AST 308 Homework Set 5
Due Tuesday Nov 1

Question 1. The R-W metric for the case $k > 0$ actually describes the equivalent of a 3D "spherical" surface embedded in a 4D space. It is easy to show this if you know the little tricks to use. The following steps will guide you through the problem. Write your answers on a separate sheet of paper, showing each intermediate result.

In Cartesian coordinates, the equation of a 1D circular "surface" of radius $\mathscr{R}$ embedded in a 2D space is: $x^2 + y^2 = \mathscr{R}^2$. The line element (or metric) in Cartesian coordinates in that 2D space is $dl^2 = dx^2 + dy^2$.

(a) Write down the equation of a 2D spherical surface of radius $\mathscr{R}$ embedded in a 3D space (this is just what we usually think of as a sphere), and of the line element, both in Cartesian coordinates in that 3D space.

(b) Write down the equation of a 3D "spherical surface" of radius $\mathscr{R}$ embedded in a 4D space, and of the line element, both in Cartesian coordinates in that 4D space. Call your dimensions x, y, z and w:

Just in case: Here is an intermediate answer, upside down and mirror-imaged.

$$dw = -\frac{r\,dr}{w} = -\frac{r\,dr}{\left(\mathscr{R}^2 - r^2\right)^{\frac{1}{2}}}$$

(c) In your equation for the 4D sphere, make the replacement $r^2 = x^2 + y^2 + z^2$. Now solve for w^2 in terms of $\mathscr{R}^2$ and r^2, and then for dw = function ($\mathscr{R}, r, dr$).

(d) In your 4D line element from step (b), replace dw with the results from step (c).

(e). Also replace the 3D part of the line element, $dx^2 + dy^2 + dz^2$ with its 3D equivalent in spherical coordinates r, θ, ϕ: *Hint:* if you don't know what these are, compare the first two equations on slide 7 of the Oct. 20 lecture notes (the slide titled "Some Metrics").

(f) Collect together the terms in dr^2 and use a little algebra. You should come up with a line element for which the coefficient of the dr^2 term is $1/(1 - r^2/\mathscr{R}^2)$.

$$dl^2 = \left(\frac{dr^2}{1 - r^2/\mathscr{R}^2}\right) + (r\,d\theta)^2 + (r\sin\theta\, d\phi)^2$$

(g) Finally, make the substitutions $k^{½}\varpi = r/\mathscr{R}$, $k^{½}d\varpi = dr/\mathscr{R}$, and $R(t) = k^{½}\mathscr{R}$. Here $R(t)$ is the scale factor and k is the constant in the RW metric as used by [CO]. Write down your final answer. How does it compare with the space-like part of the RW metric for $k = +1$?

(h) So *what in the world* just happened? Write down 1-sentence answers to the following:

- What does $\mathscr{R}$ represent physically?
- What does r represent physically?
- What does k represent in terms of this geometry?
- What new constraint allowed you to eliminate dw from the 4D line element (*i.e* from the 4D metric)? You replaced the w dimension with something, somehow, somewhere.

Question 2. Solve the Friedmann equation for the case in which the density terms ρ_{matter} and $\rho_{relativistic}$ are negligible compared to the cosmological constant Λ, in a flat universe. Show your steps.

Question 3. Suppose that we were still back in the radiation-dominated universe (the "radiation era") and a photon was just arriving from the particle horizon at that time. Call the time t_x. Calculate the path that the photon would have taken to arrive at $\varpi = 0$ at time $t = t_x$, in terms of *the proper distance* $d_p(t)$ as a function of time. What is its maximum proper distance? Show your work.