation era)

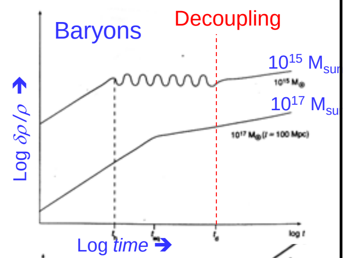
  $M_b \propto T^{-3/2}$ (matter era)

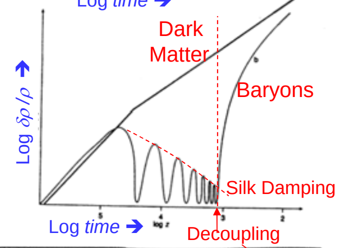


[CO Fig. 30.7]

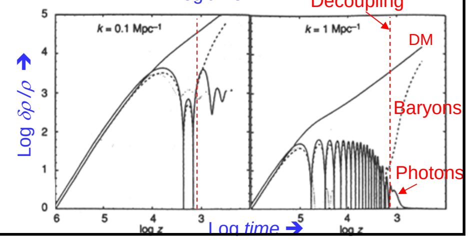


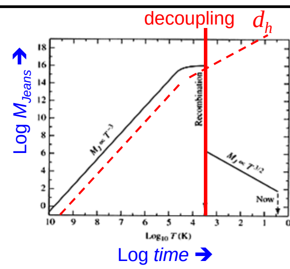
The Rest of the Story:

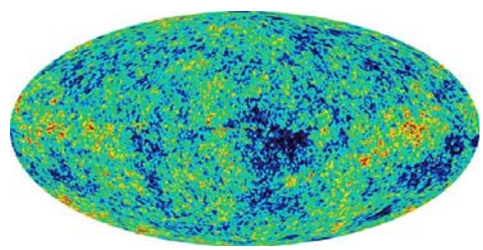




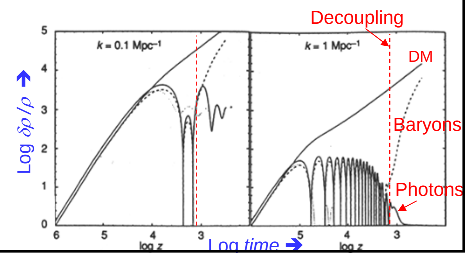
- Mass for which $\lambda = d_h$
  - $M \propto T^{-3}$ (rad. era)
  - $M \propto T^{-3/2}$ (matter era)
- At $t_{decoupling}$ this mass was ~ 10^{16} $M_\odot$
  - $M > 10^{16}$ $M_\odot$ ➔ continued growth
  - $M < 10^{16}$ $M_\odot$ ➔ oscillations once mass scale comes into particle horizon.
- But Dark Matter not subject to all this.
  - Does not feel radiation pressure.
  - Just collapses away…
- Baryons fall into Dark Matter potential wells as soon as decoupling removes photon pressure support.



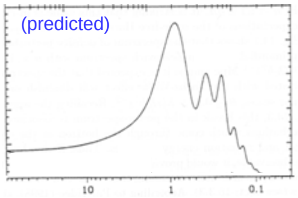




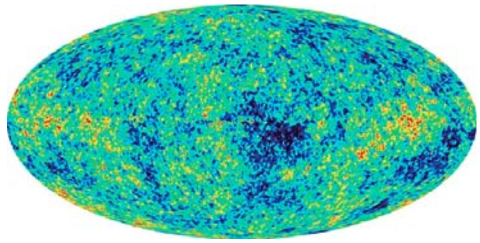
CMB Fluctuations = snapshot of oscillations at $t_{decoupling}$



Power = Average $(\delta\rho/\rho)^2$ of clouds of given size scale θ



← θ (deg)

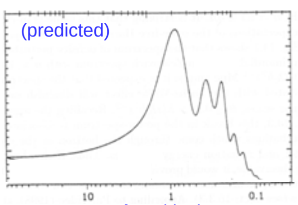


Fourier analyze WMAP image:

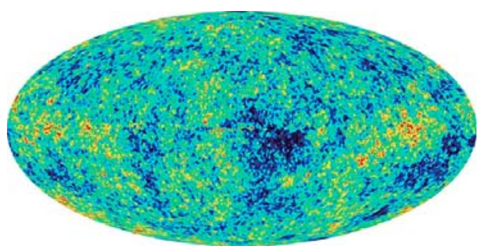
- Measures "*Power*" for each size scale θ.
- = *Power* for each mass scale *M.*

What is measured?

Power = Average $(\delta\rho/\rho)^2$ of clouds of given size scale θ



← θ (deg)



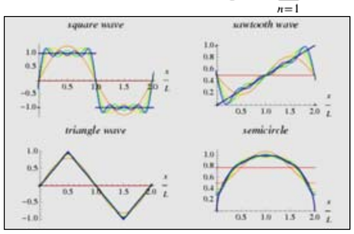
Fourier analyze WMAP image:

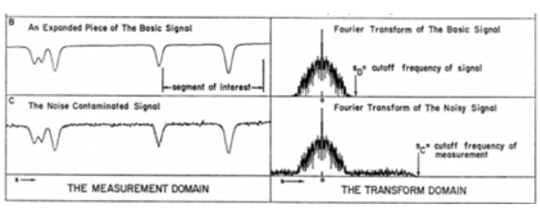
- Measures "*Power*" for each size scale θ.
- = *Power* for each mass scale *M.*

The Fourier Transform

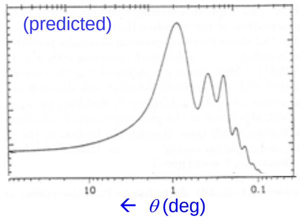
(sort of)

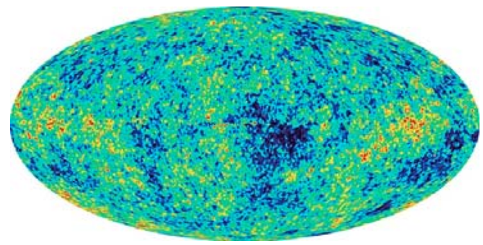
$$f(x) = \frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos(n x)$$ → *Power = a_n^2 (n)*





Power = Average $(\delta\rho/\rho)^2$ of clouds of given size scale θ

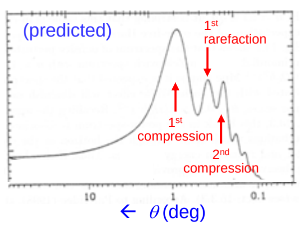


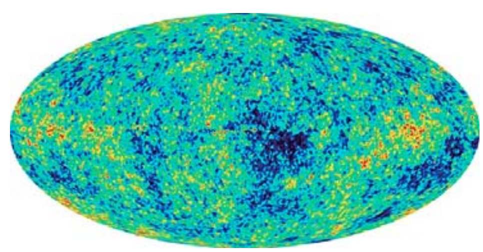


Fourier analyze WMAP image:

- Measures "*Power*" for each size scale θ.
- = *Power* for each mass scale *M.*
- *But why more power for some mass scales than others?*

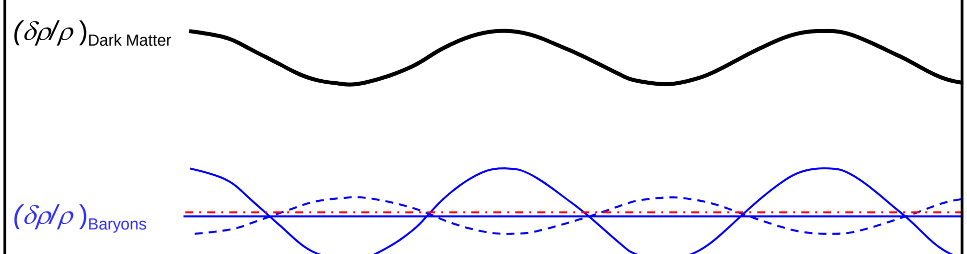
Power = Average $(\delta\rho/\rho)^2$ of clouds of given size scale θ



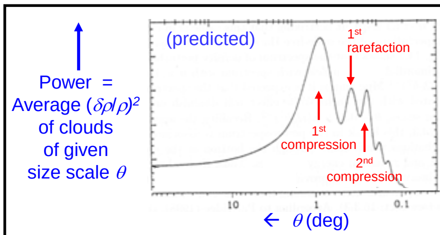


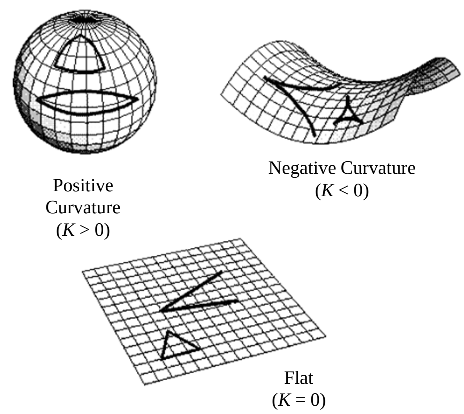
Fourier analyze WMAP image:

- Measures "*Power*" for each size scale θ.
- = *Power* for each mass scale *M.*
- *But why more power for some mass scales than others?*
- All blobs of same mass *M* oscillate synchronously.
- Peaks are for mass scales that are either fully compressed or fully rarified.



Baryons: Shorter spatial wavelengths oscillate with higher time frequency





First peak:

Size of "acoustic horizon"

$r = v_s\,(t_{\text{Decoupling}} - t_{Horizon}) = c\sqrt{3}\ t$

= linear size of perturbation

$\theta = r/\chi(d)$

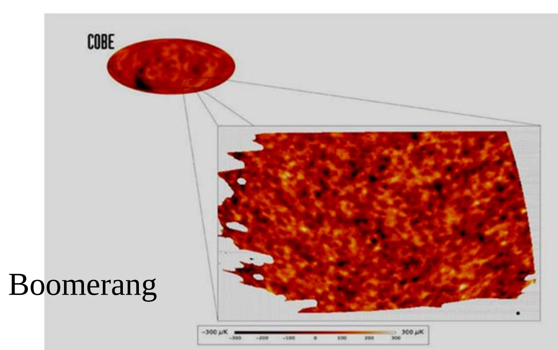
$\chi = sin(d),\ d,\ sinh(d)$

$l_{\text{peak}} = 220/\Omega_{\text{tot}}^{1/2}$ (l = multipole)

Measured $l_{\text{peak}} \rightarrow \Omega_{\text{tot}} = 1.02 \pm .02$

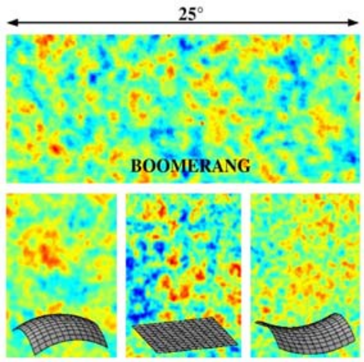
Boomerang balloon flight (1999)

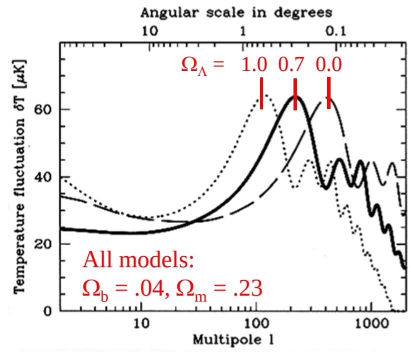
Mapped Cosmic Background Radiation with far higher angular resolution than previously available.



Launch near Mt. Erebus in Antarctica

Position of 1st peak measures curvature





First peak:

Size of "acoustic horizon"

$r = v_s\,(t_{Decoupling} - t_{Horizon}) = c\sqrt{3}\;t$

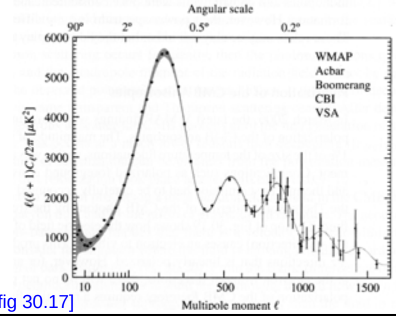
= linear size of perturbation

$\theta = r/\chi(d)$

$\chi = sin(d),\ d,\ sinh(d)$

$l_{peak} = 220/\Omega_{tot}^{1/2}$ (l = multipole)

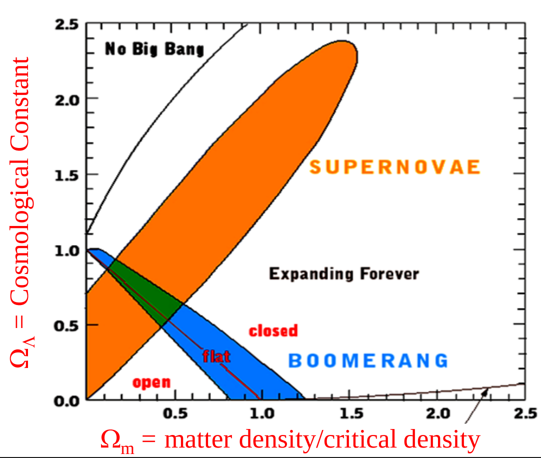
Measured l_{peak} ➔ $\Omega_{tot} = 1.02 \pm .02$



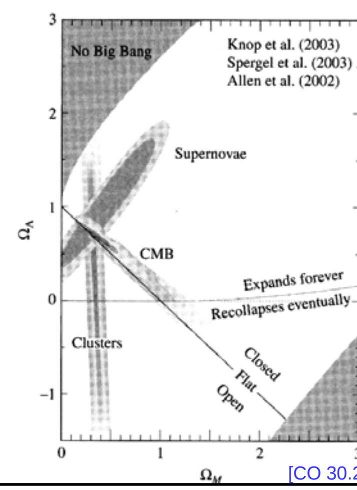
[CO fig 30.17]

The "Concordance" Cosmology (or ΛCDM)

- Type Ia Supernovae as "standard candles"
  - ➔ accelerating expansion
  - ➔ $q_o = \Omega_m/2 - \Omega_\Lambda$
- CMB anisotropy ➔ $\Omega_{total} = \Omega_m + \Omega_\Lambda$
- Can solve for Ω_m , Ω_Λ



Another independent measure:
Rate of galaxy cluster evolution



[CO 30.22]